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**SCOPE OF THE JOURNAL**

It is the endeavour of the University of Pristina to acquaint the scientific world with its achievements. We would like to affirm the intellectual potential of this region as well as natural resources of the Balkans. We would like to put forward our attitude of principle that science is universal and we invite all scientist to cooperate wherever heir scope of research may be. We are convinced that we shall contribute to the victory of science over barriers of all kinds erected throughout the Balkans.

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## IN MEMORIAM

**Dr ĐORĐIJE ĐOROVIĆ**  
**27.12.1936. - 05.06.1997.**



Iznenada, 05.06.1997. godine preminuo je dr Đorđije Đorović, profesor Poljoprivrednog fakulteta u Prištini. Dr Đorđije spada u istaknute ličnosti, koji se bavio stručnim i naučnim radom iz oblasti biotehnoških nauka.

Rođen je 27.12.1936. godine u Zahaću kod Peći. Višu realnu gimnaziju završio je u Peći, a Šumarski fakultet u Skoplju. Magistrirao je i doktorirao na Šumarskom fakultetu Univerziteta u Beogradu. Prvi je doktor šumarskih nauka na Kosovu i Metohiji. U toku svog stručnog i naučnog rada objavio je znatan broj radova iz oblasti šumarstva i pčelarstva. Svoju naučnu i stručnu delatnost usmerio je, prvenstveno, na rešavanju problema značajnih za šumarsku struku i problema iz oblasti pčelarstva. Objavio je naučnu monografiju "Biocenološki kompleks gusenica hrasta" (1992) i knjigu "Pčelarstvo" (Udžbenik) za studente Poljoprivrednog fakulteta, koju mogu da koriste i drugi stručnjaci i radnici koji se bave problemima iz oblasti pčelarske proizvodnje. Pored naučne i stručne delatnosti dr

Đorđije je pisao i poeziju, inspirisanu patriotskim osećanjima prema svom narodu i otadžbini. Posebna inspiracija mu je istorija i kultura kraja u kojem je ponikao. Poeziju piše od 1954 godine. Svoje stihove objavljuje u listovima i književnim časopisima: Jedinstvo, Prosvetni pregled, Novi svet, Omladina, Vojnik, Stremljenja i dr. Objavio je Zbirku pesama "Svi moji zaboravi" (1992) i "Potvrda Vječnosti" (1995). Pripremio je i objavio antologiju "Hvostanski krug" u kojoj su zastupljeni pesnici Peći i okoline od doba Nemanjića i prije, do današnjih dana. Radio je na još jednom sličnom delu, ali nije uspeo da ga završi. U njegovoj radnoj biografiji čitamo da je bio profesor srednje poljoprivredne i šumarske škole u Peći, da je dve i po decenije radio u Institutu za šumarstvo u Peći i da je radni vek iznenada završio kao nastavnik "Pčelarstva" na Poljoprivrednom fakultetu u Prištini. Životni put našeg profesora dr Đorđija bio je pun sadržaja, ali i nemira izazvanih okruženjem i položajem njegovog naroda u savremenom okruženju. Ovih nekoliko rečenica neka budu sećanje na tog divnog čoveka, druga i saradnika, koji nas je lepim i dobrim zadužio.

Neka je hvala i slava profesoru Đorđiju Đoroviću.

*Prof. dr Božo Radović*



# Content of Total Microflora, Fungi and Actinomyces in some Kosovo and Metohia Soils

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## ABSTRACT

**We investigated the base soil types of Kosovo and Metohia: Vertisol, Pseudogley, and Dystric Cambisol. The investigated soil types have different physical and chemical properties and, according to the conditions, different con-**

**tent of the investigated microorganism groups can be met. The highest total microflora content was observed in Vertisol, then in Pseudogley, and the lowest one in Dystric Cambisol.**

**Key words:** Microorganism, soil, Vertisol, Pseudogley, Dystric Cambisol.

## INTRODUCTION

Soil microorganisms, by their activity, take part in soil genesis and development, as well as in creation of its effective fertility. Microorganisms transform soil fresh organic matter to humus, so they represent an important factor of plant nutrition. Soil biogenity is an important parameter of its potential and effective fertility. This property mainly depends of soil type (Sarić, 1972; Mišustin, 1984). Proper understanding of soil processes is possible only on the basis of soil microorganism communities studying in every specific environment, including factors that influence on some representants of soil microorganisms and their physical and biochemical activities. The found values and character of soil microorganisms time and space variations are of the great importance for estimation of microorganisms amount, studying of soil microbiological processes dynamics, and knowing of general rules of soil microbial populations function (Zvjagincev, et al., 1976). Investigation of soil microorganisms in their natural environment is necessary for explaining of ecological relations, as well as for understanding of microorganisms number regulation mechanisms (Zvjagincev, et al., 1976).

## MATERIAL AND METHODS

The investigation presented in this paper has been carried out on different soil types as follows: the Vertisol location Laplje Selo (Priština), the Pseudogley location Vitimirica (Peć), and the Dystric Cambisol location Tankosić (Uroševac). The investigation involved so the cultivated soils as the uncultivated (virgin) ones. Soil sampling for the mentioned study was done in autumn 1994, winter 1994/95, as well as in spring and summer 1995. For every season sampling we opened new profiles. Samples were taken in the open profile, from different depths as follows: 0-25 cm,

25-50 cm, 50-75 cm, and 75-100 cm. In the laboratory, from the collected samples, were made dilutions, and the dilution of 10<sup>-4</sup> was used for inoculation. Inoculation of culture media was carried out by 0.5 ml of inoculum, and all of that was done in three repetitions. We analysed in the studied samples the following microorganisms: -The total microorganisms number on the soil agar; -Fungi on the Chapek's agar; -Actinomyces on the synthetic agar with sacharose. For preparing of soil agar soil from sampling profile was used. Number of microorganisms is expressed per a gram of absolute dry soil.

## RESULTS AND DISCUSSION

Total microflora is closely related with fresh organic matter income to soil, even during greater drought, which is confirmed by many reports (Vandecaveye and Katznelson, 1938; Eastwood et al., 1950). A high content of total microflora is characteristic for Vertisol, which is expectable because this soil has a deep humus horizon and favourable physical and chemical properties (Kiković, 1989). Plant cover and its activity have a moderate effect on microflora (Alexander, 1961), and the investigated soils were covered by grass vegetation and crops in monoculture. Relation between microorganisms and some vegetation types development obviously exist, and is specially expressed in droughty part of year (Todorović et al., 1976). Root of higher plants, or its secretions, can inhibit some and stimulate other groups of microorganisms (Raičević, 1994).

A higher total microflora content was observed in cultivated Vertisol (Tab. 1). We can say that the total microflora season dynamism of cultivated and uncultivated Vertisol were similar. A higher total microflora content observed in cultivated Vertisol was obviously

Tab. 1. Season dynamism of Vertisol total microflora (Laplje Selo)  
(in milions per a gram of absolute dry soil)

UNCULTIVATED SOIL					CULTIVATED SOIL			
Depth	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer
0-25	6.258	6.575	17.876	4.720	7.829	6.932	14.269	5.756
25-50	7.178	5.840	9.348	3.651	6.813	5.108	13.782	1.064
50-75	2.348	2.589	5.636	1.086	4.309	4.229	8.831	0.987
75-100	0.719	1.131	2.937	1.493	2.219	2.219	4.870	0.338

Tab. 2. Season dynamism of Pseudogley total microflora (Vitimirica)  
(in milions per a gram of absolute dry soil)

UNCULTIVATED SOIL					CULTIVATED SOIL			
Depth	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer
0-25	5.761	4.469	6.329	6.511	4.690	2.234	4.697	1.836
25-50	1.446	2.000	5.195	1.511	0.777	2.602	4.168	0.549
50-75	0.659	1.123	1.716	1.149	0.561	1.280	2.786	0.433
75-100	0.273	0.326	1.518	1.138	0.349	1.092	2.620	0.247

Tab. 3 Season dynamism of Dystric Cambisol total microflora (Tankosić)  
(in milions per a gram of absolute dry soil)

UNCULTIVATED SOIL					CULTIVATED SOIL			
Depth	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer
0-25	1.123	0.376	3.161	1.926	1.314	1.202	3.230	0.132
25-50	0.045	0.095	0.833	0.045	0.348	0.385	1.619	0.045
50-75	0.038	0.333	0.494	0.011	0.315	0.112	0.755	0.034
75-100	0.038	0.123	0.070	0.000	0.236	0.088	0.694	0.034

Tab. 4. Season dynamism of Vertisol fungi (Laplje Selo)  
(in thousands per a gram of absolute dry soil)

UNCULTIVATED SOIL					CULTIVATED SOIL			
Depth	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer
0-25	47.058	100.000	67.415	10.723	97.561	121.623	141.026	12.820
25-50	23.809	48.000	11.236	11.627	95.238	121.623	101.897	13.158
50-75	15.511	13.399	0.000	0.000	58.139	82.192	25.974	11.126
75-100	0.000	0.000	0.000	0.000	37.037	81.081	12.987	11.115

Tab. 5. Season dynamism of Pseudogley fungi (Vitimirica)  
(in thousands per a gram of absolute dry soil)

UNCULTIVATED SOIL					CULTIVATED SOIL			
Depth	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer
0-25	86.956	98.765	141.176	56.818	130.952	197.531	393.258	260.869
25-50	32.609	56.818	195.403	431.818	55.555	325.301	134.831	175.824
50-75	27.912	24.691	147.727	80.459	33.708	97.561	67.416	11.235
75-100	23.809	21.053	11.765	0.000	30.928	22.988	5.896	0.000

Tab. 6. Season dynamism of Dystric Cambisol fungi (Tankosić)  
(in thousands per a gram of absolute dry soil)

UNCULTIVATED SOIL					CULTIVATED SOIL			
Depth	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer
0-25	33.708	70.588	43.011	274.684	93.023	392.857	114.942	10.989
25-50	11.236	35.714	11.905	11.363	22.472	108.433	59.524	0.000
50-75	22.472	23.809	0.000	11.111	11.236	91.764	23.256	0.000
75-100	0.000	56.180	0.000	11.111	0.000	64.086	35.294	0.000

Tab. 7. Season dynamism of Vertisol actinomyces (Laplje Selo)  
(in thousands per a gram of absolute dry soil)

UNCULTIVATED SOIL					CULTIVATED SOIL			
Depth	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer
0-25	129.412	125.000	33.708	10.752	146.341	40.540	38.461	0.000
25-50	59.523	40.000	0.000	0.000	186.046	324.324	102.564	11.364
50-75	0.000	13.699	0.000	0.000	123.456	40.540	25.974	0.000
75-100	0.000	0.000	0.000	0.000	59.524	0.000	0.000	0.000

Tab. 8. Season dynamism of Pseudogley actinomyces (Vitimirica)  
(in thousands per a gram of absolute dry soil)

UNCULTIVATED SOIL					CULTIVATED SOIL			
Depth	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer
0-25	130.434	506.173	117.647	0.000	178.571	419.753	44.943	0.000
25-50	10.869	79.545	68.965	11.364	51.546	24.096	224.719	10.989
50-75	0.000	24.691	34.090	0.000	0.000	36.585	0.000	11.111
75-100	0.000	10.526	11.764	0.000	0.000	34.482	0.000	0.000

Tab. 9. Season dynamism of Dystric Cambisol actinomyces (Tankosić)  
(in thousands per a gram of absolute dry soil)

UNCULTIVATED SOIL					CULTIVATED SOIL			
Depth	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer
0-25	0.000	11.764	21.505	0.000	34.884	35.714	34.484	0.000
25-50	0.000	11.905	0.000	0.000	11.236	0.000	0.000	22.727
50-75	0.000	0.000	0.000	0.000	11.236	0.000	0.000	22.727
75-100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

induced by activity of men. A proper soil management and adequate agrotechnique measures might cause an improvement of soil biogenity, and that statement is confirmed by the investigations on the influence of some agrotechnique, amelioration, and fertilization measures on soil microflora (Tešić et al., 1976). Content of Pseudogley's total microflora was lower in comparison with Vertisol, which was expectable because this soil had worse mechanical, physical and chemical properties. Pseudogley had an acid pH value,

so the number of fungi in the total microflora was higher regarding Vertisol which had a neutral pH.

We can state, comparing biogenity of cultivated Pseudogley and uncultivated one, that uncultivated Pseudogley had a higher content of total microflora, which was not expected. A lower content of total microflora in cultivated Pseudogley was caused by many factors, such as reduction processes existing in this soil, acid pH value, monoculture, use of chemical means for crop protection, etc. Soil acidity in surface layer was higher in cultivated than in uncultivated

Pseudogley. These differences were caused by activities of men, such were deep cultivation, liming, and ploughing, but lime went to deeper layers which induced high acidity just under surface and acidity decrease by depth.

The lowest content of total microflora among the investigated soil types had Dystric Cambisol (Tab. 3), which also had the worst physical and chemical traits. A higher total microflora content was observed in cultivated Dystric Cambisol, which could be explained by a higher income of fresh organic matter, better aeration, higher nitrogen, phosphorus, and potassium content, and this is confirmed by some investigators (Sarić, 1983). Percental part of fungi in total microflora was the largest in Dystric Cambisol, which means fungi played a significant role in transformation of fresh organic matter. Fungi were well-represented in Vertisol, despite they were acidophile representants, but they also could develop in neutral environment such this one was. Fungi development can be influenced by fresh organic matter incoming to soil. Cultivated Vertisol had much higher content of fungi than the uncultivated one. Similarly to the total microflora, we observed the highest fungi content in the cultivated layer, which was natural because the greatest income of fresh organic matter, as well as the best conditions for aerobic microorganisms development, were in this layer. Fungi arrangement (Tab. 4) in the profile was determined by a row of ecological factors, as first by oxygen content and arrangement of fresh organic matter in the profile (Todorović et al., 1976; 1987).

The investigated Pseudogley had a significantly higher content of fungi in regard to Vertisol (Tab. 5). Acid pH value and relatively large income of fresh organic matter influenced the high content of this microflora. Some reports consider that fungi are in vegetative stage in soils with a higher water content, but spores are prevalent in dry soils, which enlarges their number on selective culture media (Aristovskaya, 1965).

(Tab. 6). Content of fungi was higher in cultivated soil, and their role in transformation of fresh organic matter was much more important comparing with Pseudogley.

Actinomyces were very well-represented in cultivated Vertisol (Tab. 7), but in uncultivated Vertisol their number was much lower. Seasonal variations of actinomyces content was not in accordance with the previous reports. Maximal number of these microorganisms was expected in summer period, because actinomyces were involved in transformation of hardly soluble substances (Mišustin, 1963). We also observed a high number of actinomyces in the investigated Pseudogley, despite this soil had an acid pH value. Actinomyces were placed mainly in surface layer where content of humus matters was higher.

According to some previous studies (Mišustin and Emcev, 1974; Mišustin, 1975) amount of actinomy-

ces increases in southern soils, with favourable pH value, which is caused by a great activity of microbiological processes in those soils.

Content of actinomyces, as alkalophile microflora, was low in Dystric Cambisol (Tab. 9), and reason for that was a very acid pH value. Significantly higher content of this microflora was observed in cultivated Dystric Cambisol, which is obviously related with a higher income of fresh organic matter to soil, so the amount of hardly soluble matters was also higher.

## CONCLUSION

On the basis on the results obtained by investigating season dynamism of total microflora, fungi and actinomyces content in Vertisol, Pseudogley, and Dystric Cambisol, we can point out to the following: -Vertisol is the soil having the best mechanical, physical, and chemical properties, so it was rich by microflora. -Pseudogley is a soil with a worse mechanical structure and an acid pH value, so it had much lower content of the total microflora, but higher percent of fungi in total microflora content. -The investigated Dystric Cambisol had the worst mechanical structure, an acid pH value and the lowest microflora content, as well as significantly higher percent of fungi in total microflora content.

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## REZIME

ZASTUPLJENOST UKUPNE MIKROFLORE, GLJIVA I AKTINOMICETA U NEKIM ZEMLJIŠTIMA KOSOVA I METOHİJE

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Ispitivani su osnovni tipovi zemljišta Kosova i Metohije: smonica, pseudoglej i distrični kambisol. Proučavani zemljišni tipovi su različitih fizičko-hemijskih svojstava pa, adekvatno uslovima, srećemo različitu zastupljenost ispitivanih grupa mikroorganizama. Najveću zastupljenost mikroflore srećemo kod smonice, zatim pseudogleja, a najmanju kod distričnog kambisola.

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# **Influence of Heavy Metals on *Azotobacter* sp. in the Soils Near Kosovska Mitrovica**

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## **ABSTRACT**

**Increased concentrations of heavy metals, as is the case concerning area of Kosovska Mitrovica where are located large industrial capacities, have an unfavourable effect on life communities. The investigated locations near the emission source had significantly higher con-**

**centrations of zinc, lead, nickel, and cadmium, in comparison with the lawfully allowed ones. Increased concentrations of the investigated heavy metals had an unfavourable effect on number and enzyme activity of the studied physiological microorganism groups.**

**Key words:** Soil, Vertisol, Alluvium, heavy metals, *Azotobacter* sp.

## **INTRODUCTION**

Ecological problems existing in the modern world are obviously caused by activities of men. Technological and industrial development, exploitation of various mineral materials and other natural resources, are of a special importance for the modern society and its future. Various waste gases and aerosols of industrial origin influence different intensity of soil contamination, and soil becomes less and less suitable for growing of agricultural and forest crops. On the present level of social, technological and industrial development of our country, there is a special interest for local and regional pollution. Industrial centres, like northern part of Kosovo and Metohia i.e. region of Kosovska Mitrovica in which are located chemical industry, mines, a melting house, lead and zinc production etc., have a specially high level of potential danger for the environment. It is established that heavy metals, depending of concentration and soil physical and chemical properties, in a larger or smaller extent, influence on number and activity of soil microorganisms. Non-symbiotic microflora, using suitable soil energy sources, bounds a significant amount of air nitrogen, mainly as ammonium compounds, leaving it to soil where it can be used by the other microorganisms and higher plants. This is undoubtedly a significant type of microbiological nitrogen fixation (Raičević, 1994). In fact, *Azotobacterium* in an indicator of some soil's culture state. Nitrogen-fixing microorganisms are sensitive to high doses of nitrogen fertilizers, and such nitrogen rates decrease their number in soil. It is stated in many reports that number of nitrogen-fixing bacteria is characteristic for every soil type (Paul and Newton, 1961; Klevenkaja, et al., 1970; Linge, et al., 1971). This investigation have been aimed to establish status of soil pollution and its influence on free aerobe nitrogen-fixing bacteria, at some locations placed on the different distance from the source of harmful gases and solid waste matters, as well as on different soil types (*Azotobacter* sp.).

## **MATERIAL AND METHODS**

The investigation presented in this paper has been carried out on different soil types as follows: the Alluvium locations Malo Rudare and Leposavić, and the Vertisol locations Šupkovac, Svinjare, and Trnavce. The investigation involved so the cultivated soils as the uncultivated (virgin) ones. Soil sampling for the study was done from 1993-1995. Samples were taken in the open profile, from different depths as follows: 0-10 cm, 10-20 cm, and 20-30 cm. Content of free aerobe nitrogen-fixing bacteria is determined on silica gel with standard nitrogenless solution and with manit as the only carbon source. Growing bases were sowed by putting of 25 soil granules in every Petrie's dish. Incubation was done on 24°C. Counting of the fertile granules percent was done 15 and 30 days after incubation. Soil samples for determination of the investigated heavy metals content (Cu, Co, Zn, Pb, Ni, Cd) were taken from the same profiles from that were taken samples for microbiological studying.

## **RESULTS AND DISCUSSION**

Various factors influence content of heavy metals in soil, and the most important factors are soil type and its physical and chemical properties. Heavy metals come to a soil from basic substratum during soil genesis, and by pollution. The investigated locations around Kosovska Mitrovica had a larger or smaller content of heavy metals depending of the emission source vicinity. The highest exceedings of the lawfully allowed heavy metals content were observed in the locations Malo Rudare and Šupkovac that were the nearest to the pollution source. The location Svinjare was on a higher distance from the pollution source so we observed slightly lower heavy metals content. In the Alluvium control location Leposavić lead and nickel content were higher than the lawfully allowed

Table 1. The total elements content in soils around Kosovska Mitrovica

Depth	Cu	Co	Zn	Pb	Ni	Cd
Malo Rudare						
0 - 10	9.05	3.10	127.50	263.50	11.00	1.65
10 - 20	10.25	3.10	144.50	310.00	11.00	1.40
20 - 30	9.55	3.55	96.50	292.00	10.50	0.95
Leposavić						
0 - 10	4.45	4.90	12.00	28.60	12.10	0.07
10 - 20	4.10	4.85	9.25	12.00	14.15	0.20
20 - 30	6.50	4.35	8.45	9.25	13.85	0.15
Šupkovac						
0 - 10	3.25	3.75	236.10	104.85	6.30	1.80
10 - 20	4.25	2.70	201.60	82.10	5.40	1.80
20 - 30	3.75	3.55	182.35	83.95	6.80	0.95
Svinjare						
0 - 10	2.60	2.85	96.60	115.85	10.75	1.05
10 - 20	2.45	2.95	8.10	111.70	9.60	0.30
20 - 30	2.70	1.50	9.50	112.70	8.85	0.30
Trnavce						
0 - 10	2.40	3.55	8.70	5.50	9.85	0.29
10 - 20	3.00	3.50	9.65	3.65	9.90	0.25
20 - 30	3.50	4.10	9.66	3.90	9.60	0.15

Table 2. Seasonal dynamism of *Azotobacter* sp. in Malo Rudare (percent of fertile soil granules)

UNCULTIVATED SOIL					CULTIVATED SOIL			
Depth	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring
0-10	0	0	7	0	7	3	3	53
10-20	0	0	1	5	3	8	8	53
20-30	1	0	19	0	0	0	0	7

Table 3. Seasonal dynamism of *Azotobacter* sp. in Leposavić (percent of fertile soil granules)

UNCULTIVATED SOIL					CULTIVATED SOIL			
Depth	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring
0-10	3	0	0	11	0	7	7	5
10-20	1	0	4	0	16	7	7	39
20-30	0	0	4	0	7	0	1	11

Table 4. Seasonal dynamism of *Azotobacter* sp. in Svinjare (percent of fertile soil granules)

UNCULTIVATED SOIL					CULTIVATED SOIL			
Depth	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring
0-10	0	0	0	0	0	0	0	7
10-20	0	0	0	0	1	0	8	1
20-30	0	0	0	0	0	0	1	0

Table 5. Seasonal dynamism of *Azotobacter* sp. in Trnavce (percent of fertile soil granules)

UNCULTIVATED SOIL					CULTIVATED SOIL			
Depth	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring
0-10	0	0	0	5	0	0	0	0
10-20	12	0	0	16	0	0	0	0
20-30	0	0	9	3	0	0	0	0

one, and in the control Vertisol location Trnavce all the heavy metals concentrations were within lawfully allowed levels. Content of free aerobe nitrogen-fixing bacteria (*Azotobacter* sp.) was low in the uncultivated Alluvium of Malo Rudare location, which was near to the pollution source and had significantly higher heavy metals content (especially Zn, Pb, and Cd) in regard to the lawfully allowed one. However, there was a higher content of *Azotobacter* in the cultivated Alluvium of the same location, which was expectable because this nitrogen fixation type was characteristic for cultivated and aerated soils. Free aerobe nitrogen-fixing bacteria leave to soil about 80% of the assimilated nitrogen as amoniac, hidroxyamine, amino acids, peptides, growth matters, and the other nitrogen derivatives. These secretions can be used by plants (Sarić, Z. and Sarić, M., 1976).

The Alluvium control location Leposavić had a little lower content of free aerobe nitrogen-fixing bacteria comparing with the location in the close vicinity of the pollution source, and that was not expected because this location had significantly lower content of the investigated heavy metals.

The Vertisol location Šupkovac, which is in the close vicinity of the pollution source, had an acid pH value, low humus content, low content of available phosphorus, and a higher content of heavy metals (Zn, Pb, Cd) which influenced unfavourably on microorganisms and their activity. Soil acidity also have a significant influence on heavy metals soil dynamics. In an acid environment, like this one is, higher amounts of heavy metals can be released to soil solution which increases toxicity for plants of the investigated elements. This toxic effect is temporary in light, sandy soils because of rinsing down, but in heavy, clayey soils, like this one is, the toxic effect is long-lasting. So we did not observe *Azotobacter* in this location during the whole investigation.

Another Vertisol location, which was on the higher distance from the pollution source, had a lower content of the investigated heavy metals, and similar physical and chemical properties with the above mentioned location. In uncultivated Vertisol we did not observe *Azotobacter* during the whole investigated period, and in cultivated Vertisol we observed them but in a very small percent. It was established that increased nickel concentration can decrease molecule nitrogen fixation by nitrogen-fixing bacteria (Vesper and Weidensaul, 1978).

This microflora is bad-represented in the profile of the Trnavce location uncultivated Vertisol, which was the farthest from the pollution source. In cultivated soil we did not observe *Azotobacter* during this study, inspite the fact that content of free aerobe nitrogen-fixing bacteria was mostly higher in cultivated soils (Kiković, 1989).

## CONCLUSION

Presence of higher heavy metals concentrations, as is case with locations in the close vicinity of

Kosovska Mitrovica, have an unfavourable effect on microorganism number and activity. The investigated cultivated Alluvium locations had a relatively high content of *Azotobacter*, but much lower content of this microorganism was observed in uncultivated Alluvium. The Vertisol location Šupkovac, which is the closest to the pollution source, did not contain this microflora during the whole investigated period. At the other two Vertisol locations we observed a very low content of *Azotobacter*.

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## REZIME

UTICAJ TEŠKIH METALA NA ZASTUPLJENOST AZOTOBACTERA SP. U OKOLINI KOSOVSKE MITROVICE

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Prisustvo teških metala u većim koncentracijama kao što je to slučaj sa okolinom Kosovske Mitrovice u kojoj su locirani veliki industrijski kapaciteti nepovoljno utiču na živi svet. Ispitivani lokaliteti u neposrednoj blizini izvora emisije imaju znatno veće koncentracije cinka, olova, nikla i kadmijuma od zakonom dozvoljene. Povećane koncentracije ispitivanih teških metala nepovoljno utiču na brojnost i enzimsku aktivnost ispitivanih fizioloških grupa mikroorganizama.

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# Peryphyton Algae in two small Lakes on the Spring Branch of Crnkamenska reka river on Šar-planina Mt.

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## ABSTRACT

Total 158 taxons (Cyanophyta - 14, Pyrrophyta - 1, Bacillariophyta - 74, Euglenophyta - 1 and Chlorophyta - 68) of peryphyton algae were established during the research (September, 1996) that was performed in two circ lakes located on Šar-planina Mt. They are Donje Veljinbeško jezero lake and Srednje Defsko jezero lake. Distance between lakes is 1700 m, and ecological conditions are similar. Donje Veljinbeško jezero lake peryphyton was present

with 82 taxons. Bacillariophyta with 52 taxons was dominant. In Srednje Defsko jezero lake peryphyton was present with 96 taxons, and Chlorophyta with its 55 taxons was dominant. Similarities in peryphyton flora were expressed through 20 common taxons. The water quality (1,59) in Donje Veljinbeško jezero lake is of I - II class bonity, while the water quality (1,37) in Srednje Defsko lake jezero is of I - II class bonity too.

**Key words:** Peryphyton Algae, Donje Veljinbeško circ lake, Srednje Defsko cirk lake, Saprobity.

## INTRODUCTION

Intensive research on peryphyton flora in water system of Šar-planina Mt. (Urošević, 1994a, 1994b). Is lasting for 10 years. Owing to these researches big wealth in taxons had been found and certain number of new taxons had been discovered for algae flora of Serbia. Circ, glacial, solifluctial and nivation lakes, together with some springs on Šar-planina Mt. Were included in the research. Comparing certain lakes and springs brought us to a conclusion that they have very specific and various structure. Percentage of common taxons in two nearest lakes was very small. This justifies the need for further exploration of more than 150 lakes located on Šar-planina Mt. The subjects of this work are two small circ lakes located in the south-west part of Šar-planina Mt. In the valley of Mala Vraca, 2536 m massive (Fig. 1). These lakes, together with other lakes and springs are the branch of Crnkamenska reka river. In geological sense both lakes lie on glacial basis (morena) on the slopes of Vraca Mt. Donje Veljinbeško jezero lake, 2085 n, is located on plateau Veljin Beg, in the valley on Mala Vraca Mt. This lake is in a peat and according to CORINE classification it is marked as "fens, transition mires and springs". It lies on pedological basis of peat soil of eutrophic bog peat. The lake has no tributary, so it fills with water due to melting of snow from surrounding slopes of Vraca Mt. That fresh water spring up like sublacustric springs. This lake is intensively covered with moss, from which the material of peryphyton were taken. Srednje Defsko jezero lake, 2080 m, is located on plateau Gornji Def in the valley of Donja Vraca Mt. It spreads on the south and it is 1700 m far from Donje Veljinbeško jezero lake. During the period of the highest water stage this lake is 120 m long, approximately 60 m wide, its depth is 50 cm.

## MATERIAL AND METHODS

Peryphyton was obtained through grating of sill and straining of moss. The specimens were fixed in 4% Phormaldechide on the field. For Bacillariophyta Braune et all. (1982) method was used. The rest of peryphyton was analysed while being in fixed condition. Microscopic analysis was carried out by using the "Ergeval" microscope (Zeiss, Jena) in the laboratory of Biology Departman, PMF in Pristina. Determination of material was done according to following works: Cvijan & Blaženčić (1996), Dedusenko et all.(1959), Gollerbach et all. (1953), Hustedt (1930), Hustedt (1961-1965), Hindak et all. (1975), Komarenko et all. (1975), Lazar (1969), Patrick - Reimer (1966), Pascher (1925), Vodenicharov et all. (1971), Zabelina et all. (1951), Popova (1966), Palmar - Mondviceva (1982). Indicators of water saprobity were established according to Sladeček (1973). Saprobity index was determined according to Pantle and Buck (1955).

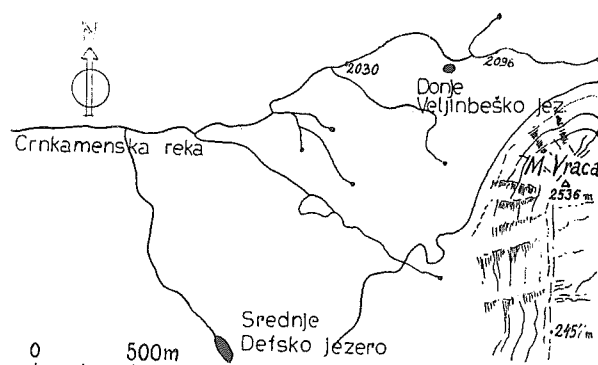


Fig. 1. Geografical position of examined lakes  
Sl. 1. Geografski položaj istraženih jezera

## RESULTS

Tab. 1. The sistematic inventori of established taxons of peryphyton Algae and frequency (skale 1 - 5) of some species that are the indicators of water saprobity (3) in the Donje Veljinbeško jezero Lake (1) and Srednje Delfsko jezero Lake (2), September 1996.

Peryphyton algae	1	2	3
<b>CYANOPHYTA</b>			
<i>Anabaena flos-aquae</i> (Lyngb.) Bréb.	1		b
<i>Aphanothece stagnina</i> (Spring.) A.Br.	1		x - o
<i>Calothrix</i> sp.		1	
<i>Chroococcus minor</i> (Kütz.) Näg.	1		
<i>Chroococcus turgidus</i> (Kütz.) Näg.		1	
<i>Coleosphaerium kützingianum</i> Näg.	1		
<i>Coleosphaerium proboscideum</i> Bohl.		1	
<i>Merismopedia tenuissima</i> Lemm.	1		b-a
<i>Microcystis aeruginosa</i> Kütz.	1		b
<i>Microcystis aeruginosa</i> f. <i>marginata</i> (Menegh.) Elenk.	5		
<i>Nostoc linckia</i> Frey		1	b
<i>Nostoc kihlmanii</i> Lemm.	1	1	
<i>Nostoc paridosum</i> Kütz.		1	
<i>Synechococcus aeruginosa</i> Näg.	1		
<b>PYRROPHYTA</b>			
<i>Peridinium willei</i> Huütf. & Kass.		1	b
<b>BACILLARIOPHYTA</b>			
<i>Achnanthes lanceolata</i> (Bréb.) Grun.	1		x-b
<i>Amphora ovalis</i> Kütz	1		o-b
<i>Anomoeoneis serians</i> var. <i>brachysira</i> (Bréb.) Cl.		1	x
<i>Caloneis silicula</i> (Ehr.) Cl.	1		o-b
<i>Caloneis silicula</i> var. <i>major</i> Skv.	1		
<i>Cyclotella bodanica</i> Eulens	1	1	o
<i>Cymbella aspera</i> (Ehr.) Cl.	1	1	b
<i>Cymbella cistula</i> (Hemp.) Grun.	1		b
<i>Cymbella cistula</i> var. <i>gibbosa</i> Brun.	1		
<i>Cymbella cuspidata</i> Kütz.	1		
<i>Cymbella cymbiformis</i> (Ag.) Kütz.)	1		
<i>Cymbella gracilis</i> (Rabenb.) Cl.	1	3	x
<i>Cymbella hebridica</i> (Greg.) Grun.	1		
<i>Cymbella heteropleura</i> Ehr.	1		
<i>Cymbella heteropleura</i> var. <i>minor</i> Cl.		1	
<i>Cymbella inaequalis</i> (Ehr.) Rabenh.	1	1	
<i>Cymbella lanceolata</i> (Ehr.) V.H.	1		b
<i>Cymbella naviculiformis</i> Auersw.	1		b
<i>Cymbella tumidula</i> Grun.	1		
<i>Cymbella ventricosa</i> Kütz.	1		b
<i>Eunotia argus</i> var. <i>angustata</i> Fricke	1		
<i>Eunotia falax</i> var. <i>gracillima</i> Krasske		1	
<i>Eunotia flexuosa</i> (Bréb.) Kütz.		1	
<i>Eunotia gracilis</i> (Ehr.) Rabenh.		1	
<i>Eunotia lunaris</i> (Ehr.) Grun.		1	o
<i>Eunotia pectinalis</i> (Dillw.) Kütz.		1	
<i>Eunotia praerupta</i> Ehr.	1		

<i>Eunotia praerupta</i> var. <i>inflata</i> Grun.	1		
<i>Eunotia praerupta</i> var. <i>musculicola</i> Boye P.	1		
<i>Eunotia robusta</i> var. <i>tetraodon</i> (Ehr.) Ralfs.		1	
<i>Eunotia valida</i> Hust.		1	
<i>Fragilaria pinnata</i> Hust.	1		
<i>Fragilaria virescens</i> Ralfs.		3	x
<i>Fragilaria virescens</i> var. <i>capitata</i> Østr.	3		
<i>Gomphonema acuminatum</i> Ehr.	1		b
<i>Gomphonema angustatum</i> (Kütz.) Rabenh.	1		o
<i>Gomphonema angustatum</i> var. <i>productum</i> Grun.	1		b-a
<i>Gomphonema constrictum</i> Ehr.	3		b
<i>Gomphonema gracile</i> Ehr.		3	
<i>Gomphonema longiceps</i> Ehr.	1		
<i>Gomphonema longiceps</i> var. <i>montanum</i> (Schum.) Cl.	1	1	x
<i>Gomphonema longiceps</i> var. <i>montanum</i> f. <i>suecicum</i> Grun.	1	1	
<i>Melosira italica</i> var. <i>valida</i> (Grun.) Hust.		1	
<i>Meridion circulare</i> Ag.	1		x-o
<i>Navicula pseudoscutiformis</i> Hust.	1		
<i>Navicula radiosa</i> Kütz.	1		o-b
<i>Nitzschia vermicularis</i> (Kütz.) Grun.	1		b
<i>Pinnularia borealis</i> Ehr.	1	1	x-o
<i>Pinnularia dactylus</i> Ehr.	1		
<i>Pinnularia distiguenda</i> Cl.	1	1	
<i>Pinnularia gibba</i> Ehr.		1	
<i>Pinnularia gibba</i> var. <i>linearis</i> Hust.		1	
<i>Pinnularia intermedia</i> Lagerst	1		
<i>Pinnularia interrupta</i> f. <i>minor</i> Boye P.		1	
<i>Pinnularia lata</i> (Bréb.) W.Sm.	1		
<i>Pinnularia lata</i> var. <i>thuringiaca</i> (Rabenh.) A.Mayer	1		
<i>Pinnularia major</i> (Kütz.) Cl.	1	1	b
<i>Pinnularia major</i> var. <i>hyalina</i> (Hust.) Skarbitsh.	1		
<i>Pinnularia major</i> var. <i>lacustris</i> Meist.	1		
<i>Pinnularia mesolepta</i> (Ehr.) W.Sm.	1		o
<i>Pinnularia microstauron</i> (Ehr.) Cl.		1	o
<i>Pinnularia stomatophora</i> Grun.		1	
<i>Pinnularia subcapitata</i> var. <i>hilseana</i> (Janisch.) O.Müll.		1	x-o
<i>Pinnularia viridis</i> (Nitzsch.) Ehr.		1	b
<i>Pinnularia viridis</i> var. <i>elliptica</i> Meist.		1	
<i>Pinnularia viridis</i> var. <i>fallax</i> Cl.		1	
<i>Pinnularia viridis</i> var. <i>leptogongyla</i> (Ehr.? Grun.) Cl.		1	
<i>Stauroneis acuta</i> W.Sm.	1		o
<i>Stauroneis anceps</i> Ehr.	1	1	b
<i>Stauroneis phoenicenteron</i> Ehr.	1		b
<i>Surirella biserata</i> var. <i>constricta</i> Grun.	1		
<i>Surirella linearis</i> var. <i>constricta</i> Grun.	1		
<i>Synedra ulna</i> (Nitzsch.) Ehr.	1		b
<i>Tabellaria flocculosa</i> (Roth.) Kütz.	1	1	o-x
EUGLENOPHYTA			
<i>Trachelomonas volvocina</i> var. <i>volvocina</i> Ehr.		1	b

CHLOROPHYTA			
<i>Asteriococcus superbus</i> (Ceink.)Scherff.		1	
<i>Bulbochete</i> sp.	1		
<i>Bulbochete</i> sp.		1	
<i>Chlamydocapsa ampla</i> (Kütz.)Fott.	1		
<i>Closterium jenerii</i> Ralfs.		1	
<i>Closterium parvulum</i> Näg.	1		b
<i>Coleastrum costatum</i> Nordsh.		1	
<i>Cosmarium angulosum</i> Bréb.		1	
<i>Cosmarium blutii</i> Wille	1	1	
<i>Cosmarium botrytis</i> Menegh.	1		a
<i>Cosmarium coelatum</i> var. <i>coelatum</i> f. <i>coelatum</i> Ralfs.		1	
<i>Cosmarium conspersum</i> var. <i>conspersum</i> Ralfs.		1	
<i>Cosmarium debaryi</i> Arch.		1	
<i>Cosmarium granatum</i> Bréb.		1	
<i>Cosmarium hexalobum</i> Nordst.		1	
<i>Cosmarium hornavense</i> Gutw.	1		
<i>Cosmarium impressulum</i> Elfv.	1	1	
<i>Cosmarium latum</i> Bréb.		1	
<i>Cosmarium margaritifera</i> Menegh.		1	
<i>Cosmarium meneghinii</i> Bréb.	1		
<i>Cosmarium orbiculare</i> (Ralfs.)Palm.-Mordv.		1	
<i>Cosmarium phaseolus</i> Bréb.	3		
<i>Cosmarium portianum</i> Arch.		1	
<i>Cosmarium pseudoamoenum</i> Wille.		1	
<i>Cosmarium pygmeum</i> Arch.		1	
<i>Cosmarium subcrenatum</i> Hantzsch.		1	
<i>Cosmarium subexclavatum</i> W. et G.S.West.		1	
<i>Cosmarium subprotundum</i> Nordst.		1	
<i>Cosmarium undulatum</i> Corda et Ralfs	1	1	
<i>Cosmarium venustum</i> (Bréb.) Arch.	1		
<i>Desmidium swartzii</i> Ag.		1	o
<i>Eremosphaera viridis</i> De Bary	1	3	
<i>Euastrum ampullaceum</i> Ralfs.		1	
<i>Euastrum ansatum</i> Ehrenb.		1	
<i>Euastrum denticulatum</i> (Kirchn.)Gaj.		1	
<i>Euastrum elegans</i> (Bréb.)Kütz.		1	o
<i>Euastrum insulare</i> (Witr.)Roy.		1	
<i>Euastrum pinnatum</i> Ralfs.		1	
<i>Euastrum verrucosum</i> Ehrenb.	1		
<i>Euastrum verrucosum</i> var. <i>alatum</i> Wolle		1	
<i>Geminella mutabilis</i> (Näg.)Wille.		1	
<i>Micrasterias papillifera</i> Bréb.		1	
<i>Micrasterias truncata</i> (Corda) Bréb.	1	1	o
<i>Mougeotia</i> sp.	1		o
<i>Mougeotia</i> sp.		1	o
<i>Netrium digitus</i> (Ehrenb. ) Jtzig. et Rothe		1	
<i>Netrium interruptum</i> (Bréb.) Lutkem.		1	
<i>Oedogonium</i> sp.		1	
<i>Pediastrum angulosum</i> (Ehrenb.)Menegh.		1	
<i>Pediastrum boryanum</i> (Turr.)Menegh.	1	1	b



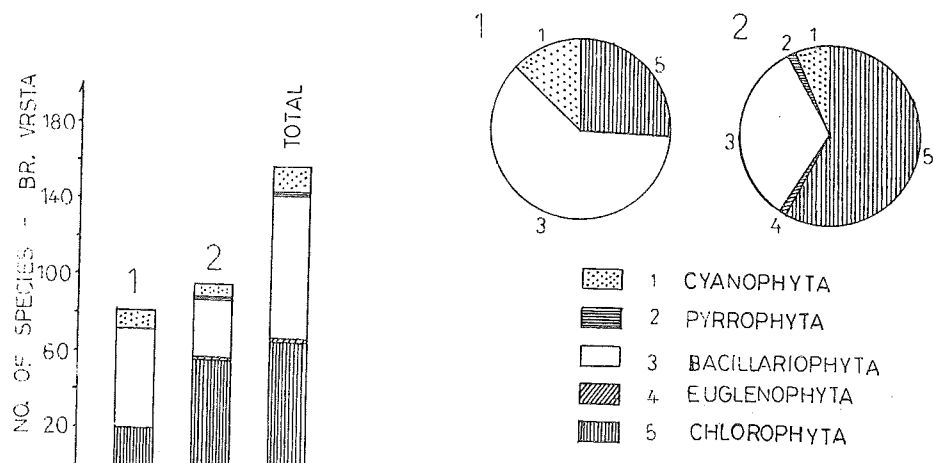
<i>Pediastrum tetras</i> (Ehrenb.) Ralfs	1	1	b
<i>Penium spirostriolatum</i> Barker		1	
<i>Pleurotaenium ehrenbergi</i> (Bréb.) De Bary		1	
<i>Pleurotaenium trabecula</i> (Ehrenb.) Näg.	1		
<i>Scenedesmus brasiliensis</i> Bohl.		1	
<i>Scenedesmus ecoris</i> (Ralfs.) Chod.	1		
<i>Scenedesmus incrassatus</i> Bohl.	1		
<i>Scenedesmus quadricauda</i> var. <i>quadricauda</i> (Turp.) Bréb.	1	1	
<i>Scenedesmus quadricauda</i> var. <i>quadricauda</i> f. <i>granulatus</i> (Hortob.) Uherkov.		1	
<i>Sphaerocystis schroëteri</i> Chod		1	o
<i>Spirogyra</i> sp.		1	
<i>Staurastrum alternans</i> Bréb.		1	
<i>Staurastrum dickiei</i> Bréb.		1	
<i>Staurastrum gladioosum</i> (Turn.) Palm.-Mordv.		1	
<i>Staurastrum meriani</i> Reinch.		1	
<i>Staurastrum polytrichum</i> (Perty.) Rabenh.		1	
<i>Staurastrum simonyi</i> (Heimerl.) Palm.-Mordv.		1	
<i>Telememorus laevis</i> (Ktz.) Ralfs.		1	

## DISCUSSION

Flora of algae in circ lakes (Donje Veljinbeško and Srednje Defsko) was represented by 158 taxons of peryphyton during the exploration period (September, 1996). Representatives from Bacillariophyta order with 74 taxons (46,8%) and Chlorophyta order with 68 taxons (43,0%) were dominant. Representatives from the following order: Cyanophyta - 14 (8,8%), Pyrrophyta - 1 (0,7%) and Euglenophyta - 1 (0,7%) were the minority. Circ lakes Donje Veljinbeško and Srednje Defsko differed in qualitative and quantitative structure of peryphyton community. Donje Veljinbeško jezero lake, poor with flora had 82 taxons: Cyanophyta - 9, Bacillariophyta - 52 and Chlorophyta - 21. Bacillariophyta (63,5%) were more dominant than Chlorophyta (25,6%). Domination of Bacillariophyta is caused by hydrochemical conditions which are related to constant flow of fresh water from lake's spring course. The epyphyt species Gomphonema constrictum found there a good conditions for its development. Chlorophyta was good presented with one species - Cosmarium phaseolus. Donje Veljinbeško jezero lake is the only deposit for this species (Urošević, 1994 - 1996). In peryphyton of this lake Cyanophyta were presented with 10,7%, with Microcystis aeruginosa f. marginata. This is a new species for algae flora of Serbia, since it was not previously recorded by Cvijan and Blaženčić (1997). Srednje Defsko jezero lake differs from Donje Veljinbeško jezero lake (Fig. 1, Fig. 2, Tab. 1) in rich flora - 96 taxons and vegetation composition: Cyanophyta - 6, Pyrrophyta - 1, Bacillariophyta - 33, Euglenophyta - 1 and Chlorophyta - 55. Repre-

sentatives of Chlorophyta (57,3%) dominated over Bacillariophyta (33,4%) representatives in this lake. Variety of peryphyton flora in Srednje Defsko jezero lake is related to complex influence of several environment factors and probably the most important is muddy and grassy base of peat. This is confirmed by the presence of green algae (Conjugatophyceae), especially by the presence of their genus rich in taxons: Cosmarium - 19, Euastrum - 7, Staurastrum - 6, Micrasterias and Netrium - 2, as a Closterium, Desmidium, Mougeotia, Penium and Spirogyra with one species. In this lake Bacillariophyta representatives were present with 33 taxons, they have not found a good conditions for their development, but the following three taxons were present in high quantity: Cymbella gracilis, Fragilaria virescens and Gomphonema gracile (Tab. 1). Participation of Cyanophyta (6,3%) was expressed through three genus: Calothrix, Chroococcus, Coleosporium (with one taxon each) and Nostoc (3 taxons). Their presence indicates on sufficient amounts of nitrogen for their feeding in Srednje Defsko jezero lake. Pyrrophyta (Peridinium willei) and Euglenophyta (Trachelomonas volvocina) in algae community of this lake were present with one taxon each. Both lakes are located on the area of Šutman (Šar-planina Mt.), lie on the same pedological basis, the distance between them is 1700 m. They are characterized by the same climate conditions: freezing of the lake, short vegetative period, poor in nutritive materies of water and small differences in altitude (5 m). Related to these above, similarity in peryphyton flora in two

Fig. 2. Number of periphyton taxa in examined lakes  
Sl. 2. Broj vrsta perifitonskih algi u istraženim jezerima



explored lakes was expressed only through 20 (12,6%) common taxons: Cyanophyta - 1, Bacillariophyta - 11 and Chlorophyta - 8. From total of 158 established taxons 49 species or 30,3% belonged to water saprobity indicators: betamesosaprobic 23, oligosaprobic 16, xenosaprobic 9 and alphamesosaprobic 1 (Tab. 1). Saprobity index was calculated through basic indicator values of algae species in Donje Veljinbeško jezero lake. It was 1,59 ( $x = 6$ ,  $o = 10$ ,  $b = 20$  and  $a = 1$ ), which indicated oligobetamesosaprobic degree, or I - II class of water bonity. Srednje Defsko jezero lake has lower saprobity index - 1,32 ( $x = 6$ ,  $o = 9$  and  $b = 9$ ), which indicate oligobetamesosaprobic degree, or I - II class of water bonity.

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## REZIME

### OBRAŠTAJNE ALGE U DVA MALA ŠARPLAN- INSKA JEZERA NA IZVORIŠNOM KRAKU CRNKAMENSKE REKE

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Analizom obraštajnih algi (septembra, 1996.) na cirknim jezerima Donjem Veljinbeškom i Srednjem Defskom (Šar-planina) utvrdili smo ukupno 158 taksona: Cyanophyta - 14, Pyrrophyta - 1, Bacillariophyta - 74, Euglenophyta - 1 i Chlorophyta - 68 (Fig. 1, 2 i Tab. 1). U florno siromašnijem Donjem Veljinbeškom jezeru (82 taksona) predstavnici Bacillariophyta (52 taksona) bili su dva i po puta brojniji od predstavnika Chlorophyta (21 takson). Dominacija silikatnih algi sa osnovnom epifitnom vrstom *Gomphonema constrictum* verovatno je u saglasnosti sa hidrološkim uslovima koji se odnose na stalni priliv hladne vode iz vedaraca sfagnumskog dna ovog jezera. Istovremeno, i zelene alge imale su dobro učestalu vrstu *Cosmarium phaseolus*. Cyanophyta po brojnom prisustvu (9 taksona) javljaju se na trećem mestu. U obraštaju pripadležnost daju masovno učestaloj vrsti *Microcystis aeruginosa* f. *marginata*. U florno bogatijem obraštaju Srednjeg Defskog jezera (96 taksona) predstavnici Chlorophyta (55 taksona) bili su za 1,6 puta brojniji od predstavnika Bacillariophyta (33 taksona). Raznovrsnija flora algi Srednjeg Defskog jezera u odnosu na prethodno može se dovesti u vezu sa kompleksnim dejstvom više faktora sredine, od kojih smatramo da ja

hidrohemijski sastav muljevitog-sfagnumskog dna ovog jezera najznačajniji. To potvrđuje brojna prisutnost najzastupljenijeg razdela, posebno njegovih taksonima raznovrsnim (Conjugatophyceae) rodovima: *Cosmarium* (19), *Euastrum* (7), *Staurastrum* (6), *Microasterias*, *Netrium* (po 2), *Closterium*, *Desmidium*, *Mougeotia*, *Penium*, *Spirogyra* (po 1). U složenom sastavu ove zajednice silikatne alge su imale tri dobro učestale vrste: *Cymbella gracilis*, *Fragilaria virescens* i *Gomphonema gracile*; dok su predstavnici Cyanophyta (6 taksona), Pyrrophyta (*Peridinium willei*) i Euglenophyta (*Trachelomonas volvocina*) nadjeni u retkim pojedinačnim primercima. Saprobiološka analiza obraštaja pokazuje da je u ispitivanom periodu voda Srednjeg Defskog jezera bila neznatno boljeg kvaliteta (indeks 1,37, I - II klase boniteta) u odnosu na vodu Donjeg Veljinbeškog jezera (indeks 1,59, I-II klase boniteta). Donje Veljinbeško jezero i Srednje Defsko jezero leže na istoj geološkoj podlozi (glacijalni nanos - morene) i pedološkoj podlozi (ranker tipičan na škriljcima), međusobno udaljena oko 1700 m. Karakteriše ih približno isti klimat (zamrzavanje jezera do dna, kratak vegetacioni period) i hidrografski uslovi (ishranjivanje snežanikom), kao i neznatna visinska razlika (oko 5 m). Florna sličnost za oba jezera ogleda se samo u prisustvu zajedničkih 20 taksona (12,6%). S ciljem detaljnijeg tumačenja rezultata potrebno je u daljim istraživanjima obaviti i analizu hemijskog sastava vode ovih algološki zanimljivih jezera.

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# Contribution to the Flora of Mt. Šar-planina

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## ABSTRACT

On the basis of floristic investigation of Mt. Šar-planina new chorological data are presented for the species *Linum spathulatum*, *Primula halleri*, *Astragalus vesicarius* ssp. *pastellianus*, *Col-*

*chicum macedonicum* and *Sesleria korabensis*. Hitherto *Linum spathulatum* and *Colchicum macedonicum* are not known from flora of Yugoslavia.

**Key words:** Mt. Šar-planina, plant species, chorological data

## INTRODUCTION

Mt. Šar-planina belongs phytogeographically to the North Scardo-Pindic province of Central European floristic region. Specific characteristics of the geological substrate and morphology of terrain conditioned a big variety of flora and vegetation in these regions.

Despite the fact that flora of Mt. Šar-planina was investigated by large number of authors and that there are a great number of literary sources about its floristic diversity (see Stevanović et al., 1989), complete list of plant species of this interesting massif has not been made yet.

By our investigations in the period from 1992-1997 we established a large number of taxons, new for the region of Mt. Šar-planina, some of which are also new for flora of Serbia and Yugoslavia (Zlatković et al., 1995; Amidžić, Krivošej, 1996, 1997).

This paper shows the data about the spreading of taxons *Linum spathulatum*, *Primula halleri*, *Astragalus vesicarius* ssp. *pastellianus*, *Colchicum macedonicum* and *Sesleria korabensis*.

## MATERIAL AND METHODS

Determination of the collected plant material was carried out according to "Flora Europaea, 1-5" (Tutin et al. (ed.), 1964-1980) and "Flora SR Srbije, 1-10" (Josifović (ed.), 1970-1977; Sarić (ed.), 1986), and nomenclature of taxons was in compliance with "Flora Europaea, 1-5" (Tutin et al. (ed.), 1964-1980). Herbarium material of shown taxons is located in private herbarium collection "Herbarium Moesiacum" in Doljevac (HMD).

## RESULTS AND DISCUSSION

### A review of species

*Linum spathulatum* (Halcsy & Bald.) Halcsy:  
Popovo prase (Ošljak, Mt. Šar-planina), alt. 1800-1850

m, S exposed limestone rocks, 14.07.1995. B. Zlatković & V. Randelović, HMD; above the village Mušnikovo (Ošljak, Mt. Šar-planina), alt. 1900 m, S exposed limestone rocks, 02.07.1997. B. Zlatković & V. Pešić, HMD.

Species *Linum spathulatum* is Scardo-Pindic floristic element, known so far only for some mountains of northern Greece, Macedonia and southern Albania (Ockendon, Walters 1968: 210; Hartvig, 1986: 562), where according to Horvat (Horvat et al., 1974: 645-647) it takes part in the creation of subalpine vegetation of the order *Daphno-Festucetalia* Quzel 1964, that is, the alliance *Astragalo-Seslerion* Quzel 1964. On Olympus this species was found in the association *Anthyllis aurea-Achillea ageratifolia* Quzel 1967, in which creation most of plant species take part, which also appear on the habitat of this species on Ošljak: *Daphne oleoides*, *Achillea ageratifolia*, *Anthyllis aurea*, *Anthyllis vulneraria*, *Saponaria belidifolia*, *Draba athoa*, *Sesleria coerulans*, *Edraianthus graminifolius* and others. It is possible that the composition that was found out on Ošljak was the fragment of this South Scardo-Pindic subalpine vegetation, but this problem should be regarded more from the phytocenological aspect.

In a taxonomic sense this species is closely related to the species *L. hirsutum* L., from which it differs by its woody stem in the base and short sterile branches in anthesis. Besides, the leaves are the broadest in the upper half, in contrast to the species *L. hirsutum* with its broadest leaves in the middle part, that is, in the lower part. By analysis of herbarium samples of the species *L. hirsutum* in the herbarium of the Institute of Botany and Botanical garden "Jevremovac" (BEOU) we concluded that the material which was collected on some mountains of the western Macedonia corresponds to the species *L. spathulatum*. Nevertheless, it can be spoken about the spreading of this species in the Balkan peninsula only after the revision of other herbarium collections as well in these regions. In the

herbarium of the Botanical garden "Jevremovac" there is also a sample whose legator is Ivo Rudski, who cites *L. hirsutum* for Ošljak (Rudski, 1936). Unfortunately, the sample contains only two blossoming branches, without the earthly part, by which these two species differ among themselves.

***Primula halleri* Gmelin:** near the peat-bog "Tija voda" (Šutman, Mt. Šar-planina), in associations *Primulo-Nardetum strictae* prov. and *Cariceto-Narthecietum scardici* Horvat 1960, alt. c. 2000 m, 1.08.1997. V. Randelović, HMD.

In the edgy part of one larger spring peat bog on the way from the peat bog "Tija voda" toward the village Brod, in the community of the type "Hygronardetum", which we provisionally named *Primulo-Nardetum strictae*, very numerous population of the species *Primula halleri* was found. However, some of the samples of this species were also found in the peat bog community *Cariceto-Narthecietum scardici*.

In the region of Serbia this species has been known only for the area of Mt. Stara-planina (eastern Serbia). Our new find on Mt. Šar-planina is the first record of the presence of this species on the territory of Kosovo and Metohia.

***Astragalus vesicarius* L. ssp. *pastellianus* (Poll.) Arcang.:** above the village Mušnikovo (Ošljak, Mt. Šar-planina), alt. 1900 m, S exposed limestone rocks, 02.07.1997. B. Zlatković & V. Pešić, HMD.

On limestone rocks of the southern slopes of Ošljak the species *Astragalus vesicarius* was found, which was known so far for the region of Serbia only on Mt. Suva-planina near Niš (Pančić, 1884, Petrović, 1885). For the same locality Diklić (1972) in accordance with the previous data, cites *Astragalus vesicarius* subsp. *pastellianus* (Poll.) Archang.

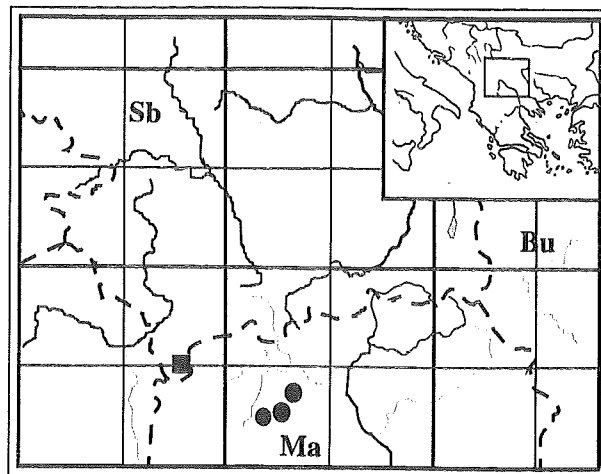
On new locality the mentioned species is shown in the vegetation of southerly exposed rocks together with the species *Athamanta haynaldi*, *Linum spathulatum*, *Ptilotrichum scardicum*, *Sempervivum kindigeri* etc. This species is new for flora of Kosovo and Metohia.

***Colchicum macedonicum* Kusanin:** on the hill Mramor (Vraca, Mt. Šar-planina), in the limestone-marble contact zone, alt. 2200 m, 30.7.1997. V. Randelović, L. Amidžić & S. Belij, HMD. (Mapp 1)

North Scardo-Pindic species *Colchicum macedonicum* Kusanin noted so far on the mountains of central Macedonia, on Mt. Jakupica, Mt. Karadžica, Mt. Solunska (Kusanin, 1911), Mt. Golešnica and Mt. Dautica (Stefanov, 1926). Our new find of *Colchicum macedonicum* on the hill Mramor (Vraca) on the southwestern part of Mt. Šar-planina significant extends northwardly the area of this species. At the same time this is also the first find of such species for flora of Yugoslavia.

Population of this species was developed in that part of the massif in which marble and limestone rocks

touch and mix among themselves, while not any sample was found on the base of pure marble. On such habitats it takes part in the creation of the association *Onobrycho-Festucetum variae* Horvat 1936.



Map 1. Distribution of the species *Colchicum macedonicum* Kusanin (? - known localities, ? - new locality)

Stefanov (1926: 40) subordinated this species to subgenus *Archicolchicum*. Brickell (1980:23) assumed that population of this species in Macedonia (Mt. Jakupica) belonged to the species *C. micranthum* Boiss. (subgen. *Eucolchicum*), and he explained it by the fact that leaves and flowers were developed at different time, what was in contrast to the original description of the species (Kusanin, 1911: 232) in which it was stated that flowers and leaves were developed at the same time. However, Kusanin presented on the picture flower, fruit and leaves on one sample, from which it could be anticipated that the leafing, fruiting and anthesis may occur at the same time to some degree. Brickell also gives such affirmation, with the note that the flowers sometimes develop together with the leaves and fruit from the previous season. By the analysis of the individuals of newly-found population we came to the conclusion that with this species it had been first developed the leaves and fruit from the previous season after snow melting and immediately after it new flowers as well. Therefore it gives such impression that this species belongs to such group of the species where the leaves and flowers appears at the same time. We think that such occurrence was caused by very short vegetation period on this height above sea level, and it is often the case with other high-mountainous species of the subgenus *Archicolchicum* (Stefanov, 1926). Subgenus *Archicolchicum* differentiates from the subgenus *Eucolchicum* according to the other characteristics also, and before all, according to the stigma form. The species from the subgenus *Archicolchicum* have punctiform stigma, while the species from the subgenus *Eucolchicum* have stigma decurrent. Just such characteristic differentiates also the species *C. macedonicum* and *C. micranthum*. Besides, with the species *Colchicum*

macedonicum the period of leafing and fruiting (February-March) has been distinctly separated from the period of anthesis (September-October). Oblanceolate leaves and prolonged neck of the tunica, which is, by its length, several times longer than the bulb length, with the species *C. macedonicum* are characteristics which also differentiate it from the species *C. micranthum*, with its linear leaves and short neck of tunica.

***Sesleria korabensis* (Kmm. et Jav.) Deyl:** Mramor (Vraca, Mt. Šar-planina), limestone rocks, alt. 2250 m, 30.07.1997. V. Randelović & L. Amidžić, HMD; Velika Vraca (Šar planina), limestone rocks, alt. 2250 m, 31.07.1997. V. Randelović, HMD; Čelepinski vrv (Šar planina), limestone rocks, alt. 2500 m, 1.08.1997. V. Randelović & L. Amidžić, HMD.

The species *Sesleria korabensis* has been unjustifiably left out during working on flora of Serbia (Tatić, 1976) because it was already known before that it had been represented on north-eastern part of Mt. Šar-planina. Namely, Rajevski (1960) in his disertation cited this species for greater number of plant communities on the eastern and north-eastern slopes of Ljuboten, what was also mentioned later by Janković (1982). New finds on Vraca, Mramor and Čelepinski vrv confirm the presence of this species on the limestone rocks of Mt. Šar-planina.

## CONCLUSION

During of the floristic investigations on Mt. Šar-planina in southern Serbia the presence of several interesting plant taksons was determined: *Linum spathulatum*, *Colchicum macedonicum*, *Primula halleri*, *Astragalus vesicarius* ssp. *pastellianus* and *Sesleria korabensis*. *Linum spathulatum* and *Colchicum macedonicum* were noted for flora of Serbia for the first time, while the species *Primula halleri* and *Astragalus vesicarius* ssp. *pastellianus* were only known for the eastern Serbia. The presence of the species *Sesleria korabensis* in flora of Serbia was confirmed, so that its omission during the working out of this paper within "Flori SR Srbije VIII" (Tatić, 1976) could be regarded as unjustified.

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## REZIME

## PRILOG FLORI ŠAR PLANINE

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Tokom istraživanja flore Šar planine u periodu od 1992-1997. godine konstatovali smo veći broj taksona novih za područje Šar planine, od kojih su neki novi i za floru Srbije i Jugoslavije. U ovom radu iznosimo podatke o rasprostranjenju taksona *Linum spathulatum*, *Primula halleri*, *Astragalus vesicarius* ssp. *pastellianus*, *Colchicum macedonicum* i *Sesleria korabensis*.

Vrsta *Linum spathulatum* je skardsko-pindski florni elemenat, do sada poznata samo za neke planine severne Grčke i južne Albanije. Na novopronađenom lokalitetu se javlja zajedno sa vrstama karakterističnim za južno skardsko-pindsku vegetaciju sveze *Astragalo-Seslerion*. U taksonomskom pogledu, ova vrsta je srodna sa vrstom *L. hirsutum* L., od koje se razlikuje po tome što ima odruvenelo stablo u osnovi i kratke sterilne izdanke u vreme cvetanja biljke.

U obodnom delu jedne veće izvorske tresave, na putu od tresave "Tija voda" prema selu Brod, u zajednici tipa "Hygronardetum", koju smo provizorno nazvali *Primulo-Nardetum strictae*, pronađena je veoma brojna populacija vrste *Primula halleri*. Osim toga, pojedini primerci ove vrste nalaženi su i u tresavskoj zajednici *Cariceto-Narthecietum scardici*. Na području Srbije ova vrsta je poznata samo za područje Stare

planine (istočna Srbija), tako da pronalazak ove vrste na Šar planini predstavlja novinu za floru Kosova i Metohije.

Na krečnjačkim stenama južnih padina Ošljaka pronađena je vrsta *Astragalus vesicarius*, do sada na području Srbije poznata samo za Suvu planinu kod Niša. Na novom lokalitetu pomenuta vrsta javlja se u vegetaciji južno eksponiranih stena zajedno sa vrstama *Athamanta haynaldi*, *Linum spathulatum*, *Ptilotrichum scardicum*, *Sempervivum kindigeri* itd. Ovaj vrsta je nova za floru Kosova i Metohije.

Severno skardsko-pindska vrsta *Colchicum macedonicum* Kosanin je do sada zabeležena na planinama centralne Makedonije (Košanin, 1911; Stefanov, 1926). Nalaz ove vrste u sklopu asocijacije *Onobrycho-Festucetum varia*e na brdu Mramor (Vraca) u jugozapadnom delu Šar planine predstavlja značajno proširenje njenog areala na sever. Ujedno ovo je i prvi nalaz ove vrste za floru Srbije.

Vrsta *Sesleria korabensis* je neopravdano izostavljena prilikom obrađivanja flore Srbije jer je od ranije poznato da je ona veoma zastupljena na severoistočnom delu Šar planine. Naime, Rajevski u svojoj doktorskoj disertaciji ovu vrstu navodi za veći broj biljnih zajednica na istočnim i severoistočnim padinama Ljubotena. Nova nalazišta na Vraci, Mramoru i Čelepinskom vrhu potvrđuju prisustvo ove vrste na krečnjačkim stenama Šar planine.

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# High-mountain Vegetation of the Circues in Suvi Potok and Durlev Potok Creek-Basins on the Sar-planina Mt. Northern Slopes

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## ABSTRACT

**Amidžić Lidija (1997): High-mountain Vegetation of the Circues in the Suvi (Dry) and Durlev Potok (creeks) creek-basins on the Sar-planina Northern Slopes. The University Thought. Natural Sciences (Special Edition). Sar-planina (mountain) presents Central-Balkan mountain massif with still visible traces, that remained from the Last Glacier Age. Mostly emphasized**

**glacial forms are presented by circues, cut in the source points of mountain creeks. The circues in Durlev and Suvi Potok belong to the group of the most attractive circues of the massif, not only from the geologic, geo-morphologic and hydro-logic point, but, also, for its high-mountain vegetation that presents the spectrum of unique combination of species.**

**Key words:** Sar-planina, circues, high-mountain vegetation

## INTRODUCTION

The Sar-planina Central-Balkan massif is sharply differentiated in morphologic sense, by surrounding low leveled ravines and counties, forming a marking barrier over the contact point of Sub-Mediterranean region to the south and continental one to the north. Glacial, fluvial, karst, gravitational and peri-glacial processes had the greatest impact on Sar-planina modern sculpture. Glacial forms of the Last Glacial Age provided its relief with a dominant morphologic characteristic. Multiple advances and retreats of its glaciers, as well as, their final melting, resulted in the loss of many circues, but, with the glacial lakes remaining (Belij et al. 1996).

The circues in the Suvi and Durlev Potok creek-basins are among the most interesting circues of the massif northern slopes. Both circues are deeply cut under sharp peaks of the main ridge. The cirque background in the Suvi Potok creek-basin is made up of mutually elevated Jezerski (Lake) peaks (2,586 m, 2,595 m, 2,597 m) and Bistra (2,651 m), while the Durlev Potok creek-basin background is made of the peaks Bela Rojina (White Erosive Soil - 2,421 m), Rafsa (2,454 m) and Crni Kamen (Black Rock - 2,536 m) that, in a ridge-like way meet with Jezerski vrhovi (Lake Peaks). The cirque, located between Jezerska Cuka (Lake Hill) and Crni Kamen, stands between the two previous circues.

Larger number of small circues, combined into a unique glacial depression, their cascade bottom with rocky bends, created by gradual movement of glaciers, steep, often vertical sides as the products of regressive erosion of glaciers (Menkovic, 1990) and moraines made of big angle-like blocks, provide major morphologic characteristic of the circues. Jazinacko Lake

(2,135 m), with its bottom deepened by the glacier gradual movement, represents a special attraction of the cirque in the Suvi Potok creek-basin. The landscape of a typical alpine region (Amidžić et Belij, 1996) was created, first of all, by later tempestuous dynamic of peri-glacial processes and as a result of the phenomenon of solifluction of rocky glaciers and the stone torrents.

From the geologic point, this area is made of regionally metamorphic Paleozoic rocks with gneiss-granite imprinted as a large magmatic mass. At the same time, significant changes occurred on surrounding rocks, manifested with various degree of schist (Koscal, 1990). Granitic steep ground and an intensive frost destruction resulted in the domination of rocky ground (lithsole) on gneiss-granite of district colluvium, as well as, poor to average skeleton humus-silicate soils (rankers) on the spots suitable for accumulation (Topalovic, 1990).

Annual average temperature of 0-2°C is somewhat higher on porous south exposed slopes of the cirque (2-3°C), than on windy ridges (below 0°C). Living conditions in this zone are severe and lowest possible. The vegetation period shortens from 160 days at 2,000 m a/s/l, to 90 days as determined in case of the peaks. The parameters indicate the prevalence of the conditions of frost-snowy, peri-glacial climate respectively.

A unique and diverse vegetation, representing a fragment of Balkan - Boreal Sub-region of the Boreal bio-geographic region, appears in these circues due to their tempestuous dynamic, although evident unfavorable conditions of survival are present.

## MATERIAL AND METHODS

The mapping of vegetation in the Sar-planina circues for the purpose of selecting the nature reservations, has been underway for three years already. Based on the existing literature as a significant foundation, we have continued our independent research aiming to amend and update the data published already. Therefore, phyto-cenologic research, according to standard method of Zurich-Maunt Pellier school (Brown - Blanquet, 1932), have been done again on the whole area marked.

## RESULTS AND DISCUSSION

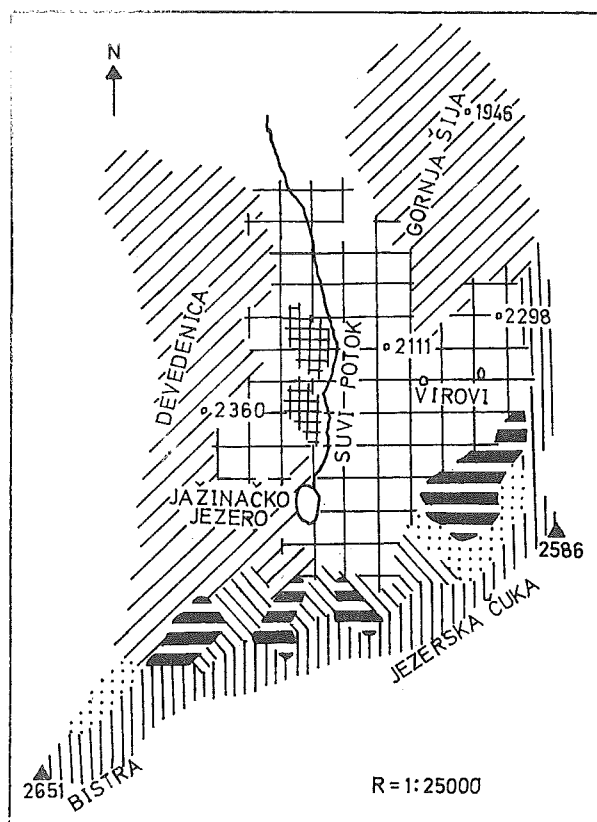
High-mountain vegetation of the circues in Suvi and Durljev Potok creek-basins is characterized by its diversity, florist resource and unique combinations of species, although, it went through the phase of anthropo-genic degradation. Negative human influence was reflected, first of all, on the lowering of upper forest border, when, the frigorific zone of coniferous forest suffered the most.

Upper forest border is quite indented in the Suvi Potok creek-basin. On the localities of Mekus Bor (Mekus Pine) and Gornja Sija (Upper Neck), it is made of climate-zonal, endemo-relict association of *Rhododendro ferruginei* - *Pinetum peucis* Jank. et Bog. 1962. This, unique community, tied to Sar-planina only, reaches up to some 2,000 m a/s/l/ on the locality of Mekus Bor, close to its natural border respectively. This border was secondarily lowered for some 100 to 200 m on the locality of Gornja Sija.

The upper forest border on the slopes of Devedenica is created by the associations of *Fagetum moesiicum montanum* Rudski 1949 and *Pineo* - *Pinetum peucis mixtum* Jank. 1976, reaching up to 1,900 m a/s/l/.

High-mountain bushy association of *Juniperus nana* - *Bruckenthalia spiculifolia* Horv., appearing in the form of two sub-associations, *vaccinetosum uliginosi* L. Raj. 1974 and *rhododendrosom ferruginei* L. Raj. 1974, spreads above the forest zone. The first sub-association spreads on Gornja Sija and on higher slopes of Devedenica, while, the other one on part of Gornja Sija, on Virovi (Whirlpool) and on lower slopes of Devedenica and Jezerska Cuka, above the Jazinacko Lake (Map 1).

An Arctic element of *Juniperus sibirica*, Mesozoic - Carpathian sub-endemic of *Bruckenthalia spiculifolia* and Boreal - Circum-polar species of *Vaccinium myrtillus* and *V. uliginosum*, representing general edifiers, dominate in the sub-association of *Juniperus nana* - *Bruckenthalia spiculifolia* Horv. *vaccinetosum uliginosi*. However, this association, rich in florist sense, is differentiated into numerous facies with significantly increased frequency of other species. *Cerastium decalvans* and *Anemone narcissiflora* predominate in the



	<i>Juniperus nana</i> - <i>bruckenthalia spiculifolia</i> Horv. subass. <i>vaccinetosum uliginosi</i> L.Raj.
	<i>Juniperus nana</i> - <i>bruckenthalia spiculifolia</i> Horv. subass. <i>rhododendrosom ferruginei</i> L.Raj.
	<i>Rumicetum alpinae</i> Horv.
	<i>Potentillo doereferii</i> - <i>Juncetum trifidii</i> V. Stev., S.Jov., D.Lak.
	<i>Ranunculetum crenati scardicum</i> Rexh.
	<i>Carex curvula</i> - <i>Sesleria comosa</i> Horv.
	<i>Lichen desert</i>

MAP. 1. VEGETATION MAP OF THE HIGH-MOUNTAIN VEGETATION OF THE CIRQUE JAZINAČKO JEZERO LAKE

Authot: dr Lidiija Amidžić

facies between Donja and Gornja Sija (Lower and Upper neck). On the edge of Gornja Sija, it is replaced by the facies with *Geum montanum*, on northern exposures of Devedenica, the facies with local endemic *Crocus scardicus*, respectively. In the sub-association of *Juniperus nana* - *Bruckenthalia spiculifolia* Horv. *Rhododendrosom ferruginei*, one of the basic residents is the Alpine relict *Rhododendron ferrugineum* that, in these circues, creates communities unique in the Balkans. Local conditions heterogeny caused the appearance of numerous facies also in this sub-association, providing it with an exceptional coloring. The facies with the species of *Primula columnae* and *Thlaspi avalunum* are distinguished on the locality of Donji Virovi (Lower Whirlpool), the facies with *Anemone narcissiflora*, *Doronicum columnae* and *Ranunculus montanus* on eastern exposed rock-blocks

above Jazinacko Lake, and the facies with *Achillea ahrotinoides* on the western exposed blocks above the lake, respectively.

The fragments of the sheepfold herbaceous vegetation, presented by the association of *Rumocetum alpinae* Horv., could be found in the zone of bushy vegetation. An Alpine - Carpathian element of *Rumex alpinus* is the general and, almost, the only creator of this, anthropo-genically conditioned community.

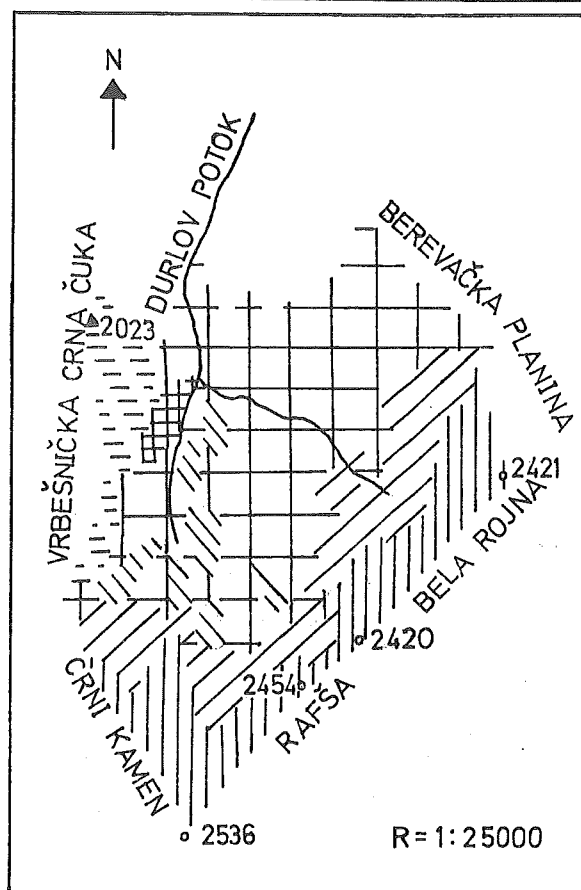
Herbaceous vegetation can be found above the zone of bushy associations, that, in places, climb up even to the very peaks. Northern exposed cliffs of Jezerska planina (Lake Mountain) represent the residence of an endemic, chasm-phyte association of *Potentilla doerflerii* - *Juncetum trifidii* Stev., Jov. et Lak. 1989, whose general characteristic is given by the rock-endemic of *Potentilla doerflerii* and Arctic - Alpine species of *Juncus trifidus*. This community gradually shifts into the fragments of the community of *Ranunculetum crenati scardicum* Rexh. 1980., around the firm snow and on humid, north-western exposed depressions. The sub-endemic of *Ranunculus crenatus* and *Cardamine glauca*, endemic *Arabis flavescens* and *Androsacea hedraantha*, as well as, the rock-endemic of *Crocus scardicus*, are its general edificers.

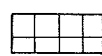
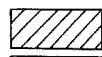



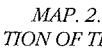
The community of *Carex curvula* - *Sesleria comosa* Horv., primarily created by the Alpine species of *Carex curvula*, *Primula minima* and *Campanula alpina*, as well as, by the endemic of *Festuca scardica* and *Sesleria korabensis*, exists on the highest ridge parts of Bistra and Jezerska planina (Lake Mountain), in the conditions of severe peri-glacial climate. This association, also, is not a uniform one, thus, the facies that could be pointed out are with *Juniperus sibirica* and *Vaccinium uliginosum*, representing a transfer toward the bushy associations, the facies dominated with *Senecio carpaticus* and *Anthemis carpatica*, the facies with the species of *Saxifraga adscendens* and the species of *Hieracium murorum*, *Paronychia kapela*, *Cetraria islandica* and *Cetraria nivalis*.

Vascular vegetation is absent on the most extreme cirque localities, on which, frost destruction initiated the phenomenon of unstable erosive rocky ground. Hard lichens on rock-blocks are the only living forms here.

High-mountain vegetation of the Durlev Potok cirque is similar to those of the cirque in the Suvi Potok creek-basin. However, the upper forest border here is lowered to an average of 1,750 m a/s/l/. It is created of, previously mentioned, endemo-relict community of *Rhododendro ferruginei* - *Pinetum peucis* Jank. et Bog. 1962, and is permeated with deciduous forest association of *Acero heldreichii* - *Fagetum moesiaceae* Jank. et Stev. 1983, created of the Mesozoic beech and a very rare mountain maple (*Acer heldreichii*).

Bushy community of *Juniperus nana* - *Bruckenthalia spiculifolia* Horv., that, as in the previous cirque, differentiate into two sub-associations, *rhododendro-*



-  *Juniperus nana* - *Bruckenthalia spiculifolia* *rhododendrosom ferruginei* L.Raj.
-  *Juniperus nana* - *Bruckenthalia spiculifolia* *vaccinetosum uliginosi* L.Raj.
-  *Rumicetum alpinae* Horv.
-  *Ranunculetum crenati scardicum* Rexh.
-  *Festucetum paniculatae* Horv.
-  *Carex curvula* - *Sesleria comosa* Horv.

MAP. 2. VEGETATION MAP OF THE HIGH-MOUNTAIN VEGETATION OF THE CIRQUE DURLEV POTOK CREEK

Author: dr Lidija Amidžić

*sum ferruginei* and *vaccinetosum uliginosi* L. Raj. 1974 (Map 2), spreads above a highly degraded forest zone. The first sub-association is located on, somewhat, lower expositions, next to Durlev Potok, on the slopes of Bela Rojna and Bervacke planine (Bervacke Mountains). The facies are differentiated there too. The species of *Polygonum bistorta*, *Trollius europaeus*, *Luzula luzuloides*, *Gentiana asclepiadea* are significantly presented on humid and lower expositions, in addition to general modifiers, while, the facies with *Nardus stricta* is presented on more dry, higher expositions, on the locations of once degraded pastures. The second sub-association, *vaccinetosum uliginosi*, as a difference to the first cirque, appears on somewhat higher expositions below the ridge parts of Bela Rojna and Rafsa. The species of *Nardus stricta* also dominates

in this community in addition to its general residents of the herbaceous flora level.

The sheepfold vegetation, presented by the association of *Rumicetum alpinae* Horv., but, also the association of *Ranunculetum crenati scardicum* Raxh. 1980, are implanted in the zone of the circue bushy vegetation, and the respective are the representatives of the firm snow vegetation. Its florist composition is identical to those of the previous circue.

An anthropo-genically degraded association of *Festucetum paniculatae* Horv. is located on the eastern slopes of Vrbesticka Crna Cuka (Vrbesticka Black Hill). Apart from the Sub-Mediterranean species of *Festuca paniculata*, significant contribution to the association composition is also given by *Nadrus stricta*, *Geum montanum*, *Potentilla ternata* and *Vaccinium myrtillus*.

The highest ridge parts in the background of the circue, comprised of the massifs of Bela Rojna, Rafsa and Crni Kamen, are covered by the association of *Carex curvula* - *Sesleria comosa* Horv., that, apart from the endemic *Sesleria comosa*, is significantly presented by the Alpine species of *Carex curvula*, *Juncus trifidus*, *Primula minima*. Certain facies could be also distinguished within this part of the Sar ridge community. The most emphasized facies with the Alpine lichens are *Cetraria islandica* and *Cetraria nivalis*, the Alpine species of *Homogyne alpina* and Arctic species of *Saxifraga adscendens*, respectively.

## CONCLUSION

General characteristics of the high-mountain vegetation of the circues in the Suvi and Durlev Potok creek-basins on the Sar-planina northern slopes are: Its diversity, unique combination of species and florist resource, especially regarding its local endemic and South-European - mountainous, Mid-European - mountainous, Arctic - Alpine and Boreal flora elements. The vegetation of the two circues whole territory, due to, the mosaic distribution and permeation of all the communities mentioned, but also, due to the influence of surrounding community elements, does have the structure of eco-tones, making the vegetation map preparation being significantly difficult.

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## REZIME

VISOKOPLANINSKA VEGETACIJA CIRKOVA U SLIVU SUVOG I DURLEVOG POTOKA NA SEVERNIM PADINAMA ŠAR-PLANINE

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Dominantno morfološko obeležje reljefu Šar-planine daju glacialni oblici poslednjeg ledenog doba. Višestruko napredovanje i povlačenje lednika kao i njihovo konačno otapanje izdubilo je brojne cirkove u kojima su zaostala glacialna jezera (Belij et al. 1996). Impozantno deluje njihovo kaskadno dno sa stenovitim pregibima nastalim slapovitim kretanjem lednika, strme, često vertikalne strane kao proizvod regresivne erozije lednika i morene od krupnih uglastih blokova

. Kasnijom burnom dinamikom pre svega , periglacialnih procesa i pojavama soliflukcije, kamenitih ledenika i struje kamenja, stvoren je pejzaž tipičnog alpskog predela (Amidžić et Belij, 1996 ).

Među najinteresantnije cirkove severnih padina Šarplanine spadaju cirkovi u slivu Suvog i Durlevog potoka , duboko uvučeni ispod strmih vrhova glavnog grebena. Zaleđe cirka u slivu Suvog potoka čine međusobno srasli Jezerski vrhovi (2586 m, 2595 m, 2597 m) i Bistre (2651 m), a cirka u slivu Durlevog potoka vrhovi Bela Rojna (2421 m ), Rafša (2454 m ) i Crni kamen (2536 m) koji se prema severozapadu grebenski nastavlja na Jezerske vrhove .

U uslovima surove mrazno - snežne , odnosno , periglacialne klime u ovim se cirkovima razvija izuzeto bogata vegetacija koja predstavlja fragment balkansko - borealnog podregiona borealnog biogeografskog regiona (Stevanović , 1995).

U slivu Suvog potoka gornju šumsku granicu gradi endemoreliktna asocijacija *Rhododendro ferruginei* - *Pinetum peucis* kao i asocijacije *Pineo* - *Pinetum peucis mixtum* i *Fagetum moesiaceum montanum*. U slivu Durlevog potoka pored već pomenute zajednice *Rhododendro ferruginei* - *Pinetum peucis* , najviši šumski pojas čini i asocijacija *Acero heldreichii*

- *Fagetum moesiaceae*. Iznad gornje, antropogeno spuštene šumske granice , nalazi se zona visokoplaninske žbunaste vegetacije koja je u oba cirka zastupljena asocijacijom *Juniperus nana* - *Bruckenthalia spiculifolia*. Ova se asocijacija diferencira u dve subasocijacije , *rhododendrosum ferruginei* i *vaccinetosum uliginosi*. Mada se žbunasta vegetacija mestimično penje do samih vrhova , na najvišem grebenskom delu u zaleđu cirkova , ipak preovlađuje zeljasta vegetacija predstavljena asocijacijama *Rumicetum alpinae* , *Festucetum paniculatae*, *Ranunculetum crenati scardicum*, *Potentillo doerflerii* - *Juncetum trifidii* i *Carex curvula* - *Sesleria comosa* . Osnovne karakteristike visokoplaninske vegetacije cirkova u slivu Suvog i Durlevog potoka jesu njena raznovrsnost , jedinstvenost u kombinaciji vrsta, i florističko bogatstvo posebno kada su u pitanju lokalni endemiti i južnoevropsko - planinski, srednjeevropsko - planinski, arktičko - alpski i borealni elementi flore . Mozaičan raspored navedenih zajednica i upliv elemenata okolnih asocijacija daje vegetaciji cirkova obeležje raskošnih ekotonova .

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## AN OCCURENCE OF *Branchiura sowerbyi* BEDDARD (Oligochaeta: Tubificidae) IN MACEDONIA

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### ABSTRACT

**The natural range of *B. sowerbyi* is the countries of Asia in which its density of population can be some greater and it is occurrence elsewhere is accidental. In Europe this unusual tubi-**

**ficid is very rare and accidentally. Such is the case with the finding just one specimen from a small irrigation canal of the city park in Skopje, Macedonia..**

**Key words:** *Branchiura sowerbyi* Bedd. (Oligochaeta; Tubificidae) , Macedonia

This gilled oligochaete *Branchiura sowerbyi*, which is only species with gills of the family Tubificidae, was first described by Beddard in 1892 from specimens found in the Royal Botanical Garden of Regent's Park in London. Among the tubificids this worm is very easy recognizable by the presence of the dorsoventral gill filaments on the posterior segment of the body. Sokolskaya (1961) contends the natural range of *B. sowerbyi* is the countries of Asia and that its occurrence elsewhere is accidental. Outside Asia, the most literature records for the presence of *B. sowerbyi* have been in America and Europe. So its occurrence in the USA was discovered first by Spencer 1932 in Ohio and nowadays it is known more than in ten states of USA (Cole, 1966) During 1964/65 working to bottom fauna of Lake Mendota in Madison (Wisconsin) we discovered the presence of *B. sowerbyi* in the lake in Yahara canal. In Europe this tubificid has been found at several states of North and Central part of the continent, especially in Ireland, Germany, Russia Belgium, Czech, France, Switzerland, Italy, Hungary. It is found in water tanks in botanical gardens, or in warmer effluent at greater rivers or in mouth of rivers. So in England it was found in the Royal Botanical Society's Gardens Regent Park in London, in warm effluents of river Thames at Reading (Mann, 1965) or occasionally in rivers of normal temperature (Brinkhurst, 1971). In the Balkans it is first known occurrence was in the Botanical garden, Zagreb in Croatia (Devidé, 1956). Then in Rumania (Busnita, 1966), Bulgaria, in field of Vojvodina, Serbia (Pujin and Djokic, 1978) and finally in Macedonia in small irrigation canal of the city park Skopje. The last finding of this tubificid species at the same time is shown its the most southeast distribution of Europe. In the present time, this unusual tubificid there is a widespread distribution, occurring on all continents of our planet. Many literature records suggest that the species is practically cosmopolitan but with a great rarity and small in numbers, respectively in the sense of Hutchinson it is a fugitive species in new habitats. Probably the

same case was the discovery of *B. sowerbyi* in Macedonia. It was found in a single specimen on May 23, 1995 from a small irrigation canal in the city park near the Zoological Garden in Skopje. Except the natural distribution, this worm spread by man introduced in botanical gardens, e.g. with botanical Society's gardens in Great Britain (Beddard, 1892) or in Botanical Garden in Zagreb, Croatia (Devidé, 1956) in the European continent. According to Cole (1954) *B. Sowerbyi* seems to be typical of artificial habitats, and in contrast to some other tubificids, of well-oxygenated waters.

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#### REZIME

BRANCHIURA SOWERBYI BEDDARD (OLIGOCHAETA: TUBIFICIDAE) U MAKEDONIJI

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Neobična tubificidna vrsta *Branchiura sowerbyi*, koja pripada monotipskom rodu, najpre je bila opisana od strane Beddard-a, 1892. Godine na primercima

sakupljenim u sedimentima akvatičnih rezervoara u Regent parku botaničke bašte u Londonu. Ona je kasnije bila pronadjena i na drugim kontinentima, a danas je rasprostranjena na svim kontinentima, izuzev na Antarktiku. Medjutim, ona se susreće vrlo retko u obliku "zalutale vrste", uglavnom pojedinačno ili sa izvanredno malom populacijom. Prema literaturnim podacima vrsta se javlja kao tipična za veštačka staništa, i suprotno drugim tubificidnim vrstama, u dobro aerisanim vodama. U Makedoniji je nadjen jedan primerak ove vrste u jednom kanalu gradskog parka u Skopju, neposredno u blizini Zoološkog vrta.

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# TOXIC SUBSTANCES AS MATERIAL BASIS OF APOSEMY AT BUTTERFLIES AND MOTHS (Insecta, Lepidoptera)

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## ABSTRACT

**Relation between some organic chemical compounds in poisonous plants, eaten by the caterpillars of some Lepidoptera species, and aposematic characteristics at those caterpillars and adult Lepidoptera species is discussed. It is proved that the chemical substances of herbs are**

**not fully dissolved by metabolic processes in digestive system of caterpillars, neither by intracellular digestive mechanism, but they get incorporated unchanged in the body and wings. In this way these toxic substances present a material aposematic base.**

**Key words:** toxic substances, aposemy, lepidoptera

## INTRODUCTION

It is known that most of the butterflies and moths have in their bodies some chemical compounds by which they repulse potential predators. Besides, those species are noticeably bright coloured black, yellow, red, white, and orange. This type of protective colouring is aposematic colouring. Very often birds and mammals learn in short time that brightly coloured butterflies and moths are not tasty and get accustomed to the relation between bright colouring and unpleasant consequences. The colour of insects comes from the integument and colours are mostly pigment and structural. The light reflects through cuticle and passing through lamellae of endocuticle creates structural colours. They seem as metal colour. However, in the cases of aposematic colouring the colours are pigment. Considering their chemical structure they are melanins, carotenoids, anthoxians and anthocyanins. For example, carotenoids belong to the class of terpenoids which are basic ingredient of the ether oil of plants. A molecule of carotene pigment contains 8 isoprenoid compounds. There are lots of plant species which contain terpenoids: *Origanum vulgare* L., *Thymus* sp., *Salvia sclarea* L., *Daucus carota* etc. Erlich and Raven (1964) put forward a hypothesis that vascular plants acquired an assortment of secondary metabolites back during the Cretaceous period. It happened, most probably, as a response to the pressure of herbivores. The evolution of new chemical defensive substances reduced pressure of herbivorous animals and made possible for plants to populate new ecological niches. For example, there are more than 5000 kinds of butterflies and moths in the Balkans, but caterpillars of only few kinds eat plants from the family of mint (*Mentha*) as it is *Spilosoma menthastri* Esper (fam. Arctiidae). Following this context we arose a question: is there any connection between secondary metabolites of plants, which are toxic for the most of the animals, and aposematically coloured kinds of butterflies and moths

whose caterpillars eat those plants? This question is important for the reason that caterpillars must have some mechanism for neutralisation of the toxins and in the same time they must be able to incorporate secondary metabolite from a plant, unchanged, into body and wings of butterfly or moth. This transition of secondary metabolites from plants to the bodies of butterflies or moths is characteristic phenomenon. Wigglesworth (1965) proved that caterpillars of butterflies and moths belong to the rear group of insects which have complete enzyme set in their digestive systems: protease, lipase, amylases, invertase and maltase. The epithelial cells in their middle intestinal (mesenteron) secrete above-mentioned ferments. The middle intestinal transmits digested food in a form of hilus which diffuses through the epithelium, resorbs it in the blood that spreads it through the body. There is a good example to prove that secondary metabolites of plants are transmitted to the body of adult butterfly and moth, unchanged by digestion in the digestive system of caterpillar: pterin, typical for Fabaceae and some other plants, are unchanged incorporated into wings of butterflies and moths, where take part in forming the white colour - leucopterin, yellow colour - xanthopterin and orange colour - eritropterin (Ristic and Jaksic, 1996). It is very easy to prove the presence of this pigment in plants as well in wings of butterflies and moths using UV photography (Ferris C.D. 1972).

## MATERIAL AND METHODS

The butterflies and moths that will be analysed are chosen on the basis of their aposematic colouring. In addition, the caterpillars of those species must be aposematically coloured and must eat plants toxic for the most of other animals. Observing the collection of lepidoptera (Jaksic P. leg. et coll.) it has been noticed

that there are few such species and they are taxonomically homogenised among butterflies (Papilionidae) and among few families of moths (mostly among species of the Arctiidae family). We had a chance to see in a field that birds avoid eating some of the species we chose. One more proof, indirect one, that they are tasteless for birds and other predators was that we caught them completely undamaged while other species we caught had damaged wings because bird attacked them.

## RESULTS

1. Milkweed or Monarch, **Danaus plexippus** L., 1758 (fam. Danaidae) The caterpillars of this kind of butterflies are distinctly aposematically coloured. The main colours are green and yellow with black stripes and marked thornlike excrescences; considering their habitus they are similar to wasps. The caterpillars eat species from the families Asclepias, Euphorbia etc. The juice of Asclepias contains caoutchouc, resin, amirin, estres, bitter substances and glycosides (Kojic and Janjic, 1991). The species from the family Euphorbia are toxic as well, because of the presence of euphorbine-euphorbine acid anhydride. Adult butterfly or moth from the family *Danaus plexippus* L. is aposematically coloured, too: wings are orange-red with marked black-and-white patterns. Many of the authors noticed that birds do not attack this kind of butterflies and moths of this kind because they are tasteless. However, the best proof that this kind is really repulsive is the fact that there is a kind *Limenitis archipus* (fam. Nymphalidae) in America that imitates its colour and pattern. In Africa, as well, the cognate species African Monarch (*D. Chrysippus linnocus*) is imitated by female butterflies and moths of another kind - *Diadem* (*Hypolimnas misippus linnocus*, fam. Nymphalidae). These are cited examples of the Batesian Mimicry (Smarth, 1977, Williams, 1978).

2. **Parnassius apollo** L., 1758 (fam. Papilionidae) The caterpillars of this species are distinctly aposematically coloured. The bodies are brightly black with marked orange spots. Their food plants are from the genus of *Sedum* sp: *Sedum album*, *Sedum telephium*, *Sedum purpurascens* etc. It is known that species of the family Crassulaceae, including the family *Sedum*, contain toxic glycosides with hydrocyanic acid in their structures (Durasin, 1988). The adult butterfly or moth *Paruassius apollo* L. is aposematically coloured as well: on the white-yellow basis red-and-black spots stand out. If one smell this butterfly he can feel intensive characteristic smell that repulses animals of prey.

3. **Parnassius mnemosyne** L., 1758 (fam. Papilionidae) The caterpillars of these butterflies are aposematically coloured, too. The body is black with bright red spots on it, what shows that they "bolt" toxins. Their food plants are from the family *Corydalis*. All parts of

the plants from this family are toxic because they contain alkaloid of unknown chemical structure. The adult butterflies are similar to the butterflies of previous species regarding to their habitus.

4. **Zerynthia polyxena** Denis und Schiffermiller, 1775 (fam. Papilionidae) The caterpillars are aposematically coloured light-brown with marked black-and-white tones. The food plants are from the family *Aristolochia*. We observed in the Prizren region that in the same time the female *Z. polyxena* and *Z. cerisyi* hatch eggs onto this plant. The species of the family *Aristolochia* contain alkaloid aristolochin as a basic substance, but its chemical structure is unknown. At this plants ether oils are in the idioblasts widened in the parenchymal tissues. Adult butterflies are markedly aposematically bright coloured and for that reason they are called "Easter butterflies".

5. **Zerynthia cerisyi** Godart, 1822 (fam. Papilionidae) Everything said about previous kind is valid for this kind. They are even rivals considering food plants what was illustrated by the example in the Prizren region. However, it is not a complete competition since their areas overlap each other only in the central Balkan region. This species does not exist across the Sava and Danube while *Z. polyxena* can be often met in Europe.

6. **Heodes virgaureae** L., 1758 (fam. Lycaenidae) The caterpillars of this kind are distinctly aposematically coloured; they are green with bright yellow markings. They eat different kinds of plants from the family of Sorrel (*Rumex*). Those plants contain carotene, oxalic acid, tannin and anthrakinson. The adult individual is aposematically coloured metallic-red.

7. **Rhyparioides metelkana** Led. (fam. Arctiidae) The caterpillars of this kind of butterflies and moths are markedly coloured as well; the basic colour is white colour like the colour of ivory. They eat plants from the families of Spurge (*Euphorbia*) and Asarabacca (*Wild Ginger*) (*Caltha*). Talking about butterflies and moths *D. Plexippus* it was said a few words about toxic substances at the Spurge. The Asarabacca (*Caltha palustris* L.) is toxic as well. Some parts of this plant contain carotene which certainly causes pigmentation of the butterflies and moths. The presence of toxic substance in the Asarabacca is proven but its chemical structure is still unknown. Butterflies and moths are distinctly aposematically coloured. The wings are yellow - orange with marked black spots.

8. **Ammobiota festiva** Hufn, (fam. Arctiidae) The caterpillars of this species are aposematically coloured. Their coverings are very hairy with marked black patterns. They eat plants from the families *Euphorbia*, *Rumex*, *Artemisia* and *Alyssum*. It is already said that plants from the families *Euphorbia* and *Rumex* contain toxic substances. The Absinthe (*Wormwood*) (*Artemisia absinthium* L.) contains bitter glycosides (absentin and anabsintin) and ether oils of this plant are toxic because of the presence of tuion (*Jancic*

et al., 1995). Adult butterflies and moths of this kind are distinctly aposematically coloured as well. Their wings are bright red with black spots.

9. **Panaxia quadripunctaria** Poda, (fam. Arctiidae) The caterpillars are marked, with white stripes and hairs all over the body. They eat plants from the families Lonicera, Rubus, and Corylos. Lonicera xylosteum L. acts toxically through xylostein. Adult butterflies and moths are distinctly aposematically coloured. Their wings are red and black.

10. **Utetheisa pulchella** L., (fam. Arctiidae) The caterpillars are aposematically coloured; they are markedly white with red pattern and very hairy. They eat plants from the families Myosotis, Echium, Borago and Anchusa. Wide-known plant is the Blueweed (Echium vulgare L.). It contains alkaloids consolidin and cynoglosin and smaller quantities of saponin. The Anchusa officinalis L. contains alkaloids whose chemical structure is still unknown, but it is proven they act toxically. Adult butterflies and moths are markedly aposematically coloured. There are red and blue spots on the yellow basis of the wings.

11. **Thyria jacobaea** L., (fam. Arctiidae) The caterpillars are aposematically coloured alternately by yellow and black colour. They eat species from the families Senecio, Tussilago and Petasites. The Senecio (Nemorensis L.) vulgaris L. contains pyrolisidin alkaloids, isoditin and retrosin. It is important to be mentioned that isoditin does not metabolise in the isolated microsomes of the liver. Beside it is toxic, it is proven it is cancerogenic substance. The Colstfoot (Tussilago farfara L.) similary contains cancerogenic substance senkirkin which causes the liver sarcoma, hepatocellular adenomas and cancers at experimental animals. Beside that, Colstfoot contains a series of other harmful substances: bitter glycosides, saponins, carotenoids, organic acids, inulin etc. At last, the Burdock contains similar substances (Petasites hybridus L.) whose petasitein and fukinotoxine (a new cancerogenic pyrolisidin alkaloid) cause the sarcoma of liver at the experimental animals (Kojic and Janjic, 1991). The wings of butterflies and moths of this kind are distinctly aposematically coloured; they are bright red with brown design.

12. **Eulype hastata** L., (fam. Geomitridae) The moths from the family of Cankerworms are rarely aposematically coloured. The caterpillars are light grey with yellow strips. They eat breach (Betula Sp.) leaves. Breaches contain a lot of compounds in the ether oils: alpha-betulenol-acetate, cariophylen, homulen, cariophylenoxid, betulenol, pinen, kamphen, bornilacetate etc. The butterflies and moths of this kind are of marked colour with black-and-white aposematic design.

## DISCUSSION AND CONCLUSIONS

Further research the relation between the plants containing secondary metabolites and butterflies and moths certainly would discover new examples which would be essentially similar to hear presented examples. However, we are of the opinion that presented examples are enough for drawing global conclusions:

1. Some plants became able during the evolution to synthesise secondary metabolites. Those metabolites are not important in physiology but they play an important role in dissuasion of most of the herbivores which avoid eating those plants because of their unpleasant smell and taste.

2. The caterpillars of certain kinds of butterflies and moths adapted themselves during the evolution to eat such kinds of toxic plants. How they succeed to neutralise plant toxins is strange to us at this moment. But this question is interesting for two reasons. The caterpillars have complete sets of enzymes in their intestinal tracts and it is amazing how those toxic substances stay undigested. It is also unsolved question how those plant toxins stay undesintegrated during the metamorphosis of the caterpillars into adult butterflies and moths.

3. It is logical to be assumed that plant secondary metabolites with toxic effect take part in forming the aposematic colour of the caterpillars and adult butterflies and moths. Future research should explain the details of this process.

4. On the basis of the presented facts, bucked up with examples, we can undoubtedly conclude that toxic substances in the plants present material basis of the aposemy at butterflies and moths.

5. The apagogic proof in favor of this assertion is the fact that among numerous groups of butterflies and moths, whose caterpillars eat plants containing no toxic secondary metabolites, there are not aposematic kinds. Such are, for example, representatives of the families Satyridae, Hesperidae, Geometridae, Noctuidae etc.

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## REZIME

TOKSIČNE SUPSTANCE U BILJKAMA KAO MATERIJALNA BAZA APOSEMIJE KOD LEPTIRA (INSECTA, LEPIDOPTERA)

JAKŠIĆ P. I RISTIĆ Gordana

Razmatran je odnos između organskih hemijskih jedinjenja u biljkama koje su otrovne i kojima se hrane gusenice pojedinih vrsta leptira i izraženosti aposematičkih svojstava kod tih gusenica i leptira. Dokazano je da se hemijske supstance iz biljaka ne razgrađuju u potpunosti metaboličkim putevima u digestivnom sistemu gusenica, niti kroz intracelularni mehanizam varenja, već se nepromenjeno inkorporiraju u strukturu tela i krila. Tako ove toksične supstance predstavljaju materijalnu osnovu aposemije.

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# THE CRESTED NEWT (*Triturus cristatus* SUPERSPECIES) IN EX-YUGOSLAVIA: MORPHOLOGICAL STRUCTURING AND DISTRIBUTION PATTERNS

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## ABSTRACT

Morphological affinities of the crested newt (*Triturus cristatus* Artenkreis) species were reconsidered on the base of 42 population samples taken from the taxonomically most perplexing area of the Balkans. The species were much more differentiated in body size and shape, than in integument characters, such as coloration. In a morphospace determined by the first two canonical axes, *T. dobrogicus* populations were on one pole, *T. carnifex* and *T. karelinii* popula-

tions, indistinguishable morphometrically, were on a opposite pole, and *T. cristatus* populations took the intermediate position. It was supposed that the phenotype of massive and longlimbed *T. carnifex* and *T. karelinii* was ancestral to smaller, more slender and shortlimbed *T. cristatus* and especially to *T. dobrogicus* phenotypes. The most up-to-date distribution map of the species localities, as well as supposed species ranges, in former Yugoslavia were presented.

**Key words:** crested newt, morphology, distribution, ex-Yugoslavia

## INTRODUCTION

It has become a common place in the newt literature to consider the crested newt (*Triturus cristatus sensu lato*) an Artenkreis (superspecies) including four species (e.g. Wallis and Arntzen, 1989; Macgregor et al., 1990). The limitations of species distributions are clearly defined within broad territory of the Artenkreis range. However, there are many vaguenesses about species ranges on the Balkans where all four species came in contact, and where was presumably a center of the crested newt origin or, at least, an area where the basic lineage of its species was limited in the late Miocene (Crnobrnja-Isailović et al., in press). Lack of detailed molecular studies together with a long-standing question of the crested newt presence in a part of the Balkans region (see below), made the species delimitation as a difficult task. The analyses of samples from ex-Yugoslavia showed the absence of nuclear gene markers (Crnobrnja et al., 1989), substantial morphological variation among populations, including within species structuring (Kalezić et al., 1990), relatively low level of the discriminatory power of some meristic characters (number of trunk vertebrae; Arntzen, personal communication; Crnobrnja-Isailović et al., in press) and the evidence of hybridization (Wallis and Arntzen, 1989).

The previous morphometric study of the *Triturus cristatus* Artenkreis from the western and central parts of the Balkan peninsula revealed an apparent similarity of *T. carnifex* and *T. karelinii* phenotypes, the existence of western and eastern groups of populations within *T. carnifex*, and a phenetic affiliation of Istria

and, geographically remote, Montenegro population groups (Kalezić et al., 1990). We showed in this paper that additional population samples established different, presumably more realistic, phenetic relationships among populations and taxa, especially those from Istria and Montenegro. Also, analysis of a number of integumental qualitative traits, specified mainly by Wolterstorff (1923) for the crested newt taxa delimitation, was performed for the first time on such a large number of relatively numerous population samples. This analysis aimed to contribute to more appropriate crested newt taxa morphological descriptions, at least for the Balkans region. Another purpose of this paper is to present the first comprehensive survey of the crested newt distribution map in the former Yugoslavia based on more than 440 finding records. Finally, using data obtained by a mitochondrial DNA restriction site analysis (Wallis and Arntzen, 1989), and synthesis of data which came from morphological investigations given here, as well as from biogeographical considerations (Crnobrnja-Isailović et al., in press), we made another sketch of supposed species ranges for this part of the Balkans.

## MATERIAL AND METHODS

### Population studied

The analysed samples were in the text and in figures assigned as *T. cristatus*, *T. dobrogicus*, *T. carnifex* and *T. karelinii* according to published data about taxonomic recognition of certain populations

and about species ranges (Wallis and Arntzen, 1989; Kalezić et al, 1990; Džukić, 1993).

Morphological variation patterns among 42 newt population samples including only mature individuals collected during the breeding seasons were studied. Population sample locations are presented in Fig. 6., while locality names, their UTM codes (10 x 10 km squares), the number of females and males collected, are listed in Appendix. A total of 743 females and 653 males were subjected to analyses. Average number of females and males per sample was  $17.69 \pm 1.21$ , and  $15.57 \pm 2.40$ , respectively. Males from Peć locality were not accessible, so that only females were scored for traits studied. Specimens were preserved in 70% ethanol for varying periods of time before morphometric and qualitative characters were scored, and that could have made some scoring errors, but hopefully not systematic ones. (Specimens were deposited at the Georg Džukić's batrachological collection, Institute for Biological Research, Belgrade.)

#### Characters studied

The following morphometric characteristics were measured with digital calliper (to the nearest 0.01mm) for each individual: L-total length, Lcp-body length (from the snout to the front edge of the cloaca basis), Lcd-tail length (from the anterior edge of the cloaca basis to the tail tip), Lsv-snout-vent length (from the snout to the posterior edge of the cloaca basis), Ltc-head width, Lcl-head length (from the snout to the gular fold), Pa-forelimb length, Pp-hindlimb length, D-distance between fore and hind extremities. The length of the tail in damaged individuals was estimated as expected value from the regression of Lcd on Lsv.

Variation of the following qualitative traits (see Kalezić and Stevanović, 1980) was studied in all population samples except in a sample from Ravenica (No. 14): A. Texture of the skin - 1. finely granulated, 2. coarsely granulated; B. Colour of the upper surface - 1. brown, 2. brownish, 3. brownish with black spots; C. White stippling along flanks - 1. present, 2. absent; D. Throat colour pattern - 1. pale background with white spots, 2. pale background with dark spots; E. Belly yellow or vermillion with - 1. lateral dark spots and/or blotches, 2. lateral and medial dark spots and/or blotches, 3. lateral and medial dark spots and/or blotches fused, 4. irregularly dispersed dark spots and/or blotches.

#### Data analyses

Analysis of variance for unbalanced data revealed significant differences between sexes for most of the analyzed characteristics. To determine the amount of differences between the sexes and among populations considering external morphology, a multivariate analysis of variance was used. All test criteria (Hotelling's trace, Wilk's lambda and Pillai's trace) showed a significant variation between both the sexes and the populations.

Canonical variate analysis, which maximized variation between the groups, was used to characterize the degree of divergence among the populations (James and McCulloch, 1990). Canonical variates were calculated and centroids of each sample population were plotted on the first two canonical axes.

Individuals received a score on each qualitative character consisting of a discrete value for the trait state. These data were statistically analyzed using a correspondence analysis following the algorithm of Greenacre (1984). The input data for this analysis were the two-way contingency table of observed frequencies of character states in analyzed populations, while the output were the coordinates of the rows (populations) and columns (states of characters) correspondence axes superimposed on the scatter. It should be pointed out that distances between the points have no straightforward interpretation in multiple correspondence analysis (Greenacre and Hastie, 1987), so the obtained results allowed only the appreciation of the relationships among populations according to features of the correspondence axes estimated by the positions of column variable on the scatter. For statistics analyses a SAS package (SAS Institute, 1989) was used, considering  $P 0.05$  as a criterion for significance.

## RESULTS

Numerical results of canonical variate analysis revealed that the first two variates depicted accounted for 84% and 71% of the total dispersion in females and males, respectively (Table 1). The pattern of correlations of characters between population variability and canonical variates was sex-dependent, especially for all characters of the third variate, and for the distance between legs (D) of the first variate (Table 2). Canonical variate analysis showed that in both males and females, the *T. cristatus* was sandwiched, between *T. dobrogicus* and the group of *T. carnifex* and *T. karelinii* populations (Figs 2 and 3, respectively). The populations from the western group of *T. carnifex* (Slovenia and Croatia) are, more or less, closer to *T. cristatus* populations than the eastern group of *T. carnifex* populations. This is especially apparent for females, a gender which better discriminates the eastern *T. carnifex* and *T. karelinii* populations. Notably, *T. carnifex* from mountain regions of Montenegro are morphometrically the most similar to *T. karelinii* populations. The position of population sample from species intergradation zone (No. 16) in CAN1 and CAN2 space is in both genders heavily discriminated by a high positive CAN2 values, while having similar CAN1 values to the group of *T. carnifex* and *T. karelinii* populations (Figs. 2 and 3).

Correspondence analysis finds 12 dimensions in the correspondence table (Table 3).  $\chi^2$  of the first four singular values depicts for 72.81% and 76.89% in females and males, respectively. Qualitative phenotypi-

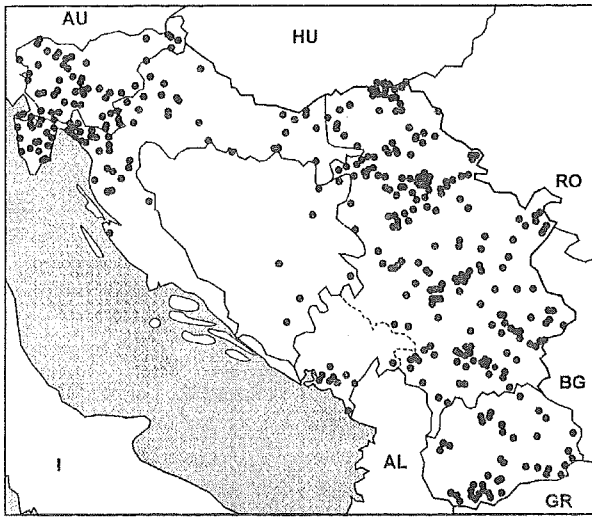


Figure 1. The distribution of the crested newt location sites in former Yugoslavia.

cal ("Gestalt") characteristics of the crested newt species, specified mainly by Wolterstorff (1923), apparently blurred on the Balkans if males are considered, including morphometrically the most distinctive species (*T. dobrogicus* and *T. cristatus*). In other words, males belong to gender with apparently more similarities in integument traits and coloration patterns, irrespective of species designations (Fig. 4). Only two populations drifted apart (*T. cristatus* from Mesić locality and *T. carnifex* from Istarske Toplice). For female population discrimination, the western group of *T. carnifex* (from Slovenia and Croatia) have the most negative DIM1 axis values and almost exclusively positive values of DIM2 axis values (Fig. 5). These populations are mainly characterized by frequent phenotypes of pale throat with white spots, brown colour of the upper body surface, and the belly with fused lateral and medial dark spots and/or blotches. *T. dobrogicus* populations form another relatively distinct group with the most frequent qualitative characteristics such as: finely granulated skin, brownish colour with black spots of the upper body surface, irregularly dispersed dark spots and/or blotches on the belly, and the absence of white stippling along flanks. *T. karelinii*, eastern *T. carnifex* and *T. cristatus* are not so distinctive by their qualitative characteristics. Only a few of mountain *T. carnifex* populations from Montenegro, and a population from Istria are quite different in phenotype frequencies probably due to drift effect.

Analysis of the body size (snout-vent length) species differences revealed that the group of the biggest newts of both sex were those of *T. karelinii* (females:  $75.45 \pm 0.79$ ; males:  $71.14 \pm 0.54$ ), and *T. carnifex* ( $76.40 \pm 0.35$ ;  $70.52 \pm 0.30$ ). Individuals from the other group of species were significantly smaller, as follows: *T. cristatus* ( $69.14 \pm 0.81$  and  $64.81 \pm 0.73$ ) and *T. dobrogicus* ( $66.44 \pm 0.66$  and  $65.29 \pm 0.56$ ). Only intergroup pairwise comparisons yielded statistically significant difference between species (Scheffe's mul-

tiply t-test). Interestingly, newts from the single population situated in a genetically confirmed interspecific transition zone (mtDNA analysis, *T. dobrogicus* and *T. karelinii*; Wallis and Arntzen, 1989), Trešnja locality, (No. 16) exceeded considerably all groups: the mean-standard error of Lsv were  $88.95 \pm 0.83$  and  $87.18 \pm 1.86$  for females and males, respectively. To the best of our knowledge, the biggest crested newt specimen ever recorded came from this population, a female with 221.2mm of the total length (Franić, unpublished data), which considerably exceeded thus far known record of the greatest total length in the crested newts of 209mm (Lanza and Campolmi, 1991).

We summarized all up-to-date known crested newt localities from former Yugoslavia, and presented the distribution map using approximately 440 location site records (Fig. 1). (For the sake of simplicity spatially adjacent sites were omitted in the map.) To provide only certain findings cites, data came from articles published in scientific and professional journals, from museum collections and its catalogues, unpublished records of our colleagues (names given in Acknowledgements) and our own data gathered during field investigations. Several localities, especially ones from which crested newt presence were recorded long ago, were destroyed by certain or there is a reasonable doubt of the newt existence at present. However, the main mystery of the crested newt presence in ex-Yugoslavia remains the area of western and central Bosnia, western Herzegovina, and the central part of Dalmatian coast from which records, older or new ones, do not exist. In spite of extensive field work in last fifteen years, we did not detect the presence of the newt in this area in spite of a number of appropriate habitats. At this moment, we do not have a reasonable explanation for the "gap", except to evoke for help some unknown historical factor(s). Also, some older records from Bosnia (Koševo lake, Sarajevo; Zelengora Mount; Baba Mount; Bolkey, 1924, 1928, 1929) had not been confirmed till now. Nevertheless, due to these records the eastern *T. carnifex* range was expanded to the west (see Figures 1 and 6). The map of supposed species ranges (Fig. 6) appeared to be close, especially for the most problematic area (Serbia), to that presented by Macgregor et al. (1990)

## DISCUSSION

The propensity for natural interspecific hybridization within the crested newt *Artenkreis* has been known for a long time and was based mostly on morphological studies. Traditionally, hybrids have been identified by use of morphology on the assumption that hybrids are intermediate to parental species. However, in genetically identified hybrids of some anuran species morphological analysis revealed an extreme range of phenotypic expression within F1 individuals and backcrosses groups (Lamb and Avise,

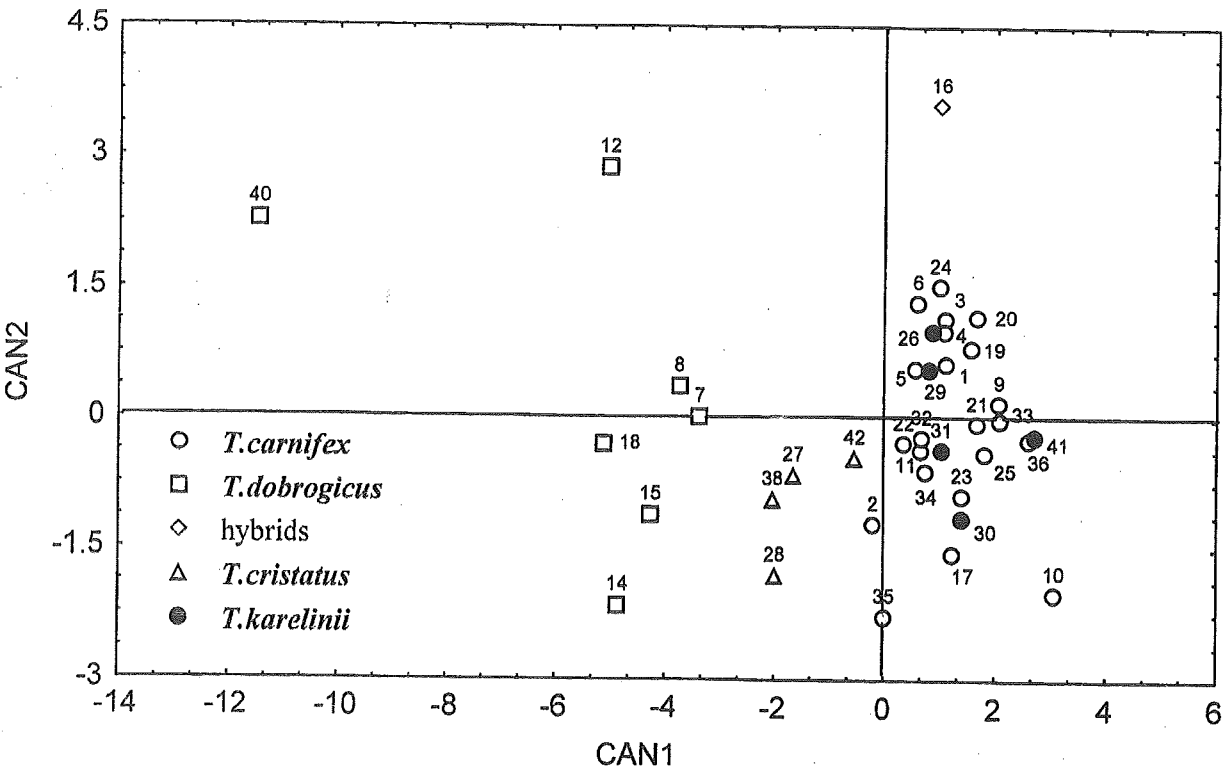


Figure 2. Plot of population centroids for males on the first and second canonical axes (CAN). Appurtenance of a particular population to following appropriate species is emphasized by symbols explained on the Figure.

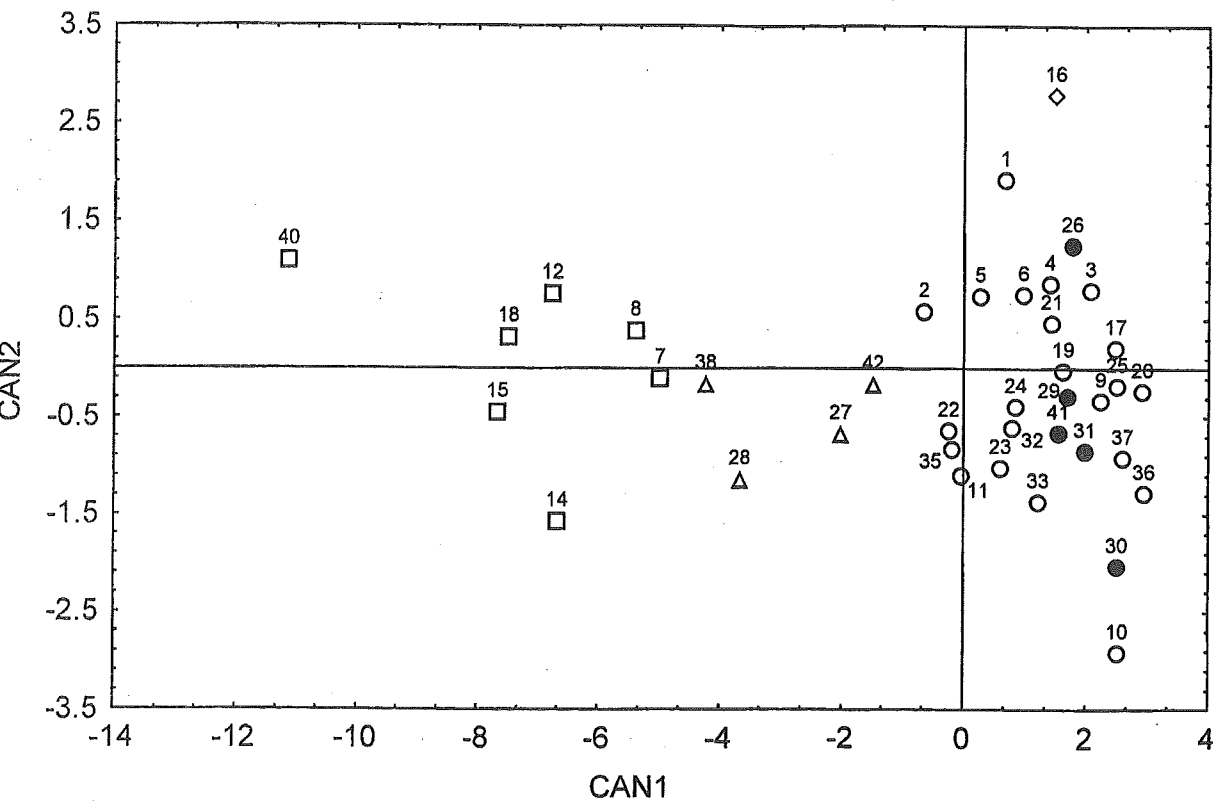


Figure 3. Plot of population centroids for females on the first and second canonical axes (CAN). Symbols of appurtenance of particular populations to appropriate species are as on Figure 2.



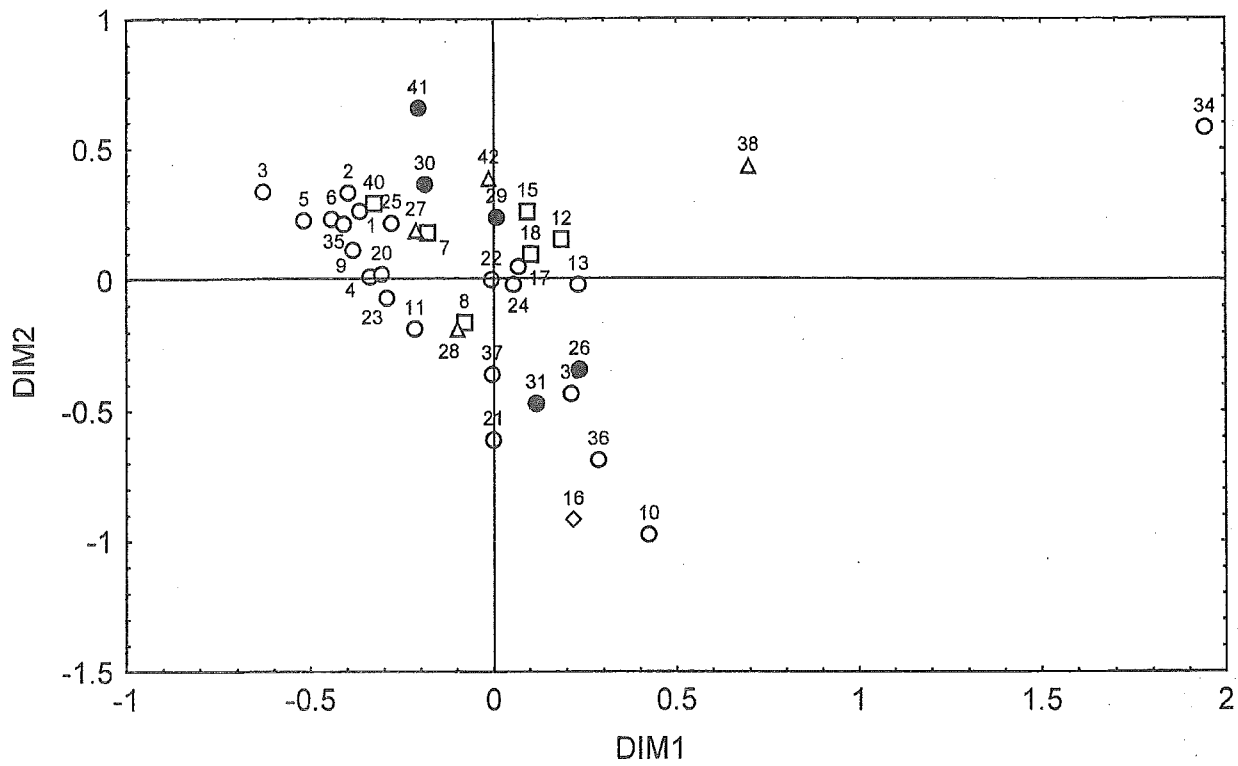


Figure 4. Plot of populations for males in the space of first and second correspondence axes (DIM). Symbols of appurtenance of particular populations to appropriate species are as on Figure 2.

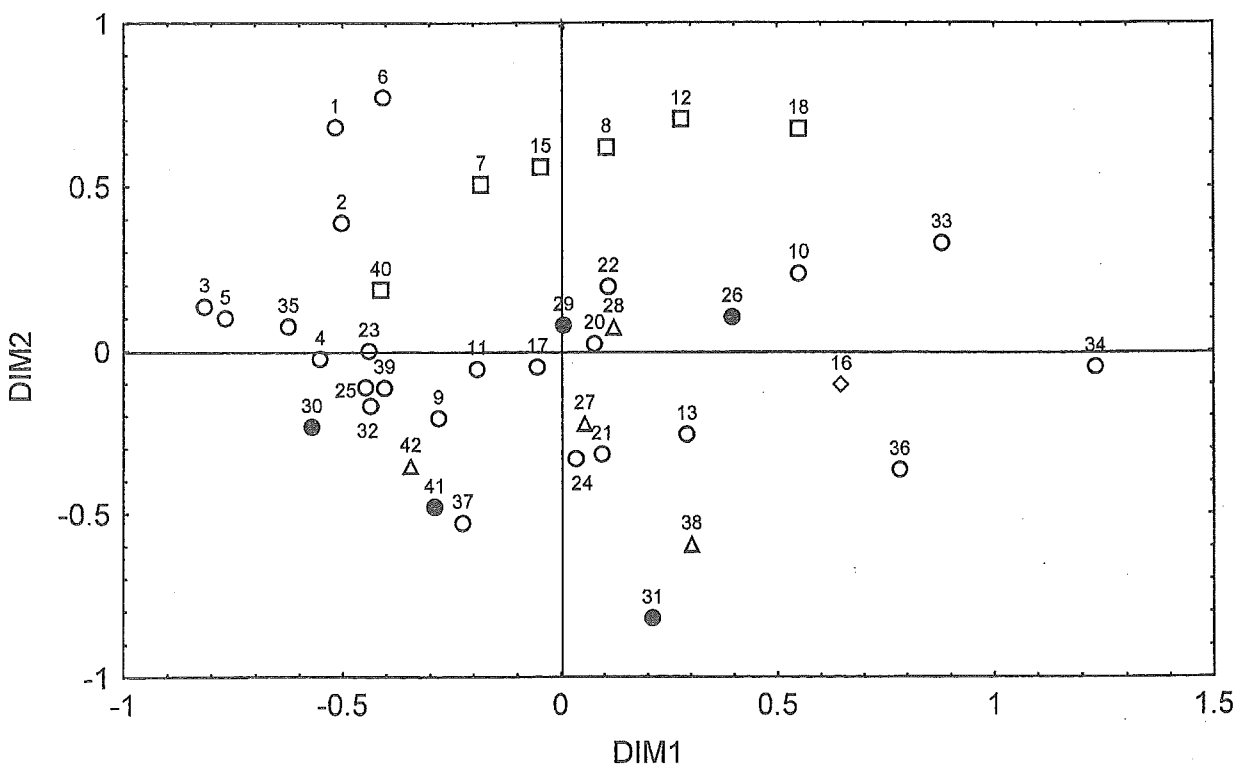


Figure 5. Plot of populations for females in the space of first and second correspondence axes (DIM). Symbols of appurtenance of particular populations to appropriate species are as on Figure 2.

1987). It seems to be the case in only one non-morphologically-confirmed population from interspecific transition zone studied here. At Trešnja lake, near Belgrade (No. 16), *T. dobrogicus* mtDNA was found in some of the specimens, all of which have *T. karelinii* phenotype (Wallis and Arntzen, 1989). Individuals of this population stand apart considerably from intermediacy of parental species in morphological space. Thus far genetically established interspecific transition zones between crested newt species unexceptionally appeared relatively narrow confirming their parapatric status (Wallis and Arntzen, 1989). Thus, in the case of crested newt species intergrading, based on morphologically intermediates, should be confirmed by genetical analysis as recently shown by Litvinchuk et al. (1994). Morphological intermediacy in the crested newt is not necessarily the consequence of species interbreeding (hybridization), especially when backcrosses are involved.

From a morphological point of view, *T. carnifex* and *T. karelinii* are clearly the most related crested newt species. They share basically the same phenotype, stout body with big head and elongate limbs, which could be very close to the ancestral crested newt phenotype. From such a phenotype differentiated, as we proposed here, smaller and differently shaped *T. cristatus* and especially *T. dobrogicus* individuals in the course of their range extensions, especially after Pleistocene glaciations (see Crnobrnja-Isailović et al., in press). Such changes in size and shape (trunk elongation with concomitant limbs length and head reduction) could emerge as a adaptation to a basically lowland plains of central, western and eastern Europe. These trends appear to be the most prominent in *T. dobrogicus* species which spreaded throughout the Pannonian and Dobrogean areas and become adapted to the most aquatic life in extensive swamps and marches of the floodplains of the river Danube, Sava and its tributaries (Fig. 6; see also Arntzen et al., 1997). Wallis and Arntzen (1989) invoked for those crested newt species high gene flow homogenizing mitochondrial genomes. Thus, in the case of crested newt species, low land and high land areas, and transition zones between them, are areas that might generate morphological patterns in response to a complex array of physical and biological gradients. In other words, the shape of organism represents an additional aspect of morphology that may influence the pattern of morphological variation along vertical gradients.

Morphological relationships between crested newt species are not congruent to their phylogenetic affinities based on allozymes, reproductive interactions, mitochondrial DNA sequencing data, cytogenetic parameters, immunological data (summarized in evolutionary tree of the genus *Triturus* spp.; Macgregor et al., 1990) where *T. cristatus* and *T. carnifex* make one, and *T. dobrogicus* and *T. karelinii* another cluster. Apparently, morphological and other aspects of the

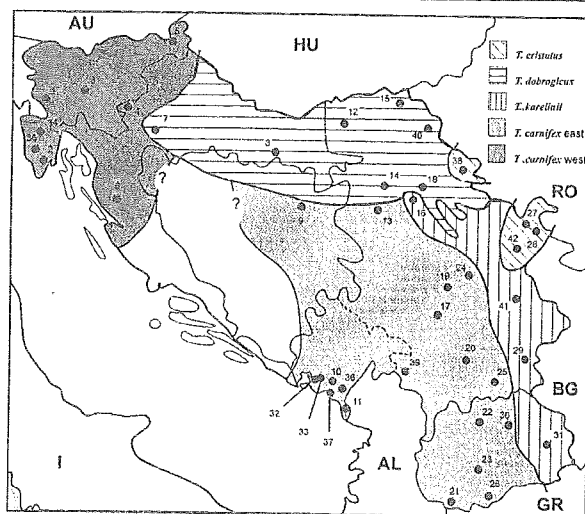


Figure 6. Distribution of the sampling sites of *Triturus cristatus* Artenkreis from Balkan peninsula included into analysis. The range of the species as explained in the figure are indicated by different patterns.

crested newt evolution did not run side by side, i.e. their were decoupled.

Structuring within *T. carnifex* on the Balkan peninsula observed in earlier studies (Kalezić and Stevanović, 1980; Wallis and Arntzen, 1989; Kalezić et al., 1990; Džukić, 1993), suggested the existence of distinguishable groups within this taxon. The results of the present study confirmed this statement. The western group of populations (from Slovenia and Croatia) appeared different in respect to morphometry and especially of qualitative characteristics in females in comparison to eastern populations (eastern Bosnia, Montenegro, western Macedonia, western and central parts of Serbia). This morphological differentiation seems to be the consequence of their spatial isolation, probably not a recent one. Namely, western and eastern groups are separated by the *T. dobrogicus* range and the considerable area devoided of the crested newts (Figs. 1 and 6). For a moment, in the absence of additional data, this divergence could be interpreted in terms of subspecific recognition. Karaman (1922) first noticed a morphological distinctness of the crested newts from western Macedonia (*terra typica* Ohrid) and named it *Molge karelinii macedonica*. According to that, the eastern form should be named *T. carnifex macedonica*, while the western form should be considered a nominotypical one (*T. carnifex carnifex*). After Karaman, the eastern form was named *T. carnifex albanicus* (Dely, 1959) and *T. carnifex kemzae*, as a *nomen illegitimum*, (Džukić, 1993).

The Balkan crested newt species harboured an appreciably high level of phenotypic variation to such an extent that morphological population affinities appeared to be sensitive to the number of population samples examined. Thus, in our previous study (Kalezić et al., 1990), which included only a few samples from Istria and Montenegro, their morphometric closeness to affiliation was proposed. However, when the

samples examined from these regions increased, populations from Istria appeared to belong to the western *T. carnifex* group and populations from Montenegro to the eastern conspecific group. Also, as stated in Results, mountainous populations from Montenegro showed morphometrical similarity to *T. karelinii* populations.

#### ACKNOWLEDGMENTS

Pim Arntzen provided pertinent suggestions of which some were incorporated in this paper, but the opinions expressed here, however, remain our own. He also shared with us his site record from Višegrad. Authors are grateful to Irena Rot for laboratory assistance. A number of our colleagues generously made access to us to mainly unpublished crested newt location sites. Concerning this, our thanks are due to: Slovenia (Boris Kryštufek, Savo Breljih, Katja Poboljšaj), Croatia (Nikola Tvrtković, Edo Kletečki), Serbia (Željko Tomanović, Nikola Pantelić, Imre Krizmanić, Ištvan Ham, Mihailo Stanković), and Macedonia (Trajan i Svetozar Petkovski, Stevan Petrušev).

#### APPENDIX

Location of populations, their altitudes, UTM code, the number of females and males collected. 1. Goče - Štanjel (Slovenia, 250m above sea level, VL17, 18 females + 13 males); 2. Salakovci (Croatia, 320m, VK38, 15+20); 3. Podstrmec (Slovenia, 560m, VL67, 19+26); 4. Budinjak - Žumberak (Croatia, 770m, WL26, 20+14); 5. Turjanci (Slovenia, 195m, WM86, 13+11); 6. Lički Osik (Croatia, 570m, WK33, 9+10); 7. Orle (Croatia, 100m, WL96, 15+16); 8. Ciglanske bare - Slavonski Brod (Croatia, 100m, BR70, 10+14); 9. Dubrave (BiH, 230m, CQ12, 17+10); 10. Rid - Dobro Selo (Montenegro, 250m, CM39, 18+15); 11. Donji Štoj (Montenegro, 0m, CM54, 13+15); 12. Svetozar Miletić (Serbia, 100m, CR67, 16+10); 13. Bukovac - Valjevo (Serbia, 400m, DP19, 13+13); 14. Ravenica - Obedska bara (Serbia, 80m, DQ15/25, 17+14); 15. Novi Kneževac (Serbia, 80m, DR39/DS30, 14+10); 16. Trešnja (Serbia, 285m, DQ63, 56+13); 17. Radošiće (Serbia, 600m, DN89, 25+11); 18. Ivanovo (Serbia, 75m, DQ75, 20+31); 19. Rataje - Aleksandrovačka Župa (Serbia, 260m, EP11, 24+33); 20. Novo Brdo (Serbia, 800m, EN31, 20+20); 21. Kostobačilo - Mount Galičica (Macedonia, 1550m, DL84, 32+30); 22. Petrovec - Skoplje (Macedonia, 225m, EM54, 9+13); 23. Prilep (Macedonia, 640m, EL47, 11+16); 24. Rtanj (Serbia, 620m, EP74, 19+16); 25. Sveti Ilija - Vranje (Serbia, 1120m, EN61, 11+21); 26. Visoka Čuka - Mount Kožuf (Macedonia, 1200m, FL06, 22+16); 27. Crepana - Štubik (Serbia, 210m, FQ00, 15+14); 28. Negotin (Serbia, 35m, FP29, 16+13); 29. Stojkovića mahala - Vlasinsko jezero (Serbia, 1340m, FN12/13, 17+17); 30. Lesnovo (Macedonia, 940m, FM05, 13+11); 31. Smojmirovo - Berovo (Macedonia,

810m, FM52, 13+9); 32. Jezero - Lovćen (Montenegro, 1355m, CM29, 19+13); 33. Bjeloši - Lovćen (Montenegro, 850m, CN29, 21+14); 34. Istarske Toplice (Croatia, 20m, VL12, 17+26); 35. Svetvinčenat (Croatia, 300m, VK19, 16+20); 36. Donji Lokanj - Brežine (Montenegro, 210m, CM49, 21+19); 37. Pijavica - Gornji Ceklin (Montenegro, 250m, CM39, 29+20); 38. Mesić (Serbia, 175m, EQ39, 15+12); 39. Karagač - Peć (Serbia, 525m, DN42, 14+0); 40. Radojevo (Serbia, 80m, DR86, 10+10); 41. Vrtovac (Serbia, 400m, FP10, 16+5); 42. Manastirište - Zlot (Serbia, 260m, EP77, 15+20).

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## REZIME

VELIKI MRMO LJAK (*Triturus cristatus* SUPER-SPECIES) U BIVŠOJ JUGOSLAVIJI: MORFOLOŠKA STRUKTURIRANOST I DISTRIBUCIJA

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Oblast Balkanskog poluostrva koja je pripadala bivšoj Jugoslaviji je teritorija gde se sučeljavaju sve četiri vrste velikog mrmoljka (*Triturus cristatus* supervrsta) koje međusobno hibridiziraju i imaju parapatričku distribuciju. To je deo arela velikog mrmoljka sa mnogim nerešenim problemima u pogledu razgraničenja vrsta, posebno na osnovu morfoloških odlika, kao i u odnosu areala vrsta. U ovom radu istraživane su morfometrijske i kvalitativne karakteristike jedinki 42 populacije ovog mrmoljka. Utvrđena je velika morfološka sličnost vrsta *Triturus carnifex* i *T. karelinii*, koje su se u odnosu na druge dve vrste (*T. dobrogicus* i *T. cristatus*) diferencirale najvećim delom u obliku i veličini tela, a u manjoj meri u karakteristikama integumenta (pre svega obojenosti). Pretpostavlja se da je fenotip prve dve vrste predački u odnosu na fenotip druge dve vrste. Utvrđena je fragmentisanost areala vrste *T. carnifex* i stepen morfoloških razlika između zapadnih i istočnih populacija, koji sugerise postojanje dve podvrste ove vrste. Prikazana je distribucija do sada poznatih lokaliteta velikog mrmoljka na teritoriji bivše Jugoslavije, kao i mogući areali vrsta.

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Table 1. Results of canonical discriminant analysis for both sexes of the crested newt samples. R - canonical correlations; SE - standard error of canonical correlations;  $\lambda$  - eigenvalues; Cum - cumulative proportion of eigenvalues.

	males				females			
	R	SE	$\lambda$	Cum	R	SE	$\lambda$	Cum
1	0.936	0.005	7.062	0.58	0.958	0.003	11.059	0.74
2	0.773	0.016	1.489	0.71	0.779	0.014	1.541	0.84
3	0.742	0.018	1.222	0.81	0.650	0.021	0.733	0.89
4	0.704	0.020	0.983	0.89	0.598	0.023	0.557	0.93
5	0.612	0.024	0.599	0.94	0.532	0.026	0.395	0.96
6	0.498	0.029	0.330	0.96	0.446	0.029	0.249	0.97
7	0.426	0.032	0.221	0.98	0.393	0.031	0.182	0.99
8	0.322	0.035	0.116	0.99	0.319	0.033	0.113	0.99
9	0.277	0.036	0.083	1.00	0.248	0.034	0.065	1.00

Table 2. Correlations of the measured traits with the between population variability of analysed traits on the first three canonical axes (CAN) for both sexes in the crested newt samples.

Char	males			females		
	CAN1	CAN2	CAN3	CAN1	CAN2	CAN3
L	0.361	0.645	-0.161	0.725	0.462	0.070
Lcp	0.375	0.740	-0.448	0.641	0.592	0.045
Lcd	0.424	0.814	-0.065	0.770	0.324	0.077
Lsv	0.405	0.837	-0.300	0.665	0.559	0.055
Ltc	0.927	0.336	-0.052	0.959	0.153	0.129
Lcl	0.873	0.285	-0.077	0.932	0.122	0.173
Pa	0.757	0.535	-0.347	0.913	0.363	-0.101
Pp	0.682	0.615	-0.359	0.898	0.386	-0.047
D	-0.077	0.908	-0.095	0.240	0.867	0.213

Table 3. Results of singular value decomposition of the phenotype frequencies contingency table in analysed populations of the crested newt samples. Dim - some of extracted dimensions with a nonzero singular value;  $\lambda$  - eigenvalues (principal inertias);  $\chi^2$  - Chi-square of extracted dimensions and % - percent of Chi-square contribution of extracted dimension to the total chi-square.

Dim	males			females		
	$\lambda$	$\chi^2$	%	$\lambda$	$\chi^2$	%
1	0.237	711.28	30.46	0.224	743.35	28.10
2	0.144	431.95	18.50	0.135	445.81	16.85
3	0.129	387.12	16.58	0.120	397.73	15.04
4	0.088	264.97	11.35	0.102	339.21	12.82
5	0.057	169.74	7.27	0.069	227.51	8.60
6	0.044	130.83	5.60	0.057	188.06	7.11
7	0.036	109.18	4.68	0.046	152.50	5.77
8	0.023	68.06	2.91	0.026	87.64	3.31
9	0.021	61.83	2.65	0.019	63.39	2.40



# LAKES IN THE KALUDERSKA REKA RIVER DRAINAGE AREA ON SAR-PLANINA MOUNTAIN

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## ABSTRACT

Shown results are obtained on the basis of research done into the Livadicko jezero Lake and Malo (Little) Livadicko jezero Lake during August of 1974 and July of 1975 and 1985. Both of the lakes are situated in the cirque of the Kaluderska reka River source on the NW part of Sar-planina Mt., Serbia, Yugoslavia. Basin genesis of both lakes, their location and their morphometrical data were established during field investigations; the coastal line of the Livadicko jezero Lake was

also mapped than. On the basis of mentioned data and barimetric measurement, an isobathic map of the Livadicko jezero Lake was drawn and its along profile is graphically shown. Isobathic map is used for calculating quantitative indexes that are important for knowledge of the lake and its basin. The temperature and optical characteristics of the water of both lakes are established as well as genetic and hydrographical classification of both lakes.

**Key words:** Livadičko jezero Lake, lake genesis, lake classification, Sar-planina Mt.

## INTRODUCTION

Sar-planina Mt. covers 1602 km<sup>2</sup> and extends along SW Serbia and NW Macedonia and it is surrounded by big tectonic entities - Kosovska kotlina ravine in the north, Metohijsko-prizrenska kotlina ravine in the northwest, Poloska kotlina ravine in the southeast and Mavrovska kotlina ravine in the south.

The Livadicko jezero Lake and Malo Livadicko jezero Lake are situated on the NW side of Sar-planina Mt., in the Kaluderska reka River basin.

### History of research

The Livadicko jezero lake is mentioned by several travel authors as lake "at the top of the peak Ljuboten", "near Ljuboten" or "quarter-hour walk from Ljuboten". The authors wrote what they heard from their guides or from the inhabitants (M. Makenzi and P. Irbi, 1868; Sava archimandrite, 1888; T. Stankovic, 1910).

Jovan Cvijic was the first our scientist who pointed to this lake calling it "Sarsko jezero Lake". He observed this lake from Ljuboten in 1890. He noticed some cirques as well, so he put forward a hypothesis about the old glaciation of Sar-planina Mt. in a paper published in 1891 in Vienna (J. Cvijic, 1891). After that finding Jovan Cvijic devoted himself to research of glacial relief, now fossilised, of high mountains of the Balkan Peninsula. He found numerous, diverse traces of old glacial relief on Rila Mountain in Bulgaria in 1896 (J. Cvijic, 1897) as well as on the mountains of Bosnia, Herzegovina, Montenegro and Macedonia during the following years (J. Cvijic, 1903). All of that helped him to prove the truth of his visionary hypothesis about glacial traces in the Balkan Peninsula relief offering numerous evidence. This caused a sensation in European scientific circles and it was seen as an epoch-mak-

ing discovery since the general opinion of the scientists of that time was that the Balkan Peninsula wasn't exposed to the Pleistocene glaciation.

K. Oestreich mentioned the Livadicko jezero Lake in 1899 in a paper about Vilayet Kosovo (K. Oestreich, 1899). In 1902 V. K. Petkovic observed the cirque where the lake, considered by J. Cvijic as glacial one, is situated. He stated that the cirque was built up in filitic shists emphasising that the lake is situated in such rock, too (V. K. Petkovic, 1903).

Studying the glacial relief on Sar-planina Mt. in 1911 R.T. Nikolic observed the cirque "where the Demir-kapijsko jezero Lake or Livadicko jezero Lake is situated". He concluded that the Livadicko jezero Lake is in a glacial cirque "that is especially distinctive and extended toward the NW, with high scarps and cirque saddle that connects it with the next cirque on the SW. Nikolic noticed that the lake is recharged by snow-packs and a tributary and he roughly judged the lake was 130m long and 60-70m wide. He wrote that the lake was dammed by 2m high moraine ramparts and that it wasn't very deep (R.T. Nikolic, 1912, 1912a).

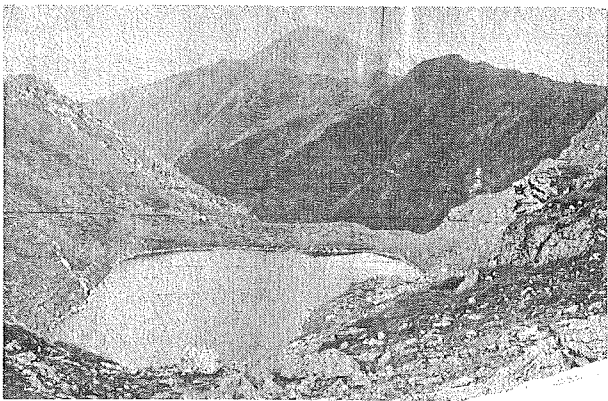
B.Z. Milojevic says that the Livadicko jezero Lake is in a glacial cirque, deepened into solid shists, that its NE side is full of scarps and detritus while the NW side is gentler. He adds: "there is a valley behind the low rung that fences in the lake; it firstly extends to the NE but than it bends to the N and NW". The valley is very wide, its slopes are steep and it can be said it is with glacial profile. The bottom of the valley is steep and there are some scarps. Milojevic further says "there is a brook that recharges the lake running from the SW" and "when snow melts the water level rises for 1.5m and a surface arm of the lake occurs as a result" but

there is a subterranean arm of the lake as well and it comes to the surface on NE slope". He doesn't present any of morphometrical data of the lake (B.Z. Milojevic, 1937).

D.S. Krivokapic researched the Livadicko jezero Lake in 1953. He described the location of the lake and found it was 228m long and 120m wide. He found as well it was 7.3m deep considering its cross profile and its area covered 20740m<sup>2</sup>. He presented some facts about "small clear lake" in a depression situated on the plateau where the arm of the Livadicko jezero Lake used to run (D.S. Krivokapic, 1959).

Method of the work

Using some methods established in the theory of lakes we measured length (L), maximal breadth (B<sub>max</sub>) and length of the coastal line of the Livadicko jezero



The Livadicko jezero Lake (2200m a.s.l.), view from the massif Livadice (2497m a.s.l.)

The massif Tumba (2346m a.s.l.) is on the left, the slope Pod (2231m a.s.l.) in the middle, on the right is the slope Krivi Rid Marsh with the peak Kula (2324m a.s.l.) on the main ridge of Sar-planina Mt.

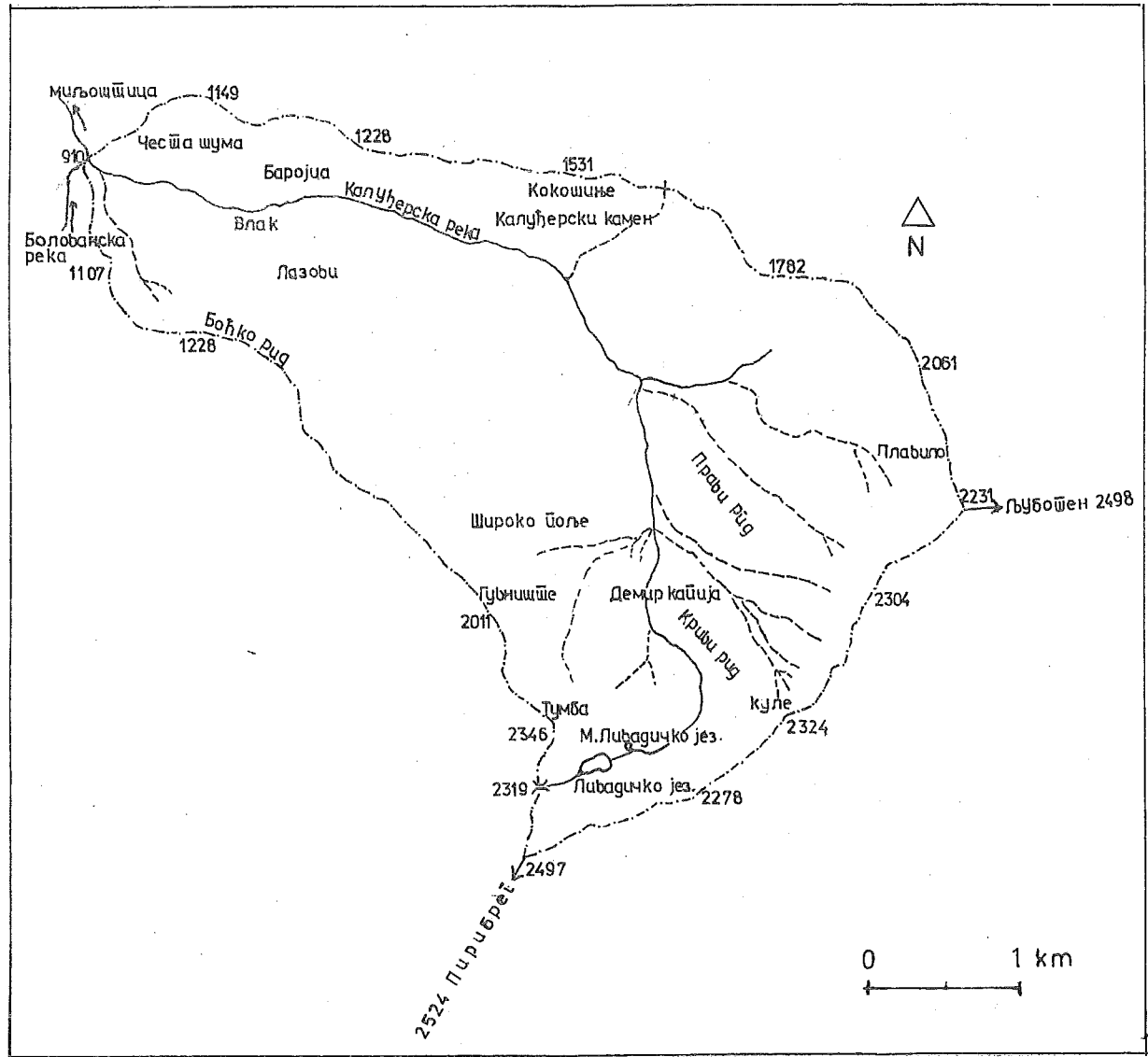


Figure 1. The Kaluderska reka River drainage area on the NW part of Sar-planina Mt.



Lake and Malo Livadicko jezero Lake. We did that with the aid of a nonelastic rope marked by aluminium bars with numbers of meters on them. After we measured radius vectors between one static point and chosen points on the coastal line and found their azimuths by compass, we were able to draw on the map the coastal line of the Livadicko jezero Lake. Radius vectors we measured were in the plane parallel to the surface of the lake. We corrected obtained plan according to the natural contour of the lake. The depth of the lake was measured by the rope with plumb at the end. We used a two-person rubber boat moving along the stretched rope and dropping the plumb at each 5-meter distance. On the basis of those data we drew up an isobathic map of the Livadicko jezero Lake and its along profile (A-B). We used that map to calculate the volume of water mass in the basin and water volume under and between certain isobaths. We drew up a consumption curve and a hypsographic curve of the lake, too. The temperature of the water was measured by a limnological thermometer whose each degree centigrade was divided into five parts. A mercury thermometer scaled in degrees centigrade measured the air temperature. Using the Seki's disc the water transparency was established.

## RESULTS

### Location of the drainage basin and the studied lakes

The Kaluderska reka River is the right tributary of the Miljostica River that flows into the River Lepenac; the last one flows into the River Vardar and belongs to the Aegean Sea basin. The Kaluderska reka River drainage basin is created on the furthest part of NW side of Sar-planina Mt. The SE watershed, 4 km long, extends along the main ridge of Sar-planina Mt., from the peak Livadice (2497m above sea level) to the peak on the slope Pod (2231m a.s.l.), located in the W going from the peak Ljuboten (2498m a.s.l.). The NE and N watershed is represented by the slopes that incline to the village Strpce in the bottom of Sirinicka korutina valley. This watershed is 7.5km long. The W and SW sides are formed by the watershed between the Kaluderska reka River and the Bolovanska reka River, the left tributary of the Miljostica River, from the peak Livadice to the juncture of the Kaluderska reka River and the Bolovanska reka River (at 910m a.s.l.). The watershed is 6.5 km long. (MAP 1.)

The Kaluderska reka River is 7.5km long and its drainage basin covers 15.5 km<sup>2</sup>.

Geological material of the drainage basin highest parts is composed of the Palaeozoic metamorphits - albit-chlorite sericiteous shists - while in the lower parts it can be found: Triassic marbly limestone, serpentinite, harzburgite, dunite, quartz-micaceous shists (M. Koscal, 1990).

The examined lakes are situated in the Kaluderska reka River source. The Livadicko jezero Lake is situated at 21° 04' 43" of eastern longitude and 42° 11' 27" of northern latitude. The Malo Livadicko jezero Lake is situated at 21° 04' 53" of eastern longitude and 42° 11' 30" of northern latitude.

One can get to the lakes going along the footpath that begins at the village of Strpce. The footpath mostly extends along the top of the slope that represents a watershed between the Kaluderska reka River and Bolovanska reka River, further till the end of the Bockov rid marsh where it turns toward Siroko polje and Demir kapija below the Krivi rid marsh. From that point the path extends along the valley of the Kaluderska reka River. The path is 13km long.

The lakes can be reached via the winter-sport centre "Stojkova kuca" (1750m a.s.l.) using the asphalt road from Brezovica. The footpath from this centre goes through the Crvena karpa rock, Mance pass (2317m a.s.l.) and Siljegarnik to the cirque where the studied lake is. The path is 3.5km long.

### Genesis of the lakes basins

The basins of the Livadicko jezero Lake and Malo Livadicko jezero Lake are in the spacious Pleistocene cirque in the source of the Kaluderska reka River, expanded to the NE. The cirque is created between the main ridge of Sar-planina Mt. and the slope Tumba that spreads from the Livadice peak (2497m a.s.l.) on the main ridge to the peak elevation 2346 on the NNE where it is divided into two forks - the shorter one spreads to the NE while the longer one to the NW. Considering other cirques it can be said this cirque is specially located because the main ridge of the mountain represents its right side. The sides of the cirque are high and steep with sharp and rocky scarps. The glacier of this cirque and glacier of the cirque Siljegarnik, in the Bolovanska reka River source, have built a saddle 2319m high. The cirque bottom is at 2200m a.s.l. The glacier has deposited two moraine ramparts at the end of the cirque; one over the rocky rung that fences in the Livadicko jezero Lake and the other behind the scarp. Behind the second rampart there is a valley with steep slopes. It spreads firstly toward the NE and turns, behind the next scarp, to the N and NW. Two scarps more can be seen looking at the along profile of this valley. One of them is low while the other one, near Demir kapija and Copur, is 100m high and positioned at 1750m a.s.l. The rocks in the valley are plated in some parts while the bottom, up to the scarp near Demir kapija is covered by moraines. Most of them are created during some kricional processes. (Figure 2 and Figure 3)

The glacier of the Kaluderska reka River had a tributary so it belongs to the group of composed valley glaciers. The glacier that came from the cirque in the source of the Pravovidski potok spring flowed to the glacier of the Kaluderska reka River, below the scarp

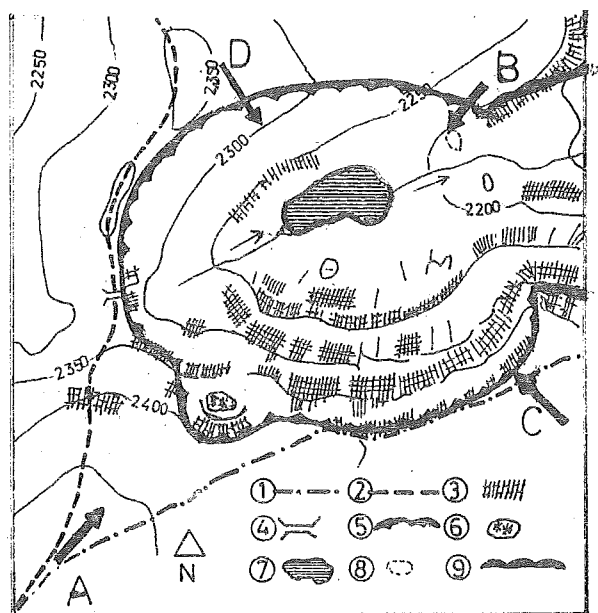


Figure 2. Cirque of the Livadicko jezero Lake 1. Main ridge of Sar-planina Mt. 2. Ridge - watershed of the Kaluderska reka River drainage area 3. Scraps 4. Cirque pass 5. Glacial cirque 6. Cirque with snow-pack 7. The Livadicko jezero Lake 8. Periodical Malo Livadicko jezero Lake 9. Valley with glacier profile

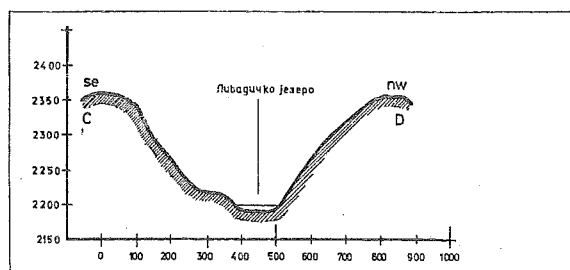
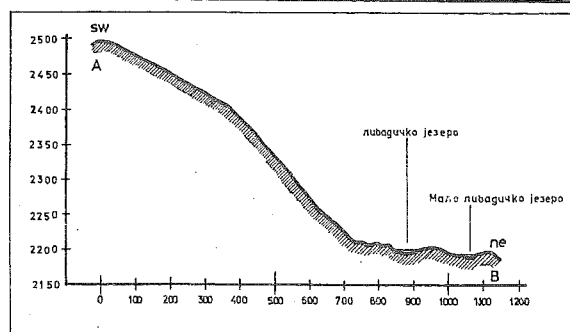


Figure 3. Two profiles of the cirque in the Kaluderska reka River source - along profile (A-B) and cross profile (C-D)

Copur near Demir kapija. There is another cirque in the northwest. The cirque has been positioned toward the north and has been hollowed out near the slope Tumba in the south going from Demir kapija. It has been stated that there are series of glacial forms built up by erosive and accumulative effect of glaciers in the high parts of this part of Sar-planina Mt. At the place where the river Miljostica flows into the Lepenac there are four glaciofluvial terraces discovered by J. Cvijic in 1900.

The genesis of the Livadicko jezero Lake basin has been greatly helped by accumulative effect of the glacier. The glacier has withdrawn to the higher parts and caused two moraine ramparts to be deposited.

The crucial process in genesis of the Malo Livadicko jezero Lake basin was the nivational process. The basin is at north side of the plateau between two moraine ramparts which have been almost parallelly deposited. Large moraine blocks have formed the plateau after glacier had melted. It was ideal place for snow to be amassed.

A surface covered by snow is very well conserved while the snow-packs, scattered through the mountain, damage it and get deepen into it. The surface of this part has been exposed to the snow-pack erosion, i.e. aggressive snow water has chemically dissolved it and frost caused the moraine layer to be decomposed. The water of melted snow-pack took dissolved and decomposed material to the depth of the moraine during the hot days so gaps between debris and large moraine blocks were filled and sealed. Lengthy effect of the recent nivation has created a depression and water of melted snow-pack has made a lake in it.

Gradually, after a watertight layer was made and by enlarging the capacity of the snow-pack, melted snow has created a shallow arm of the Malo Livadicko jezero Lake. It flows into the arm of the Livadicko jezero Lake and meandering and flooding out of shallow riverbed recharges, from time to time, the little lake.

The Livadicko jezero Lake and Malo Livadicko jezero Lake are in the zone of alpine climate conditions with fresh summers and cold winters. The air temperature was measured in Tetovo (462m a.s.l.) and on Popova Sapka (1750m a.s.l.), on the NE side of Sar-planina Mt. On the basis of that we counted that the average year air temperature at 2200m a.s.l. is  $2.9^{\circ}\text{C}$ . We found as well that average year air temperature is below  $0^{\circ}\text{C}$  during four months, it is around  $0^{\circ}\text{C}$  during two months and during the rest of the year it is between  $4.8^{\circ}\text{C}$  and  $12.6^{\circ}\text{C}$ . The average year air temperature is  $1.5^{\circ}\text{C}$  in the highest part of the cirque; it is below  $0^{\circ}\text{C}$  during six months and during another six months ranges from  $3.5^{\circ}\text{C}$  to  $10.9^{\circ}\text{C}$ . The average year air temperature is  $0.3^{\circ}\text{C}$  in the highest part of this mountain. All of that shows that those parts of the mountain are in the zone of nivation, i.e. in the active periglacial zone and there are numerous processes caused by frosty erosion (erosive and accumulative) and solifluctional moving and skimming blocks as well. Periglacial processes are more often in the spring and autumn when positive (during the day) and negative (during the night) air temperatures change alternately. The air temperature decreases going up the mountain during these periods since the temperature gradients than are bigger than ones in January, February and December are.

## THE LIVADICKO JEZERO LAKE<sup>1</sup>

Morphometry of the lake The Livadicko jezero Lake is at the height of 2200m above sea level<sup>2</sup>. On the day we measured its length, July 10, 1975, it was 220m long. Maximal width was 125m while average width was 93.5m. The coastal line was 683m long. The highest water level is during Jun and July when the lake level rises and all parameters change. This change can be considered as a little one since periodically flooded sides of the lake basin and land zone as well are rather stopped except the part near west side of the circue which gently inclines. That slope, in some parts almost vertical, stops water to run horizontally.

The coastline of the lake is heavily jagged. On the SE part of the lake some slope material moved the coastline for 22m toward the centre of the lake and made a peninsula. The coastline of the NW part of the lake, below the slope Tumba, is mostly straight while it is semicircular in the NE part. The SE coastline, where the peninsula extends out into the lake, is jagged than the SW side is. On that side there is a well-rounded fan like alluvium of flood material deposited by a tributary, so the coastline is convex, bent to the central part of the lake. The area of the Livadicko jezero Lake we measured covered 20580m<sup>2</sup>. The greatest depth was 6.7m while average depth was 3.6m. When the water level is highest the greatest depth is 7.5m. The inclination of the lake basin is not the same all around. The NE and SW parts gently slope, NW part is steep while some parts of SE side are very steep and isobaths 1.5-4.5m long coincide there. A huge stone is inversely leaned over the sublacustric cavity. The relief of the lake basin is turbulent considering its along profile. There is a sudden transition of the bottom surface 70m far from the SW coastline. A part of the bottom at the depth of 4.4m suddenly rises up for 1.2m forming a bend 40m long, gently sloped to the NE. Next to this elevation there is a recess similar to a big kettle; the greatest depth of 6.7m was measured in the centre of it. (Figure 4 and Figure 5) Cross profile shows gradual increase in depth and there is not any transition; mutual distance between two adjacent isobaths is rather equal. The greatest depth of this profile is 6.2m and it is measured in the middle.

The bottom of the lake covers 20880m<sup>2</sup> and it is 1.47% larger than the area of the aquatory.

Area of the bottom bordered by 6m long isobath covers 3200m<sup>2</sup> and it can be conditionally concerned

as a central plane of the lake. It was accumulated 74715m<sup>3</sup> of water on the day it was measured. The greatest amount of water was between the surface of the lake and an imaginary flat at the depths of 1.5m. Below this isobath the amount of water almost regularly has decreased and it can be seen in Tab.1.

Table 1. Area and volume of water below and between particular isobaths of the Livadicko jezero Lake (10 July 1975)

Isobata	F/m <sup>2</sup>	W/m <sup>3</sup>	Isobata	F/m <sup>2</sup>	W/m <sup>3</sup>
0	20.580	74.715	0-1,5	4.178	27.736
1,5	16.402	46.979	1,5-3,0	3.622	21.886
3,0	12.780	25.093	3,0-4,5	4.480	15.810
4,5	8.300	9.883	4,5-6,0	5.520	8.310
6,0	2,780	973	6,0-6,7	2.780	973

The Livadicko jezero Lake is recharged by the tributary, precipitation and sublacustric source. The lake is situated in the area of Sar-planina Mt. boarded by 1250 and 1500mm isochiats. The amount of precipitation that falls on its aquatory ranges between 25725m<sup>3</sup> and 30800m<sup>3</sup> of water. Precipitation that falls on the NE and SW parts as well on the bottom of the circue fills the lake. There are several sublacustric sources, especially on the NE part of the lake basin. Some of them are thin but strong streams that spread out over the surface of the aquatory. We noticed some sublacustric sources in few other lakes on Sar-planina Mt. The water they give can be hardly measured but, obviously, they compensate certain amount of water that flows off at the surface and drains through the moraine rampart. The lake tributary is formed by numerous snow-packs and great number of springs on the sides of the circue extended toward the NE and N. Wide and powerful snow-packs mostly stay in the highest parts of the circue only during the summer time while some of them last for several years. We found 20 snow-packs on July 10, 1975. One of them was in the circue at 2325m a.s.l., toward the SE from the lake. It was 70m long, 15m wide and 2m thick. A year before, on August 30, 1975, this snow-pack was larger and thicker: it was 85m long, 25m wide and 3m thick. We noticed than and in 1975 as well, that the lower layers of snow were taken hold of the strong ice-snow forming process. It has happened due to the pressure of the upper layers of snow and under the influence of the snow water arose by melting the upper layers during

1 There are few alter names of the lake: Lake Livadice, the Lake of Satorica, the Strbacko jezero Lake, the Demirkapijsko jezero Lake, the Sarsko jezero Lake or just Lake. Scientists and people recently use the name the Livadicko jezero Lake. That name comes from the name of the peak Livadice (2497m height above sea level) since the lake is in the immediate vicinity of it. Some people helped us in gathering morphometrical data of the Lake: Borislav and Momir Cukic (1974), Dragoljub Labus, Zarija Mladenovic, Novica Krstic and Stanimir Simic (1975) and Dusan Petrovic (1985) as well. We thank them this way.

2 This height above sea level is taken from a topographic map 1:25000 from 1973. R.T. Nikolic wrote that the lake was at 2112m height a.s.l. (R.T. Nikolic, 1912), D.S. Krivokapic stated the height was at 2173m a.s.l. (D.S. Krivokapic, 1959) while B.Z. Milojevic said the bottom of the circue was at 2220m a.s.l. and "the Livadicko jezero Lake basin is in the NE, below the steep slope" (B.Z. Milojevic, 1937). The appreciable difference between heights above sea level established by him and by the authors mentioned above appears because old topographic maps were used.

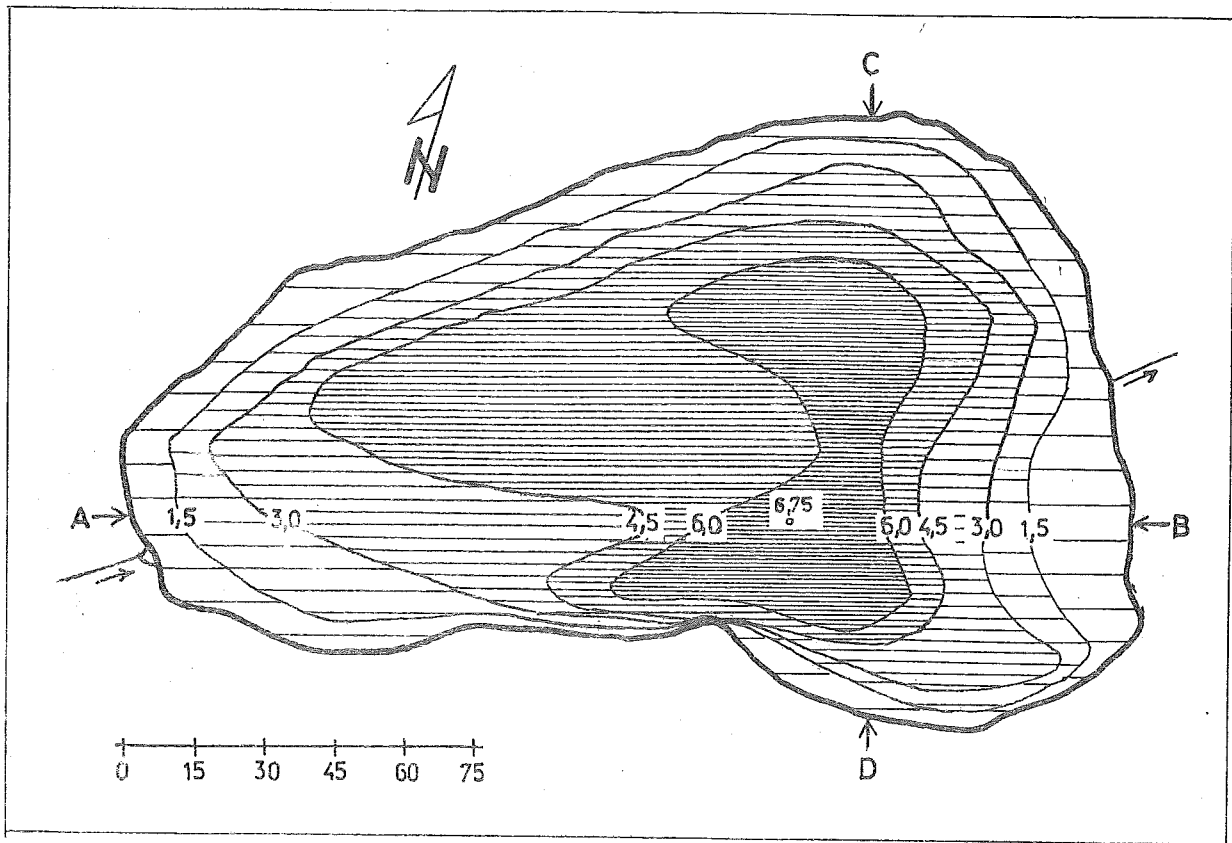


Figure 4.Isobathic map of the Livadicko jezero Lake on Sar-planina Mt.

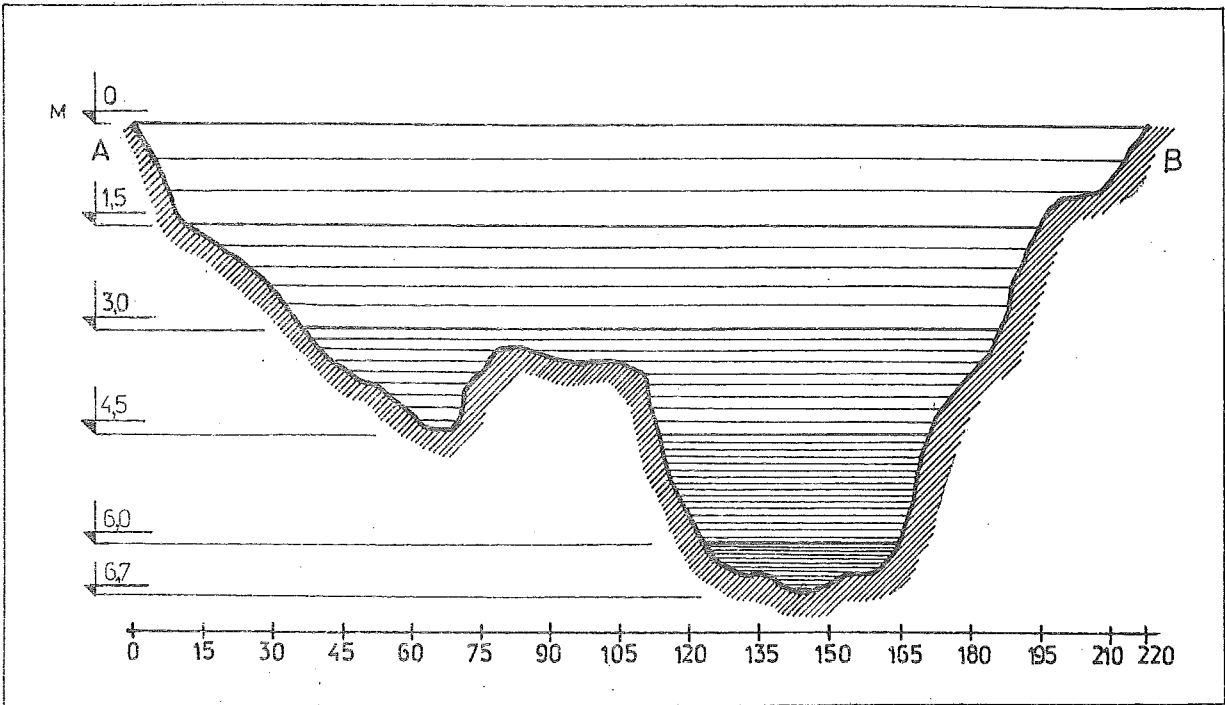


Figure 5.Along profile of the Livadicko jezero Lake bottom on July 10, 1975

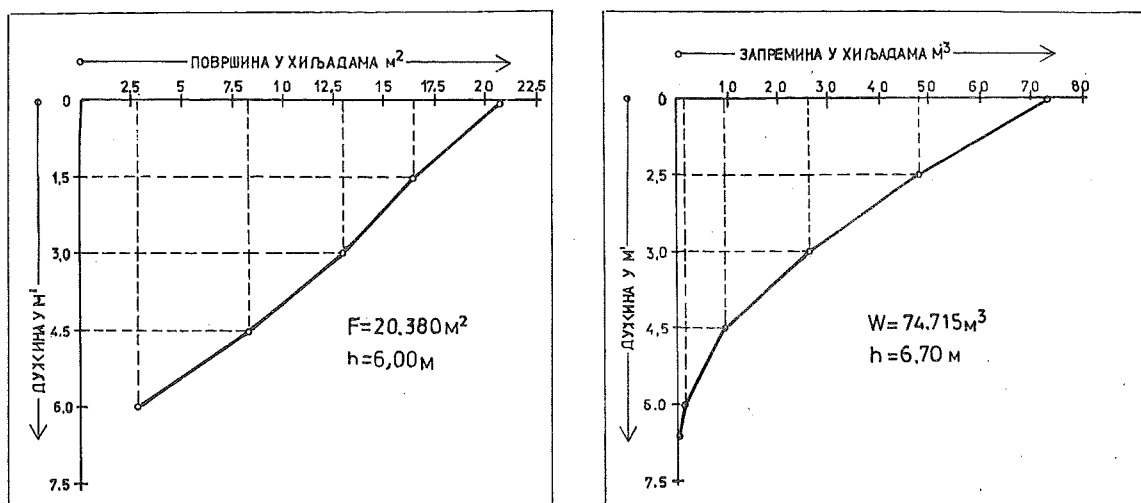


Figure 6. Curve of the area and curve of the volume of the Livadicko jezero Lake on Sar-planina Mt.

the hot days. It is possible that snow-packs, if they are well sheltered, last even during very drought periods. Consequently, we found during the drought period, on July 26, 1985, 11 snow-packs in the circue of the Livadicko jezero Lake.

R.T. Nikolic counted 12 snow-packs in the zone upper from the Livadicko jezero Lake, on August 7, 1911, while B.Z. Milojevic saw three of snow-packs on the SE side of the circue in August 1935 and below the lake, at both side of the arm, only one snow-pack. D.S. Krivokapic found 18 snow-packs in July 1953; 8 of them were on the right side of the circue. Two snow-packs, one of them was 1.9m thick, were situated at the coastline of the lake, and water eroded them (D.S. Krivokapic, 1959).

The lake tributary, recharged by these snow-packs and springs, has carved a valley. It is divided after a short flow into two branches and it fills the lake with considerable amount of water.

The lake loses water because water drains through the moraine rampart and arm that appears from time to time and has carved in the lake a shallow riverbed. The valley is narrow and shallow on the interior side of the rampart while it is wider and somewhat dipper on the exterior side. An arm appears when the water level is highest - at the end of May, in Jun and sometimes in July, as well during extrimely rainy autumns. When the water level is below the bottom of the surface arm valley the lake loses water only because it drains through the moraine rampart. A plentiful spring in the valley, around 200m below another rampart, proves it. When the water level falls hydrostatic pressure reduces and smaller water quantity drains.

On July 7, 1911 R.T. Nikolic observed that the lake water level was 1.5-2m below maximal level, while on July 22, 1934, B.Z. Milojevic observed it was 1.5m lower than the level when the surface arm occurs. He found that the lake depth ranges from 5.3 to 4.8m when the aquatory level is lowest. He based his opin-

ion on the strongest influence of waves on the lake cavity ceiling in the rock that extends out deeply into the lake. According to his data the lake water level decreases for 2-2.5m. We are of the opinion that this assumption given by D.S. Krivokapic can be accepted with correction that difference between lowest and highest water level during draughts can be even 3m. Evaporation and water loss trough this way are negligible since the water temperature is relatively low.

### Temperature, transparency and optical properties of the water

Temperature of the water in shallow coastal zone in the NW part of the lake was  $15.5^{\circ}\text{C}$  measured on July 10, 1975 at 11:30, while in the centre of the lake it was  $13.5^{\circ}\text{C}$ ; temperature of the air was  $18^{\circ}\text{C}$ . The temperature of the water at 6m depth in the centre of the lake was  $9^{\circ}\text{C}$ . The temperature of this lake water was measured in 1911, 1934 and 1953 and results are given in the Table2.

These temperature values show that water temperature of the Livadicko jezero Lake decreases during summer going from the coast to the central part as well as from the surface to the bottom. Also, it depends of water quantity brought by the tributary: the temperature of water was measured when the water level was highest (D.S. Krivokapic, 1959), due to the big tributary and the arm, while the lake water was fast exchanging. The water was few degrees Centigrade ( $3.4-5.5^{\circ}\text{C}$ ) colder than. The temperature of the spring water in the circue, measured on July 10, 1975 ranged from  $3^{\circ}\text{C}$  to  $6^{\circ}\text{C}$ ; D.S. Krivokapic stated the same in July 1953. From the end of autumn, during winter and spring, the lake is under a thick ice and covered by a great amount of snow (it falls, wind blows it into piles and it slips from the circue sides). The inverse temperature stratification goes on then. The water of the Livadicko jezero Lake is clear, clean, green-blue while the deepest parts are dark blue. The watercolour nuances change during a day and season, also they depend on the Sun position

Table 2. Temperature of air and water of the lake.

Author	Data	Time (h)	Temperature °C				
			Air	Waters			
				Bentos	In the middle of lake		
					Surface	6 m deep	7,3 deep
R.T. Nikolić	07. 07. 1911.	9,30	/	13,4	/	/	/
B.Ž. Milojević	22. 07. 1934	10,00	19,0	14,0	/	/	/
D.S. Krivokapić	8.-10. 07. 1953	On 4h	/	10,0	6,0	/	5,0
D. Ćukić	10. 07. 1975	11,30	18,0	15,5	13,5	9,0	/

and shadows produced by peaks and crests that surround the aquatory of the lake. We measured transparency of the water in different points and found that it follows the depth. However, D.S. Krivokapic said that "the water is the most transparent in the centre of the lake, measured by white ceramic tile" and "transparency is 4.7m". He found that when the greatest depth of the lake was 7.3m. By comparing these two data it can be concluded that depth of intransparent layer of the water was 2.6m. Our and his water transparency indexes are not equal, certainly because he measured water transparency by a "ceramic tile 15x15cm" while we measured it by Seki's disc (sheet metal plate of 30cm in diameter, covered by white enamel; there is a 0.5kg weight fixed on the bottom while nonelastic rope, scaled in decimetres, is fixed on the top). The bottom of the lake is mostly covered by flat block of stones. Such stones can be seen in the coastal zone of the lake.

### THE MALO LIVADICKO JEZERO LAKE

The lake is situated at height of 2190m a.s.l. As D.S. Krivokapic found it is almost elliptical, 24m long and 10m wide. Its longer axes is along NW-SSE direction, coastline is 58m long and poorly jagged. The area of the lake covers 169m<sup>2</sup> and its maximal depth is 1.3m as D.S. Krivokapic stated. The lake is recharged by snow water that comes from the melted snow-packs. The snow packs remain on the both moraine ramparts that surround the lake and on the base of the crest Tumba that is on the left side of the cirque. During the period of the highest water level the Livadicko jezero Lake is recharged by its arm as well, because it floods out of the riverbed. The lake loses water through the arm which empties into the Kaluderska reka River at 10m distance.

### DISCUSSION

The Livadicko jezero Lake is glacial and of a cirque shape. It was created in the overhollowed part of the cirque bottom for what the accumulation of the moraine material over the rung of underlying rock was

very important. This rung has been created by the selective erosion and behind it the melted mass of the last glacier has been collected. Therefore, the lake is of glacial-erosive-accumulative origin. Dry land zone is 4m higher than the maximal level of the aquatory is. The parts of the moraine rampart that hasn't been destroyed by water overflowing immediately after the glacier had melted, when the lake was considerably deeper, is of the same height but somewhere higher. The arm riverbed that can be seen nowadays is shallow and is deeply cut into the part of the rampart that has been rather eaten through by water overflowing.

The Malo Livadicko jezero Lake is nivational erosive one. It is in the depression that has been created under the effects of snow-packs to the underlying moraine in the cirque front part, between two deposited moraine ramparts. It happened during two last withdrawals of the valley glacier to the hypsometrically higher parts. The Livadicko jezero Lake has a permanent surface tributary while surface arm occurs periodically. It has only a tributary during the period of low water level, but during the period when the water level is higher than the bottom of the riverbed it has a tributary at one end but at the other it has an arm as well.

The way of recharging and losing water of periodical Malo Livadicko jezero Lake is different from the hydrologic and time aspect. At the end of the spring and beginning of the summer, when the Livadicko jezero Lake still hasn't a surface arm, this little lake has an arm recharged, during the day, by snow water of the great snow-pack that has kept in its little basin and close vicinity. The lake only loses water then. When the Livadicko jezero Lake, which is hypsometrically higher, has an arm, the Malo Livadicko jezero Lake is recharged by it when it withdraws from the riverbed that has been cut in the first moraine rampart. This little lake is of flowing type than. Therefore, it is, from the hydrologic and genesis aspect, very interesting small periodical hydrologic structure.

The Livadicko jezero Lake belongs into the group of largest Sar-planina's Mt. lakes, considering the area of aquatory. There are only three lakes whose aquatories are larger than that one of the Livadicko jezero Lake is. Two of them - the Bogovinsko jezero Lake and

Crno (Black) Lake - are permanently larger while the Lake of Veliki (big) dol at Karanikolica only during the period of the highest water level. These three lakes lay on the part of Sar-planina Mt. that belongs to Macedonia.

The Livadičko jezero Lake takes the second place among all the lakes on Sar-planina Mt. according to the depth at maximal water level. Only the Veliko (Big) Juzinacko jezero Lake is deeper at the lowest water levels (8,4m) as well at the highest (12,5m). It is on the NW side of Sar-planina Mt., Serbia, Yugoslavia.

According to area of aquatory the Malo Livadičko jezero Lake is one of the middle size lakes, but according to depth it falls into the group of deepest nivational lakes on Sar-planina Mt.

Most of the glacial lakes on Sar-planina Mt. are situated at heights that range from 2101m to 2300m a.s.l. The same is with the Livadičko jezero Lake but it is at 210m lower height than the highest - Golemo jezero Lake is. It is in the spring branch of the River Prizrenska Bistrica at 2410m a.s.l., Serbia, Yugoslavia. The Malo Livadičko jezero Lake is for 230m lower than the highest nivational lake - Priribresko jezero Lake, that is at 2420m a.s.l. The Livadičko jezero Lake and Malo Livadičko jezero Lake belong to the group of evolutionary and hydrographic structures. For the reason they are important elements of the natural environment as well as parts of the national park "Sar-planina" it is necessary to be protected, reclaimed and valorised undertaking appropriate measures.

The methods and ways of protection should be chosen after the natural processes that ruin the lakes are established. Eroded and ruined material fills in the Livadičko jezero Lake so its accumulation capability reduces. The brook, flowing into the lake, deposits fine material not only at the mouth but also on the lake bottom. The similar process is common for the left side of the cirque. Other sides are characterised by numerous screes as well as by "stony glaciers". The last ones are created by the crionival processes by which ruined blocks and detritus fall to the cirque bottom, reaching the lake as well. The arm that appears periodically deepens the riverbed so water flows out from the lake more and more. The water also drains through the moraine rampart taking off fine clay material and widens the fissures between blocks what makes the water level lower.

The protective measures should include a consolidation of the moraine rampart and rebuilding its eroded parts. On that way the area of the lake and its volume would be considerably enlarged and amplitude of the water level would be reduced; on the other side the moraine rampart would have characteristics from the earliest time - the certain height and watertightness. A structure that would collect deposits should be constructed in the riverbed. It could prevent or at least reduce accumulation of alluvium in the lake.

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## REZIME

### JEZERA U SLIVU KALUĐERSKE REKE NA ŠAR-PLANINI

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Livadičko jezero (2200 m n.v.) i Malo Livadičko jezero (2190 m n.v.) nalaze se u NW delu Šar-planine, u izvorištu Kaludjerske reke, desne sastavnice Miljoštice, koja se uliva u Lepenac i pripada egejskom slivu (Sl. 1).

Basen Livadičkog jezera nastao je ledničkom erozijom NW dela dna prostranog cirka i nagomilavanjem morenskog materijala na prečagi koja ga zagrađuje. Ona je nastala selektivnom erozijom lednika (Sl. 2 i 3). Prema tome, ovo jezero je glacijalno, cirkno, erozivno-akumulativnog porekla. Na dan merenja, 10

jula 1975. godine, bilo je dugačko 220 m, maksimalno široko 125 m, a prosečno 93,5 m. Obalska linija je dugačka 683 m (Sl. 4 i 5). Pri najvišem vodostaju, početkom leta, nivo jezera se povećava, te to utiče na promenu ovih i drugih morfometrijskih pokazatelja, ali ne mnogo, jer strme, a u pojedinim delovima i vertikalne, strane jezerskog basena i njegovog kopnenog pojasa to onemogućavaju. Površina jezera u vreme našeg merenja iznosila je 20580 m<sup>2</sup>.

Najveća dubina jezera, utvrđena na dan našeg merenja, bila je 6,7 m, a prosečna 3,6 m. Pri najvišem vodostaju jezero je dublje za 0,8 m, te mu je tada najveća dubina 7,5 m, a prosečna 4,4 m. U basenu Livadičkog jezera bilo je akumulirano 74715 m<sup>3</sup> vode (Sl. 6).

Livadičko jezero se hrani vodom padavina na njegov akvatorij, kao i na levu stranu i deo padina u zaleđu cirka. Vodu dobija i od sublakustriskih izvora i kratke pritoke. Pritoku hrani snežnica zaostalih snežanika i izvori u cirku. Jezero gubi vodu procedjivanjem kroz morenski bedem i povremenom otokom, koja je u njega usekla plitko korito. Otoka se javlja samo pri najvišem vodostaju jezera.

Temperatura vode jezera je, 10. jula 1975. godine u 11,30 časova, u plitkom priobalnom pojasu iznosila 15,5°C, na njegovoj sredini u površinskom sloju 13,5°C, a na dubini od 6 m je bila 9,0°C.

Voda jezera je bistra, čista, zelenkasto-plave boje, a u delu sa najvećom dubinom zatvoreno-plave boje.

Providna je do dna, na kome ima dosta stenovitog materijala.

Malo Livadičko jezero je nivaciono, erozivnog porekla. Smešteno je na zaravni između dva morenska bedema. Dugačko je 24 m, a široko 10 m. Obalska linija je dugačka 58 m a duža osa se pruža pravcem NNW-SSE. Površina mu iznosi 169 m, a najveća dubina 1,5 m. Hrani se snežnicom, a pri najvišem vodostaju Livadičkog jezera i vodom njegove otoke koja se preliiva preko plitkih strana korita. Vodu gubi plitko usečenim koritom svoje sasvim kratke otoke.

Livadičko jezero je u vreme niskog vodostaja uvorno, a pri najvišem vodostaju dobija površinsku otoku, te je tada protočno. Malo Livadičko jezero je otočno, a u vreme kada ga vodom hrani otoka livadičkog jezera protočno.

Oba jezera su vrlo evolutivni hidrografski objekti. Pošto su značajni element prirodne sredine, potrebno je da se preduzmu odgovarajuće mere za njihovu zaštitu. U tom sklopu potrebna je sanacija morenskog bedema Livadičkog jezera u cilju obezbeđivanja njegovog visokog vodostaja, kao i zaštita njegovog basena od unošenja erodiranog materijala sa strane cirka i vučenog i lebdećeg materijala koji u njega unosi pritoka.

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# INFLUENCE OF PARAMETERS OF LIGNITE OXIDATION WITH NITRIC ACID ON YIELD OF HUMIC ACIDS

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## ABSTRACT

The sample of Kosovo lignite from the excavation site "Dobro Selo" containing 46,79% of humic acids was oxidated with nitric acid in order humic acids to be enriched. Applying different conditions of lignite oxidation with nitric

acid we have come to conclusion that maximal yield of humic acids (73,15%) is obtained during oxidation of raw lignite with 20% nitric acid at temperature of 20 °C for a time of 16 hours.

**Key words:** humic acid, oxidation, nitric acid.

## INTRODUCTION

Economy in producing humic fertilizers on the basis of coal depends on humic acid content above all (Kuharenko and Šapiro, 1957) since the greater content of humic acids in lignite the greater economy in producing fertilizers.

Content of humic acids in coals is very different. Some samples e.g. naturally oxidated coal contains even 80% of humic substances. However most of coals contain much less humic acids even below 10%. Such content of humic acids can be increased by coal oxidation treatment e.g. air effect, treatment by acid oxidation remedies (for example by nitric acid) or by treatment of some oxidation remedies in alkali milieu (air, hydrogen-peroxide, potassium permanganate etc.).

Oxidation of coal by nitric acid solutions attracted attention of many researchers (Negro, 1967, Arita and co. 1970, Hortisuka and Gato, 1971, Auguston and co. 1972 etc.) since during oxidation of coal by nitric acid two kinds of reaction happen (Charbury and Polanski, 1957): coal is oxidated-enriched in humic acids and at the same time nitrogen is introduced into structure of humic acids.

Yield of humic acids during oxidation depends on type of coal, temperature, pressure, kind and concentration of oxidation remedies as well as on granulation of coal.

The basic aim of this work was, in Kosovo lignite containing humic acids of 46,79% by oxidation of nitric acid solutions, to increase content of humic acids so that to increase possibility of applying Kosovo lignite as the raw material to produce humic fertilizers.

## MATERIAL AND METHODS

We have tested Kosovo lignite from the surface excavation site "Dobro Selo" position 401, east shart, peak elevation 497, II floor. The basic characteristics of the tested sample of lignite were as follows: humidity 43,87%, ash 19,59%, DTM 7079 KJ/kg, evaporable-matters 23,10%, combustible matters 36,54%, content of humic acids 46,79% (calculated for lignite without humidity and ash). Oxidation of raw lignite was followed in function of:

- a) concentration of nitric acid solution,
- b) reaction temperature,
- c) duration of oxidation and
- d) volume of nitric acid solution.

### a) Influence of concentration of nitric acid solution

During the first series of testing concentration of nitric acid solution was changed and other parameters as temperature, duration of ammonization and coal / volume of nitric acid ratio were kept constant. Oxidation of raw lignite (10g) was performed with 5%, 10%, 20%, 25%, 35%, 40% and 45% of nitric acid. Experiments were conducted under the same conditions (temperature 20 °C, oxidation time 16 hours, volume of nitric acid solution 125 ml).

### b) Influence of temperature

Testing of temperature influence on oxidation of raw lignite was performed at 20 °C, 40 °C, 60 °C, 80 °C and 100 °C while all other parameters (nitric acid concentration, oxidation time, nitric acid volume) were constant.

### c) Influence of oxidation time

Oxidation of raw lignite was performed in function of duration of reaction time (1, 4, 8, 12 and 16 hours) and other parameters were kept constant.

**d) Influence of volume of nitric acid solution**

During these tests oxidation of raw lignite was performed in function of nitric acid volume (25, 50, 75, 125 and 250ml), and all other parameters were kept constant.

After completion of oxidation, oxidated lignite was separated from solution by centrifugation, rinsed with water up to neutral reaction and dried at 105 0C. In all samples of oxidated lignite content of humic acids was determined by extraction with natrium-hydroxide solutions under the same treatment and under the same conditions. Content of humic acids was calculated in relation to lignite without ash and humidity.

GRATITUDE: We thank Ministry of Science and Technology of Republic of Serbia for material help to make this paper work.

**RESULTS AND DISCUSSION**

When studying parameters of raw lignite oxidation with nitric acid we have concluded that change of oxidation parameters has essentially influenced on content of humic acids in lignite.

a) During the first series of tests on how nitric acid concentration affects oxidation of raw lignite and yield of humic acids we have stated (table 1) that by increasing nitric acid concentration from 5% to 20% content of humic acids in oxidated lignite gradually increases. Maximal yield of humic acids (73,15%) is obtained by oxidation of lignite with 20% nitric acid. However during oxidation of lignite with more concentrated solution of nitric acid lesser yield of humic acids is obtained.

Table 1. Influence of nitric acid concentration on yield of humic acids

HNO <sub>3</sub> %	HUMIC ACIDS %
Raw lignite	46,79
5	51,20
10	64,17
15	66,26
20	73,15
25	71,15
35	63,15
40	60,35
45	53,24

So during oxidation of lignite with 45% nitric acid very small yield was obtained almost as when lignite was oxidated with 5% nitric acid solution. b) During tests on how temperature affects oxidation of lignite with nitric acid (table 2) while other parameters were kept constant (125 ml, 20 %HNO3, time duration of 16 hours) we have noticed that with increase of temperature, content of humic acids in lignite decreases.

Table 2. Influence of temperature on yield of humic acids

TEMPERATURE 0C	HUMIC ACIDS %
Raw lignite	46,79
20	73,15
40	70,26
60	65,18
80	62,20
100	61,77

c) In the series of lignite oxidation tests with nitric acid in function of reaction duration when optimal parameters from previous tests were kept constant (20%HNO3, 20 0C) we have noticed that yield of humic acids gradually increases by prolongation of time for lignite oxidation from 1 hour to 16 hours (table 3). However if further prolongation of oxidation time is applied yield of humic acids decreases. So almost the same yield of humic acids is obtained during 12 and 20 hours of oxidation of lignite.

Table 3. Influence of time duration of lignite oxidation on yield of humic acids

TIME h	HUMIC ACIDS %
Raw lignite	46,79
1	49,19
4	56,31
8	66,25
12	69,76
16	73,15
20	70,41

d) Tests on how nitric acid volume affects oxidation of lignite in order content of humic acids to be obtained where optimal parameters from the previous tests were kept constant (20% HNO3, 20 0C, 16 hours) we have stated (table 4) that by increasing volume of 20% nitric acid from 25 ml to 125 ml yield of humic acids also increases. However with greater quantity of nitric acid yield of humic acids decreases. Table 4. Influence of nitric acid volume to yield of humic acids

Table 4. Influence of nitric acid volume to yield of humic acids

VOLUME ml	HUMIC ACIDS %
Raw lignite	46,79
25	51,45
50	64,55
75	68,02
125	73,15
250	62,29

**CONCLUSION**

By studying parameters of lignite oxidation with nitric acid (acid concentration, temperature, reaction time and nitric acid volume) we have conluded the following: 1) Kosovo lignite from the excavation site "Dobro selo" containing 46,79% of humic acids in oxidation treatment with nitric acids can be enriched by humic acids. 2) By inrasing acid concentration from 5 to 20%, content of humic acids in lignite also increases. By further increase of nitric acid concentration content of humic acids in lignite decreases. 3) By increasing temperature of lignite oxidation with nitric acid, content of humic acids in lignite decreases. 4) By prolongation of oxidation time from 1 to 16 hours, content of humic acids in lignite increases but further prolongation of reaction time decreases the yield of humic acids. 5) By inreasing nitric acid volume from 25 ml to 125 ml the yield of humic acids in lignite is increased, but by further increase of volume the yield is decrease. 6) Maximal yield of humic acids (73,15%) is obtained during oxidation of 10 g raw lignite in 125 ml nitric acid at 20 0C from 16 hours.

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## REZIME

# UTICAJ PARAMETARA OKSIDACIJE LIGNITA SA AZOTNOM KISELINOM NA PRINOS HUMINSKIH KISELINA

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Ispitivani su parametri oksidacije kosovskog lignita s ciljem povećanja sadržaja huminskih kiselina u lignitu, čime bi se povećala njegova mogućnost primene kao sirovine za proizvodnju huminskih đubriva. Oksidacija sirovog lignita je praćena u funkciji koncentracije rastvora azotne kiseline, reakcione temperature, dužine vremena oksidacije i količine rastvora azotne kiseline. Primenjujući različite uslove oksidacije lignita sa azotnom kiselinom došlo se do konstatacije da se maksimalan prinos huminskih kiselina (73,15%) dobija pri oksidaciji sirovog lignita sa 20% azotnom kiselinom, na temperaturi od 20 °C i u vremenu od 16 časova.



# PRODUCTION OF AMMONIUM-NITRO-HUMAT AND TEST ON ITS BIOLOGICAL EFFECT TO GROWTH OF PLANTS

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## ABSTRACT

By ammoniation of oxidated lignite from Kosovo with ammonium-hydroxide solutions subjected under different reaction conditions three products of ammonium-nitro-humat are obtained. test results show that the yield of ammonium-nitro-humat, nitrogen content and humic acids depends on the applied conditions of ammoniation. Biological i.e. stimulative effect of ammonium-nitro-humat is entrusted to methods

of germination of seeds: wheat, radish and mangel, and experiment with pepper in water culture. Test results of stimulative effect of ammonium-nitro-humat to germination of seeds have not shown the same effect of stimulation to seed germination. However all the tested substances have shown positive stimulative effect to growth of pepper - to its root system as well as to stem and leaf.

**Key words:** coal, ammoniation, humic acids, ammonium-nitro-humat.

## INTRODUCTION

In intensive agricultural production no soil has sufficient reserves of any of elements. It is therefore necessary in modern agricultural production to add to the soil all kinds of fertilizers, organic and inorganic. One-sided long - standing application of mineral fertilizers has shown its shortages in fertility of soil as well as to organisms in the soil. This has induced combination of humic and mineral components in organic - mineral fertilizers. Among many materials, potential changes for usual organic fertilizers coal is one of the as it contains humic acids and its stimulative effect to growth and development of plants has been proved (Aleksandrov and co.1992, Pokol and co. 1995, Xiaoyan, 1994, Berecz and co. 1995, Matsuo and Hanaoko, 1996). Procedure in producing fertilizers on the basis of coal can be very different (Kuhrenko and Šapiro, 1957, Charbury and Polansky, 1957, Negro, 1965, Juan and co. 1990, Calmma, Piccolo, Rauso, 1992 etc.), and procedure above all depends of kind of coal, level of coal carbonification, content of humic acids and on the applied technological parameters. In reference with the above, the aim of this work was to produce ammonium-nitro-humat on the basis of Kosovo lignite and test its biological effect to germination of seeds and growth of plants in the laboratory conditions.

## MATERIAL AND METHODS

Kosovo lignite from the excavation site "Dobro Selo" previously oxidated with nitric acid and containing 73,15% humic acids was taken for test. Ammonium-nitro-humat was produced on the basis of oxidated

lignite by ammoniation with ammonium-hydroxide solution under the following conditions: a) oxidated lignite (10 g) was ammoniated in 75 ml of 15% ammonium-hydroxide solution at 20 °C for 16 hours. b) oxidated lignite (10 g) was ammoniated in 225 ml of 15% ammonium -hydroxide solution at 20 °C for 16 hours, and c) oxidated lignite together with solution of nitric acid remains from oxidation of raw lignite, was ammoniated in 125 ml of 20% ammonium -hydroxide solution at 20 °C for 16 hours. After completion of ammoniation the obtained products were steamed at the water bath and dried at 105 °C. In all samples of ammonium-nitro-humat content of humic acids and nitrogen content were determined by Dumas method. Biological i.e. stimulative effect of ammonium-nitro-humat in laboratory conditions was performed by the following methods: - test on seed germination: wheat, radish and mangel - experiment with pepper in water culture Experiment on germination test were conducted according to Pranishnikov procedure (1964). The applied concentration of solution was 2.10-2 mg/ml. Biological experiment with pepper in water culture was conducted according to Witha and co. (1971) procedure with nutritious foundation containing all necessary elements for normal growth of plants. The prepared solutions of ammonium-nitro-humat were added to nutritious solution in concentration of 0,5.10-2 mg/ml, 10-2 mg/ml and 2.10-2 mg/ml.

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## RESULTS AND DISCUSSION

By applying different ammoniation conditions of oxidated lignite with ammonium-hydroxide solution three samples of ammonium-nitro-humat (ANH) were obtained. Characteristics given in table 1.

Table 1. Basic characteristics of ammonium-nitro-humat

Sample	ANH %	HA %	N %
ANH <sub>1</sub>	115,44	65,54	4,2
ANH <sub>2</sub>	321,47	32,68	5,18
ANH <sub>3</sub>	590,56	12,81	28,39

When analysing the yield of ammonium-nitro-humat obtained during different ammoniation conditions of oxidated lignite it can be seen that the least yield of ammonium-nitro-humat (ANH1) is obtained during ammoniation of 10 g oxidated lignite in 75 ml of 15% ammonium-hydroxide solution. However during ammoniation of 10 g oxidated lignite in 225 ml of 15% ammonium-hydroxide solution the yield of ammonium-nitro-humat (ANH2) was about two times greater.

The greatest yield of ammonium-nitro-humat (ANH3) 590,56% (calculated in relation to lignite without humidity and ash) was obtained during ammoniation of oxidated lignite together with solution from nitric acid remains. Such a high yield of ammonium-nitro-humat can be explained by the fact that during ammoniation of oxidated lignite ammonium-nitro-humat and ammonium-nitrate is formed (built). In that way the obtained product represents mixture of ammonium-nitro-humat and ammonium-nitrate.

When analysing content of humic acids (HA) in the obtained products it can be noticed that it is in inverse proportion to the yield of ammonium-nitro-humat i.e. the greatest content of humic acids was found in the sample of ammonium-nitro-humat which is obtained in the least yield and the least in the sample which is obtained in the greatest yield.

Content of total nitrogen is also proportional to the yield of ammonium-nitro-humat. The sample with the greatest yield of ammonium-nitro-humat also contains the greatest quantity of the total nitrogen 28,39%, and the sample with the least yield of ammonium-nitro-humat contains the least quantity of nitrogen. Biological i.e. stimulative effect of the obtained ammonium-nitro-humates was performed by the following methods:

- tests on seed germination: wheat, radish and mangel and
- experiment with pepper in water culture.

Results on wheat seed germination tests (table 2) show that ammonium-nitro-humates (ANH1 and ANH2) only slightly stimulate wheat seed germination in relation to control test. More over, solution of combined ammonium-nitro-humat and ammonium-nitrate (ANH3) in this experiment showed the same effect as the control solution.

Table 2. Indices on stimulative effect of ammonium-nitro-humat to wheat seed germination

Substance	Number of germinated corns	%
Control	90	100
ANH <sub>1</sub>	94	104
ANH <sub>2</sub>	91	101
ANH <sub>3</sub>	90	100

When testing radish seed germination (Table 3) it was noticed that only solution of combined ammonium-nitro-humat and ammonium-nitrate (ANH3) showed insignificant stimulation of seed germination. By comparison of the obtained results for radish seed germination with results on radish seed germination obtained by Petrović, Vitorović and Jablanović (1982) it can be seen that humic acids solutions had greater effect to germination than solution of ammonium-nitro-humat.

Table 3. Indices on stimulative effect of ammonium-nitro-humat to radish seed germination

Substance	Number of germinated corns	%
Control	88	100
ANH <sub>1</sub>	82	93
ANH <sub>2</sub>	88	100
ANH <sub>3</sub>	89	101

In experiments of mangel seed germination tests all solutions of ammonium-nitro-humat showed stimulation of seed germination in relation to control test (Table 4).

Table 4. Influence of nitric acid volume to yield of humic acids

VOLUME ml	HUMIC ACIDS %
Raw lignite	46,79
25	51,45
50	64,55
75	68,02
125	73,15
250	62,29

When analysing data on seed germination tests we have noticed that the tested substances haven't shown the same stimulative effect on seed germination. Ammonium-nitro-humates showed the poorest effect to radish seed germination and the most effective to mangel seed germination.

Such unequal results point to possibility that some seeds of some plants have their own rich biological and energetic potential so that for seeds to swell up and germinate water is sufficient, as the activator of this beginning physiological process, or more diluted solution as nutritious foundation. Moreover higher concentration of humic substances in the solution may cause the contrary effect i.e. deceleration of seed germination (Table 3, ANH1).

Biological experiment with pepper in water culture was conducting according to Witham and co. procedure (1971) with nutritious foundation containing all necessary elements for normal growth of plants. The following combination of solutions were prepared for the experiment:

- 1) nutritious solution
- 2) nutritious solutions without substances which contain nitrogen

3) solutions of ammonium-nitro-humates in concentration of 5,10 and 20 mg/l with nutritious solution

4) solutions of ammonium-nitro-humat with nutritious solution without substances which contain nitrogen and

5) control test with distilled water. When analysing test results (Table 5) we have concluded that solutions of ammonium-nitro-humat stimulate extension of overground part of pepper as well as its root system increasing length of root, stem and leaf. When observing pepper plants we have noticed that stems and leaves of pepper where the root system was growing in the solutions of the tested substances were not only more branchy but also firmer and dark green in comparison to plants from the control group.

Table 5. Indices on stimulative effect of ammonium-nitro-humat to growth of pepper

Substance	Root length	Stem length	Leaf length
1. (Control)	100,00	100,00	100,00
2. Nutr. solut. HR)	571,59	376,02	245,73
3. Nutr. solut-nitrogen (N)	167,44	166,69	134,92
4. ANH <sub>1</sub> (5 mg/l) + HR	379,77	266,43	205,78
5. ANH <sub>1</sub> (10 mg/l) + HR	186,74	225,04	166,08
6. ANH <sub>1</sub> (20 mg/l) + HR	292,33	143,34	131,16
7. ANH <sub>2</sub> (5 mg/l) + HR	482,57	354,53	251,87
8. ANH <sub>2</sub> (10 mg/l) + HR	145,81	129,66	117,59
9. ANH <sub>2</sub> (20 mg/l) + HR	289,07	193,07	174,12
10. ANH <sub>3</sub> (5 mg/l) + HR	238,37	152,75	157,29
11. ANH <sub>3</sub> (10 mg/l) + HR	416,98	386,58	256,53
12. ANH <sub>3</sub> (20 mg/l) + HR	182,09	189,52	143,97
13. ANH <sub>1</sub> (5 mg/l) - N	151,16	110,12	103,27
14. ANH <sub>1</sub> (10 mg/l) - N	469,07	118,47	114,57
15. ANH <sub>1</sub> (20 mg/l) - N	300,00	115,45	113,32
16. ANH <sub>2</sub> (5 mg/l) - N	296,05	131,97	108,04
17. ANH <sub>2</sub> (10 mg/l) - N	250,00	86,15	86,93
18. ANH <sub>2</sub> (20 mg/l) - N	379,07	118,47	114,82
19. ANH <sub>3</sub> (5 mg/l) - N	436,51	126,11	112,81
20. ANH <sub>3</sub> (10 mg/l) - N	222,56	112,79	118,09
21. ANH <sub>3</sub> (20 mg/l) - N	218,60	76,19	114,82

When observing pepper plants in the phase of growth we have also noticed that the fastest progress happened to those plants growing in the solutions of the tested substances together with nutritious solution so that these plants were firmer and in darker colour than those growing in dishes which containing complete nutritious solution. Solutions of ammonium-nitro-humat together with the nutritious solutions showed better stimulative effect to growth of pepper than nutritious solution not containing nitrogen substances and in some cases stimulative effect was better than the nutritious solution itself. Bringing closer the values of root length at solution of ammonium-nitro-humat without nitrogen to the values of effect of nutritious solution shows good possibility of using nitrogen from ammonium-nitro-humat by plants. Solutions of ammonium-nitro-humat with nutritious solution not containing nitrogen substances had better stimulative effect to growth of pepper root system than nutritious solution not containing substances of nitrogen. When analysing influence on concentration of solutions in the tested substances with nutritious solution without having nitrogen substances we could not see any dependance of stimulative effect on ammonium-nitro-humat concentration.

## CONCLUSION

In studying possibility of producing ammonium-nitro-humat by ammoniation of Kosovo lignite and during tests on its stimulative effect to growth of plants we have concluded the following:

1) The yield of ammonium-nitro-humat, content of nitrogen and humic acids depends on the applied conditions of ammoniation of oxidated lignite.

2) The combined ammonium-nitro-humat with ammonium-nitrate obtained by ammoniation of oxidated lignite together with nitric acid remains after lignite oxidation was obtained in the greatest yield and with the highest nitrogen content (28,39%).

3) Test results of stimulative effect of ammonium-nitro-humat to germination of wheat, radish and mangel have not shown the same stimulative effect to seed germination. The poorest effect of ammonium-nitro-humat was to radish seed germination and the greatest to mangel seed germination. All the tested substances also had stimulative effect to growth of pepper - to its root system as well as to stem and leaf.

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# **REZIME**

## **PROIZVODNJA AMONIJUM-NITRO-HUMATA I ISPI- TIVANJE NJEGOVOG BIOLOŠKOG DEJSTVA NA RAST BILJAKA**

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Postupkom amonizacije oksidovanog lignita sa rastvorom amonijum-hidroksida, pri različitim reakcionim uslovima, dobijeno je tri proizvoda amonijum-nitro-humata. Po završetku amonizacije, dobijeni proizvodi uparavani su na vodenom kupatilu, a zatim sušeni u sušnici na 105 °C. Kod svih uzoraka amonijum-nitro-humata određen je sadržaj huminskih kise-

lina i sadržaj azota Dumas-ovom metodom. Rezultati ispitivanja pokazuju da prinos amonijum nitro-humata, sadržaj azota i huminskih kiselina u dobijenim proizvodima zavise od primenjenih uslova amonizacije oksidovanog lignita. Biološko, tj. stimulativno dejstvo amonijum-nitro-humata provereno je metodom ispitivanja klijavosti semena: pšenice, rotkvice i blitve i ogledom sa paprikom u vodenoj kulturi. Rezultati ispitivanja stimulativnog dejstva amonijum-nitro-humata na klijavost semena nisu pokazali isti efekat stimulacije klijanja semena. Međutim, sve ispitivane supstance imale su pozitivno stimulativno dejstvo na rast paprike kako korenovog sistema tako i stabla i lista.

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# APPLICATION OF ECOLOGICALLY ACCEPTABLE ADDITIVES - WHITE FILLERS IN RUBBER INDUSTRY

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## ABSTRACT

**The influence of selected types of carbon black and white fillers, on cure characteristics and physical-mechanical properties of the natural rubber (NR) vulcanisates were investigated. The possibility of substitution of some types of black fillers with white fillers as ecologically more acceptable additives, was examined.**

**Cure characteristics of the NR rubber compounds were studied using Monsanto Rheometer 100S at 140° C. The physical-mechanical measurements were done by standard methods used in rubber industry.**

**Obtained results have shown that the substitution of carbon black type N990 with the white fillers used can be successfully made.**

Key words: elastomer, ecologically acceptable additives, carbon black, white fillers, physical-mechanical properties

## INTRODUCTION

In order to obtain an elastomer of chosen physical-mechanical properties, different types of organic and inorganic additives are added, the most important among them being fillers (Hoffman, 1968., Šušterič, 1980.). The reinforcement effect of fillers on an elastomer are reflected in the following improvements of the physical-mechanical properties: hardness, modulus 200% and 300%, tensile strength, elongation at break etc. The improvements of physical-mechanical properties in the vulcanised rubber are explained by the crosslink density and ingredient interaction with NR rubber. The extend of the reinforcement effects of additives depend on the specific surface of the filler used, as well as its texture and chemical properties. The grade of the effects it also depends upon the applied recipe.

In order to obtain better technical properties of NR compounds, a number of different types of carbon black fillers are used. However, their toxical influence on human health and natural environment provokes their substitution with ecologically more acceptable white fillers. Appart from ecological and qualitative effects there are economical aspects which are also very important (Dogatkin et al., 1954).

The aim of this work was a comparative analysis of the effects of chosen types of carbon black and white fillers on the cure characteristic and physical-mechanical properties of the selected NR vulcanisate.

## MATERIALS AND METHODS

The following materials have been used in preparation of the samples:

- NR rubber type SMR CV50 (Malesia);
- carbon black N330, N550 (Cabot-Italy) and N990 (Russia);
- white filler Vulcasil C (Bayer-Germany), Silteg AS-7 (Degussa-Germany), and Activated Opalised White Tuff - AOWT (Macedonia);
- cure system: Vulkacit Thiuram, Vulkacit MOZ, Zinc oxide (Bayer-Germany) and Sulfur (Zorka-Yugoslavia).

Compositions of the experimental samples prepared for the measurements are show in Table 1.

All the samples were prepared on a laboratory mixing 2-roll mill, dimensions 152 x 300 mm, with speed ratio of rollers of  $n_1/n_2 = 21/28$  at the rollers temperature of 50° C. The rubbers were masticated for a few minutes. Then the activators, stearic acid and zinc oxide were added, together with an aging preventer and a filler. After mixing, the elastomer compounds were moulded into 2 mm thick slabs to be used for determination of the original and aged properties. The slabs were moulded in a hydraulic press (at 140° C during 25 min.) using moulding conditional previously determined from the torque data, obtained by a Monsanto Rheometer 100S at 140° C during 60 min. The cure time for each compound sample was defined as the time interval at which 90% of its maximum torque is reached.

Physical-mechanical measurements were done by standard methods used in the rubber industry (Popović et al., 1991). Aging resistance was determined by maintaining the samples at 70° C for 70 h in an air oven and then measuring the change in their technical properties.

Table 1. Compositions of the NR rubber compounds used.  
Tabela 1. Recepture smeša na bazi NR kaučuka za testiranje.

Ingredients, phr	sample					
	1	2	3	4	5	6
SMR CV50	100	100	100	100	100	100
Zinc oxide <sup>a</sup>	5	5	5	5	5	5
Stearic acid <sup>b</sup>	2,5	2,5	2,5	2,5	2,5	2,5
N330	60	-	-	-	-	-
N550	-	70	-	-	-	-
N990	-	-	90	-	-	-
Vulkasil C	-	-	-	60	-	-
Silteg AS-7	-	-	-	-	60	-
AOWT	-	-	-	-	-	60
Gumiol HG <sup>b</sup>	10	10	10	10	10	10
Agerite resin <sup>c</sup>	1	1	1	1	1	1
Riowax 721 <sup>d</sup>	1	1	1	1	1	1
Santoflex IPPD <sup>d</sup>	2,5	2,5	2,5	2,5	2,5	2,5
Santogard PVI <sup>e</sup>	0,5	0,5	0,5	0,5	0,5	0,5
Vulkacit MOZ <sup>f</sup>	0,7	0,7	0,7	0,7	0,7	0,7
Vulkacit thiuram <sup>f</sup>	0,2	0,2	0,2	0,2	0,2	0,2
Sulfur	2,2	2,2	2,2	2,2	2,2	2,2

<sup>a</sup> activator; <sup>b</sup> plasticizer; <sup>c</sup> antioxidant; <sup>d</sup> aging prevented; <sup>e</sup> retarder; <sup>f</sup> accelerator.

RESULTS AND DISCUSSIONS

Cure characteristics of the NR rubber compounds (see Table 1.) measured by Monsanto Rheometer 100S at 140<sup>o</sup> C are presented in Table 2.

Table 2. Monsanto cure Rheometer Data at 140<sup>o</sup> C for the samples prepared according to Table 1.  
Tabela 2. Vulkanizacije karakteristike na 140<sup>o</sup> C za uzorke pripremljene po Tabeli 1.

Cure characteristics	sample					
	1	2	3	4	5	6
MI (Nm)	6,5	5,0	2,4	3,8	3,2	2,1
Mh (Nm)	38,9	40,8	30,9	32,8	30,4	25,9
t <sub>2</sub> (min)	11,02	8,44	13,16	12,15	6,07	11,14
t <sub>90</sub> (min)	15,58	12,22	17,03	16,49	10,02	15,50
CRI (min <sup>-1</sup> )	11,30	14,10	9,90	9,30	9,80	6,70

The data obtained from the rheometer curves of the examined compounds, Table 2., have shown that samples 4-6, made with white fillers, had similar scorch times (ts2) and optimum vulcanisation times (tc90) as samples 1-3, made with carbon black fillers. From the results obtained it was also quite clear that cure rate constants of samples 4-6 were smaller than those of samples 1-3 (carbon black fillers). This can be explained by the structure of white filler and its interaction with NR rubber.

The results of the physical-mechanical properties measurements on samples used are given in Table 3.

Comparison of the physical-mechanical properties of all the samples used leads to the observation that samples 4-6 generally exhibit values smaller than those for the samples 1 and 2. Also, the samples made with white fillers (samples 4-6) exhibit the values close to those of the sample 3 (N990). Therefore, it could be concluded that it is possible to substitute some of the of carbon black fillers with appropriate white fillers.

Table 3. Physical-mechanical properties of NR vulcanizates (samples 1-6).

Tabela 3. Fizičko-mehanička svojstva NR vulkanizata (uzorci 1-6).

Physical-mechanical properties	sample					
	1	2	3	4	5	6
Modulus 200% (MPa)	110	60	68	41	33	32
Modulus 300% (MPa)	171	-	103	68	56	50
Tensile strenght (MPa)	204	185	125	158	158	125
Elongation at break (%)	370	290	390	540	540	510
Hardness, Shore A (Sh <sup>b</sup> )	70	72	61	57	53	49
Agular tearing (N/mm)	128	110	64	130	90	43
Resilience (%)	27	32	42	36	42	50
Abrasion loss (mm <sup>3</sup> )	103	115	241	199	92	110

Rubber characteristics recorded for the samples after they were exposed to the air atmosphere for

certain period of time are considered very important (Flajšman, 1988). Therefore physical-mechanical properties of the samples were measured after aging at 70°C during 70 h in the atmosphere of air. The results of these measurements are given in Table 4.

Table 4. Physical-mechanical properties of the NR vulcanisate (sample 1-6) after aging.

Tabela 4. Fizičko-mehanička svojstva uzoraka NR vulkanizata posle starenja.

Physical-mechanical properties	sample					
	1	2	3	4	5	6
Modulus 200% (MPa)	119	-	80	46	42	37
Modulus 300% (MPa)	181	-	111	74	68	56
Tensile strenght (MPa)	202	169	113	162	159	130
Elongation at break (%)	350	260	320	510	500	500
Hardness, Shore A (Sh <sup>A</sup> )	74	75	60	57	55	51
Agular tearing (N/mm)	100	98	64	116	108	42
Resilience (%)	27	31	45	35	40	49

Comparison of the values of physical-mechanical properties of all NR vulcanisates examined, before and after aging, leads to the conclusion that samples 4-6 are less prone to aging than samples 1-3 (made with carbon black fillers).

## CONCLUSIONS

The results obtained allowed for the following conclusions to be made concerning cure characteristics and physical-mechanical properties of the NR vulcanisates samples examined:

1. The samples made with white fillers exhibited scorch time values and optimum vulcanisation time values similar to those for the samples made with carbon black fillers;

2. Samples 4-6 (with white fillers) showed to be less prone to aging than samples 1-3 (with black fillers);

3. The substitution of carbon black filler type N990 (in sample 3) with the white fillers (sample 4-6), can be made without a substantial loss to the measured rubber characteristics.

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## REZIME

PRIMENA EKOLOŠKI OPRAVDANIH ADITIVA-BELIH PUNILA U GUMARSKOJ INDUSTRIJI

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Aditivi koji predstavljaju glavni faktor u kreiranju svojstava gume su punila. Upotrebom čađi dobijaju se gume kvalitetnih osobina ali njihova toksičnost je navodila na traženje ekološki opravdanih punila koja će dati gumu sa sličnim osobinama. Zbog negativnog uticaja čađi na zdravlje čoveka i njegovu okolinu ispitivana je mogućnost njihove zamene sa belim punilima.

Sa tim ciljem pripremljeno je šest smesa sa punilima različite aktivnosti, od kojih tri sa čađima (N330, N550 i N990) uzorci 1-3 i tri sa belim punilima (Vulkasil C, Silteg AS-7 i Aktivirani Opalizirani Beli Tuf-AOWT) uzorci 4-6. Fizičko-mehanička svojstva su merena standardnim metodama koje se primenjuju u gumarској industriji.

Na osnovu merenih fizičko-mehaničkih svojstava ispitivanih uzoraka utvrđeno je da je pod određenim uslovima moguće izvršiti delimičnu zamenu skupih i štetnih čađi sa jeftinijim i ekološki prihvatljivijim belim punilima, bez velike štete po tražene osobine krajnjeg proizvoda.

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# On the Theoretical proceeding for constructing the vawe function of electron

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**Abstract.** In order to define the Hamiltonian operator of the electron in the standstill, an approximate estimate of the possible aspect of the electron has been given. The wave function has been constructed and a distribution of the electrical current density in the interior of the electron has been computed. The average value for the magnetic flux operator has been computed, which coincides with the quantum of magnetic flux obtained on the basis of BCS theory. In accordance with the quantum mechanical theory, the cause of the appearance of the quantum of magnetic flux on the macroscopic level, that has predicted by BCS theory, could be this one obtained on the microscopic level. In favor of that claim, goes the Quantum Mechanical fact, that the source of the quantum of magnetic flux in Cooper pair is the magnetic momentum of a single electron [ 7].

In the contemporary Physics there exist a well-known Einstein's relation  $m_0 c^2$ . This formula is a simple product of Einstein's intuition in the frame of the classical physics. In the proposed quantum mechanical theory as a result there appears the supplementary energy member which corresponds to the null energy of the linear harmonic oscilator. It is in full agreement with the General quantum mechanical theory which says that the appearance of the null energy is connection with the Heisenberg's uncertainty principle in regard to the wave feature of a microparticle as introduced by Louis de Brolie. It means that Einstein's relation for the electron in the standstill on the level of Quantum Mechanics must be reduced into  $m_0 c^2 + \frac{1}{2} \hbar \omega_0$ . One of the consequence of the proposed theory is in relation to the possible aspect of the electron which couldn't be the spherical symmetric particle.

**Key words :** electron, Quantum of magnetic flux

## 1. Introduction.

One of the result of theory of superconductivity, better known as BCS theory, is the quantum of magnetic flux. According to Sokolov [7] the source of that flux is the magnetic momentum of a single electron in Cooper pair. We will compute the quantum of the magnetic flux that represents the cause of that predicted by BCS theory. The enormous efforts have been made by the Theoretical Physicists in this century in order to solve some problems in connection with the electron as an elementary particle. None of them has thought that the crucial idea is to bring up a question of the internal structure of the electron [14, 15].

## 2. Methods

The decisive step for solving some experimentally confirmed results by utilising theoretical methods connected with the electron such as : the quantum of magnetic flux [4], Bohr's magneton, the proof for the stability of the electron as a elementary particle, the proof that the Ohm's law with the Maxwell's equations and the Poisson's theorem are in effect, is the idea of entering the electron by the Schrödinger equation [13].

## 3. Equation of motion

In order to define the Hamiltonian operator for the electron we will draw its possible aspect, As shown in Fig 1.

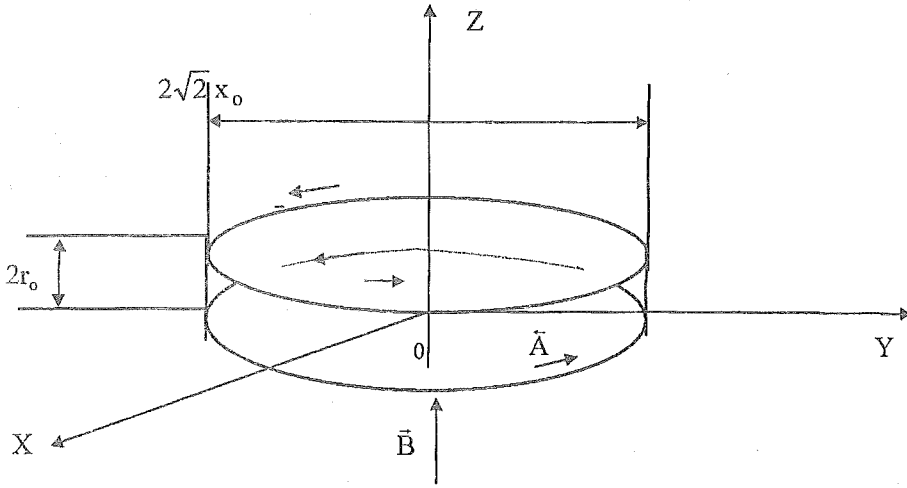


Fig.1

In accordance to Fig.1 we can write the Hamiltonian operator

$$\hat{H} = \frac{1}{2m_0} (\hat{p} + e\vec{A} + e\vec{A})^2 \quad (1)$$

or

$$\hat{H} = \frac{1}{2m_0} (\hat{p} + e2\vec{A})^2 \quad (2)$$

The space dimensions of the electron can be estimated by using the relations

$$V_0 = \frac{\mu_B}{H_e} = (2r_0) \pi (\sqrt{2} x_0)^2, \quad H_e = \frac{I_e}{2r_0}$$

where  $\mu_B$  is Bor's magneton,  $r_0$  is value equal to classical radius of electron.

$$\Phi_0 = \pi \frac{\hbar}{e} = \mu_0 \frac{\mu_B}{2r_0} = \mu_0 \frac{I_e}{2r_0} S_e = \mu_e H_e S_e$$

where  $\Phi_0$  is the quantum of magnetic flux.

$$\mu_0 \Phi_0 = \mu_B B S_e = m_0 c^2 \pi (\sqrt{2} x_0)^2$$

In our consideration we will get by means of the wave function  $\Psi$  of the electron, electrical current  $I_e$  and surface  $S_e$  within which there appears the quantum of magnetic flux.

Taking into account  $\nabla \vec{A} = 0$  we come to the Schrödinger equation

$$\left( \frac{1}{2m_0} \hat{p}^2 + 2 \frac{e}{m_0} \vec{A} \hat{p} + \frac{4e^2 A^2}{2m_0} \right) \Psi = E \Psi \quad (3)$$

where  $(\vec{A} \hat{p}) \Psi$  is

$$\begin{aligned} (\vec{A} \hat{p}) \Psi &= -i\hbar \frac{1}{2} B \rho \frac{1}{\rho} \frac{\partial}{\partial \varphi} \Psi \\ &= -i\hbar \frac{B}{2} \frac{\partial}{\partial \varphi} \Psi = -i\hbar \frac{1}{2} B \rho \frac{1}{\rho} \frac{\partial}{\partial \varphi} \Psi = -i\hbar \frac{1}{2} B \rho \frac{1}{\rho} \frac{\partial}{\partial \varphi} \Psi = 0 \end{aligned}$$

For finding a solution of this equation we shall use the method for the separation of variables in cylinder coordinates

$$\Psi(\rho, \varphi, z) = R(\rho) \Phi(\varphi) Z(z) \quad (4)$$

we assume the angle wave function in the form

$$\Phi(\varphi) = \frac{1}{\sqrt{2\pi}} e^{i0\varphi} = \frac{1}{\sqrt{2\pi}} \quad (6)$$

Because of the arbitrary motion along z-axis [10] we choose

$$Z(z) = \text{const.} \quad (7)$$

To find new form of  $\hat{H}$  we use a well-known expression for the Laplace operator in cylinder coordinates and the fact that  $\vec{A} \hat{p} \Psi = 0$

this yields

$$-\frac{\hbar^2}{2m_0} \left( \frac{d^2}{d\rho^2} + \frac{1}{\rho} \frac{d}{d\rho} \right) R(\rho) + \frac{e^2 B^2}{2m_0} \rho^2 R(\rho) = ER(\rho) \quad (8)$$

where vector potential

$$A_\varphi = \frac{1}{2} B \rho, A_z = 0, A_\rho = 0 \quad (9)$$

where

$\vec{\rho}$  is a radius vector of the electron in cylindrical coordinate system.

For  $\rho = \alpha \xi$  this equation is worded like this :

$$\frac{d^2 R}{d\xi^2} + \frac{1}{\xi} \frac{dR}{d\xi} + \left( \frac{2m_0 E}{\hbar^2} \alpha^2 - \frac{e^2 B^2}{\hbar^2} \alpha^4 \xi^2 \right) R = 0 \quad (10)$$

The solution for this equation will be sought in the form :

$$R(\xi) = e^{-\frac{\xi^2}{2}} f(\xi) \quad (11)$$

In that case the equation (10) transforms into equation

$$\frac{d^2 f}{d\xi^2} - 2\xi \frac{df}{d\xi} + \frac{1}{\xi} \frac{df}{d\xi} + \xi^2 \left( 1 - \frac{e^2 B^2}{\hbar^2} \alpha^4 \right) f + \left( \frac{2m_0 E}{\hbar^2} \alpha^2 - 2 \right) f = 0 \quad (12)$$

Let us seek the solution of equation (11) by series

$$f(\xi) = \sum_{v=0}^{\infty} a_{k+v} \xi^{k+v} \quad (13)$$

When the series substitutes into equation (12) we get

$$\begin{aligned} & \sum_{v=0}^{\infty} (k+v)(k+v-1) a_{k+v} \xi^{k+v-2} - 2 \sum_{v=0}^{\infty} (k+v) a_{k+v} \xi^{k+v} + \\ & + \sum_{v=0}^{\infty} (k+v) a_{k+v} \xi^{k+v-2} + \sum_{v=0}^{\infty} a_{k+v} \xi^{k+v+2} - 2 \sum_{v=0}^{\infty} a_{k+v} \xi^{k+v} + \\ & + \frac{2m_0 E}{\hbar^2} \alpha^2 \sum_{v=0}^{\infty} a_{k+v} \xi^{k+v} - \frac{e^2 B^2}{\hbar^2} \alpha^4 \sum_{v=0}^{\infty} a_{k+v} \xi^{k+v+2} = 0 \end{aligned} \quad (14)$$



The coefficients with  $\xi^{k+v+2}$  must be equal. Then is

$$\frac{e^2 B^2}{\hbar^2} \alpha^4 = 1 \quad (15)$$

and

$$\alpha^2 = \frac{\hbar}{eB} = 2X_0^2 \quad (15a)$$

In that case the equation (14) becomes

$$\begin{aligned} \sum_{v=0}^{\infty} (k+v)(k+v-1) a_{k+v} \xi^{k+v-2} + \sum_{v=0}^{\infty} (k+v) a_{k+v} \xi^{k+v-2} = \\ 2 \sum_{v=0}^{\infty} (k+v) a_{k+v} \xi^{k+v} + 2 \sum_{v=0}^{\infty} a_{k+v} \xi^{k+v} - \frac{2m_0 E}{\hbar^2} \alpha^2 \sum_{v=0}^{\infty} a_{k+v} \xi^{k+v} \end{aligned} \quad (16)$$

For  $v=0$  we find the smallest power of the polynomials (13)

$$k(k-1) + k] a_k \xi^{k-2} = \left( 2k+2 - \frac{2m_0 E}{\hbar^2} \alpha^2 \right) a_k \xi^k \quad (17)$$

From this follows

$$k(k-1) + k = 0 \Rightarrow k = 0 \quad (18)$$

In accordance with (18) the series will be

$$f(\xi) = \sum_{v=0}^{\infty} a_v \xi^v, (v = 0, 1, 2, \dots) \quad (19)$$

The relation for quantified energy states follows from the expression

$$a_{v+2} = \frac{2 + 2v - \frac{2m_0 E}{\hbar^2} \alpha^2}{(v+2)^2} a_v \quad (20)$$

and the energy states are

$$E_v = 2m_0 c^2 (v+1), (v = 0, 1, 2, \dots) \quad (21)$$

If we introduce the series (19) into (12) we get

$$\begin{aligned}
& \left( 2a_2 + 6a_3\xi + 12a_4\xi^2 + 20a_5\xi^3 + 30a_6\xi^4 + \dots \right) + \\
& \left( -2a_1\xi - 4a_2\xi^2 - 6a_3\xi^3 - 8a_4\xi^4 - \dots \right) + \\
& \left( \frac{a_1}{\xi} + 2a_2 + 3a_3\xi + 4a_4\xi^2 + 5a_5\xi^3 + 6a_6\xi^4 + \dots \right) + \\
& \left( a_0\xi^2 + a_1\xi^3 + a_2\xi^4 + \dots \right) \left( 1 - \frac{e^2 B^2}{\hbar^2} \alpha^4 \right) + \\
& \left( a_0 + a_1\xi + a_2\xi^2 + a_3\xi^3 + a_4\xi^4 + \dots \right) \left( \frac{2m_0 E}{\hbar^2} \alpha^2 - 2 \right) = 0
\end{aligned} \tag{22}$$

In order to avoid the singularity in this equation must be  $a_1 = 0$ . The consequence of this fact is that all next coefficients of the series (19) are equated to null. Then the series (19) reduces into

$$f(\xi) = a_0 \tag{23}$$

On the basis of obtained results the normalized form of the radial wave function will be

$$R(\rho) = \frac{1}{X_0} e^{-\frac{\rho^2}{4X_0^2}} \tag{24}$$

and the energy state is

$$E_0 = 2m_0 c^2 \tag{25}$$

### 3. The calculation of the quantum of the magnetic flux and Heisenberg's uncertainty relation

Eigenfunction and eigenvalue of the Hamiltonian operator apparently are :

$$R_0(\rho) = \frac{1}{X_0} e^{-\frac{\rho^2}{4X_0^2}}$$

and

$$E_0 = 2m_0c^2$$

Indeed, if we find the appropriate derivations of the wave function (24)

$$\frac{1}{\rho} \frac{d}{d\rho} R(\rho) = -\frac{1}{2X_0^2} R(\rho) \quad (26)$$

$$\frac{d^2}{d\rho^2} R(\rho) = \left( -\frac{1}{2X_0^2} + \frac{1}{4X_0^4} \rho^2 \right) R(\rho) \quad (27)$$

and then introduce into equation (8)

$$-\frac{\hbar^2}{2m_0} \left( -\frac{1}{X_0^2} + \frac{1}{4X_0^4} \rho^2 \right) R(\rho) + \frac{m_0c^2}{2X_0^2} \rho^2 R(\rho) = E_0 R(\rho) \quad (28)$$

then we get

$$\left( \frac{\hbar^2}{2m_0X_0^2} - \frac{m_0c^2}{2X_0^2} \rho^2 + \frac{m_0c^2}{2X_0^2} \rho^2 \right) R(\rho) = E_0 R(\rho) \quad (29)$$

where is

$$\frac{e^2B^2}{2m_0} = \frac{m_0c^2}{2X_0^2} \quad (30)$$

From (29) immediate follows the eigenvalue for the Hamiltonian operator

$$E_0 = \frac{\hbar^2}{2m_0X_0^2} = 2m_0c^2 \quad (31)$$

The average energy of the electron is determined by relation

$$\langle \Psi | \hat{H} | \Psi \rangle = \langle E \rangle = \int_{-\infty}^{\infty} \Psi^* (H\Psi) d\tau \quad (32)$$

$$\begin{aligned} \langle E \rangle &= \frac{1}{2m_0} \int_0^{\infty} R \left[ \left( \hat{p} + 2e\bar{A} \right)^2 R \right] \rho d\rho = \frac{1}{2m_0} \int_0^{\infty} R \left\{ \left[ -\hbar^2 \left( \frac{d^2}{d\rho^2} + \frac{1}{\rho} \frac{d}{d\rho} \right) + e^2B^2\rho^2 \right] R \right\} \rho d\rho = \\ &= 2m_0c^2 \end{aligned} \quad (33)$$

The average value for the magnetic flux operator is

$$\langle \Psi | \hat{\Phi} | \Psi \rangle = \langle \Phi \rangle = \langle R | \pi \hat{\rho}^2 B | R \rangle = \pi B \int_0^\infty R \rho^2 R \rho d\rho = \pi \frac{\hbar}{e} \quad (34)$$

The average value for the coordinate operator is

$$\langle \Psi | \hat{\rho} | \Psi \rangle = \langle R | \hat{\rho} | R \rangle = \langle \rho \rangle = \int_0^\infty R \rho R \rho d\rho = \frac{\sqrt{2\pi}}{2} X_0 \quad (35)$$

$$\langle \Psi | \hat{\rho}^2 | \Psi \rangle = \langle R | \hat{\rho}^2 | R \rangle = \langle \rho^2 \rangle = \int_0^\infty R \rho^2 R \rho d\rho = 2X_0^2 \quad (36)$$

The average value for the appropriate momentum operators are

$$\begin{aligned} \langle \Psi | \hat{P} | \Psi \rangle &= \langle R | \hat{p} + 2e\vec{A} | R \rangle = \int_0^\infty R \left( -i\hbar \frac{\partial}{\partial \rho} \bar{e}_\rho + eB\rho \bar{e}_\varphi \right) R \rho d\rho \\ &= \langle P \rangle = i\hbar \frac{\sqrt{2\pi}}{4X_0} \bar{e}_\rho + \hbar \frac{\sqrt{2\pi}}{4X_0} \bar{e}_\varphi \end{aligned} \quad (37)$$

$$\langle P \rangle^2 = -\hbar^2 \frac{2\pi}{16X_0^2} + \hbar^2 \frac{2\pi}{16X_0^2} = 0 \quad (38)$$

$$\langle \Psi | \hat{P}^2 | \Psi \rangle = \langle R | \hat{P}^2 | R \rangle = \int_0^\infty R \left[ -\hbar^2 \left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} \right) + e^2 B^2 \rho^2 \right] R \rho d\rho = \frac{\hbar^2}{X_0^2} \quad (39)$$

The amounts of the appropriate fluctuation will be

$$\langle \Delta \rho^2 \rangle = \langle \rho^2 \rangle - \langle \rho \rangle^2 = \frac{4-\pi}{2} X_0^2 \quad (40)$$

$$\langle \Delta P^2 \rangle = \langle P^2 \rangle - \langle P \rangle^2 = \frac{\hbar^2}{X_0^2} \quad (41)$$

Now we can make the product

$$\langle \Delta P^2 \rangle \langle \Delta \rho^2 \rangle = \frac{\hbar^2 (4-\pi)}{2} \quad (42)$$

and the Heisenberg uncertainty relation will be satisfied:

$$\sqrt{\langle \Delta P^2 \rangle} \sqrt{\langle \Delta p^2 \rangle} = 1.3 \frac{\hbar}{2} \quad (43)$$

#### 4. Concluding remarks

The wave feature of the particles in Quantum Mechanics, introduced by Luis de Broglie and Heisenberg's uncertainty principle comprehend of being the null energy, expressed by relation [3].

$$W_0 = \frac{1}{2} \hbar \omega$$

Applying that fact to the whole energy of electron in the standstill (25,33) equal follows that it must consist of  $E = 2m_0 c^2$  two parts

$$E = m_0 c^2 + W_0 = m_0 c^2 + \frac{1}{2} \hbar \omega_0, \quad \omega_0 = \frac{c}{x_0} \quad (45)$$

The first part is well known Einstein's energy of electron in the standstill. This value is a product of Einstein's invitation on the level of classical Physics. The similar case happened with the Bohr's quantum theory for harmonic oscillator.

N. Bohr introduced the rules of quantization :

$$\oint p_x dx = 2\pi \hbar n$$

$$E_{n_B} = n \hbar \omega, \quad (n = 0, 1, 2, \dots) \quad (46)$$

The rigorous quantum theory of harmonic oscillator gives exact formula for energy levels

$$E_n = \hbar \omega \left( n + \frac{1}{2} \right) = E_{n_B} + \frac{1}{2} \hbar \omega, \quad (n = 0, 1, 2, \dots) \quad (47)$$

Where appears the energy member of  $\frac{1}{2} \hbar \omega$  that expresses null energy of the harmonic oscillator.  $N_0$  matter of the stand of motion of a particle that member remains a constant.

#### 4. Derivation of vector field of the probability current density

$$i \hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi \quad (47)$$

$$-i \hbar \frac{\partial \Psi^*}{\partial t} = \hat{H}^* \Psi^* \quad (48)$$

$$\hat{H} = \frac{1}{2m_0} (\hat{p} + 2e\vec{A})^2, \quad \hat{H}^* = \frac{1}{2m_0} (\hat{p}^* + 2e\vec{A})^2 \quad (49)$$

$$(\hat{p} + 2e\vec{A})^2 = (\hat{p} + 2e\vec{A}) (\hat{p} + 2e\vec{A}) = p^2 + 2e\hat{p}\vec{A} + 2e\vec{p}\vec{A} + 4e^2 A^2 \quad (50)$$

Taking into account

$$[\hat{p}, \vec{A}] = \hat{p}\vec{A} - \vec{A}\hat{p} = -i\hbar \nabla \vec{A} \quad (51)$$

and  $\nabla \vec{A} = 0$

We will get :

$$\begin{aligned} (\hat{p} + 2e\vec{A})^2 &= p^2 + 2e(-i\hbar \nabla \vec{A} + \vec{A}\hat{p}) + 2e\vec{A}\hat{p} + 4e^2 A^2 \\ &= p^2 + 4e\vec{A}\hat{p} + 4e^2 A^2 = -\hbar^2 \nabla^2 - i\hbar 4e\vec{A}\nabla + 4e^2 A^2 \end{aligned} \quad (52)$$

$$(\hat{p}^* + 2e\vec{A})^2 = -\hbar^2 \nabla^2 + i\hbar 4e\vec{A}\nabla + 4e^2 A^2 \quad (53)$$

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{1}{2m_0} (-\hbar^2 \nabla^2 - i\hbar 4e\vec{A}\nabla + 4e^2 A^2) \Psi \quad (54)$$

$$-i\hbar \frac{\partial \Psi^*}{\partial t} = \frac{1}{2m_0} (-\hbar^2 \nabla^2 + i\hbar 4e\vec{A}\nabla + 4e^2 A^2) \Psi^* \quad (55)$$

$$i\hbar \frac{\partial \Psi}{\partial t} \Psi^* = -\frac{\hbar^2}{2m_0} (\nabla^2 \Psi) \Psi^* - \frac{i\hbar}{m_0} 2e\vec{A}(\nabla \Psi) \Psi^* + \frac{2e^2 A^2}{m_0} \Psi \Psi^* \quad (56)$$

$$-i\hbar \frac{\partial \Psi^*}{\partial t} \Psi = -\frac{\hbar^2}{2m_0} (\nabla^2 \Psi^*) \Psi + \frac{i\hbar}{m_0} 2e\vec{A}(\nabla \Psi^*) \Psi + \frac{2e^2 A^2}{m_0} \Psi^* \Psi \quad (57)$$

Subtracting equation (57) and (56) one obtains :

$$\begin{aligned} -i\hbar \frac{\partial \Psi}{\partial t} \Psi^* + i\hbar \frac{\partial \Psi^*}{\partial t} \Psi &= -\frac{\hbar^2}{2m_0} [(\nabla^2 \Psi) \Psi^* - (\nabla^2 \Psi^*) \Psi] - \frac{i\hbar}{m_0} 2e\vec{A}[(\nabla \Psi) \Psi^* + \\ &\quad + (\nabla \Psi^*) \Psi] + \frac{2e^2 A^2}{m_0} (\Psi \Psi^* - \Psi^* \Psi) \end{aligned} \quad (58)$$

$$i\hbar \frac{\partial}{\partial t} (\Psi^* \Psi) = -\frac{\hbar^2}{2m_0} \nabla [(\nabla \Psi) \Psi^* - (\nabla \Psi^*) \Psi] - \frac{i\hbar}{m_0} 2e\vec{A} \nabla (\Psi^* \Psi) \quad (59)$$

$$\frac{\partial}{\partial t} (\Psi^* \Psi) + \nabla \cdot \left\{ -\frac{i\hbar}{2m_0} [(\nabla \Psi) \Psi^* - (\nabla \Psi^*) \Psi] + \frac{2e\vec{A}}{m_0} \Psi^* \Psi \right\} = 0, \quad \frac{\partial}{\partial t} (\Psi^* \Psi) + \nabla \cdot \vec{j}_e = \quad (60)$$

This is the equation of continuity where the vector of probability current density is expressed by relation :

$$\vec{j}_e = -\frac{i\hbar}{2m_0}(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) + \frac{2e\vec{A}}{m_0} \Psi^* \Psi \quad (61)$$

Electrical current density of the electron will be :

$$\vec{J}_e = e \vec{j}_e = \frac{ie\hbar}{2m_0}(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) - \frac{2e\vec{A}}{m_0} \Psi^* \Psi \quad (62)$$

Taking into account that the wave function of the electron are real functions we will get

$$\vec{J}_e = -2 \frac{e^2 \vec{A}}{m_0} \Psi^* \Psi = -2 \frac{e^2 A_\phi}{m_0} \Psi^* \Psi \vec{e}_\phi \quad (63)$$

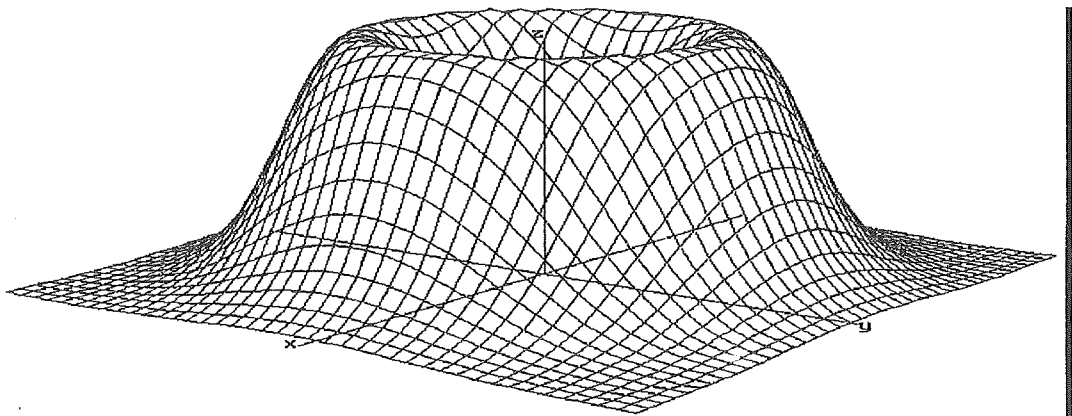
The reason of appearing the minus sign in equation (63) can be seen from the Maxwell's equation :

$$\begin{aligned} \nabla \times \vec{H}_e &= \begin{vmatrix} \frac{1}{\rho} \vec{e}_\rho & \vec{e}_\phi & \frac{1}{\rho} \vec{e}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & H_e \end{vmatrix} = \frac{1}{\rho} \vec{e}_\rho \frac{\partial}{\partial \phi} H + \vec{e}_\phi \left( -\frac{\partial}{\partial \rho} H \right) + \frac{1}{\rho} \vec{e}_z \left( \frac{\partial}{\partial \rho} 0 - \frac{\partial}{\partial \phi} 0 \right) = \\ &= -\frac{\partial}{\partial \rho} H \vec{e}_\phi = -2 \frac{e^2 A_\phi}{m_0} \Psi^* \Psi \vec{e}_\phi \end{aligned} \quad (64)$$

Apparently from this relation follows :

$$dH = 2 \frac{e^2 A_\phi}{m_0} \Psi^* \Psi d\rho \quad (65)$$

In Fig. 1. ,the distribution of the electrical current density (63) in the interior of the electron is shown.



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$$J_e = 1.8 \times 10^{43} \sqrt{x^2 + y^2} e^{-\frac{(x^2 + y^2)}{2(193 \times 10^{-15})^2}} \left( \frac{A}{m^2} \right)$$

Fig. 1. Distribution of the electrical current density in the interior of the electron

The second way of obtaining this field is (65) by using the differential element of the electrical current intensity :

$$dI_e = J_e dpdz = \vec{J}_e d\vec{\sigma} = J_e dpdz = 2 \frac{e^2 A_\phi}{m_0} \Psi^* \Psi dp dz = dH dz \quad (66)$$

From that one obtains the formula immediately (66).

#### Acknowledgment

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## Rezime

### O teorijskom postupku konstrukcije talasne funkcije

Todorović, Z.

Fundamentalni problem pri rešavanju bilo kojeg kvantnomehaničkog sistema u domenu mikročestica je vezan za određivanje talasne funkcije  $\Psi$ . Ovde je primenjen klasičan postupak u kvantnoj mehanici kojim se došlo do konstrukcije talasne funkcije za elektron. Međutim, ovde je bila ključna ideja u određivanju Hamiltonovog operatora za elektron koji je bio presudan za izračunavanje talasne funkcije  $\Psi$ . Zatim je ta talasna funkcija iskorišćena za izračunavanje, pte svega, Hajzenbergove relacije neodređenosti koja je izražena formulom (43). Dobijeni rezultat je u saglasnosti sa zakonima kvantne mehanike koji kažu da proizvod kvantnih fluktuacija operatora impulsa i odgovarajuće koordinate mora biti veći ili jednak  $\epsilon/2$ . Drugi važan rezultat primene talasne funkcije  $\Psi$ , se odnosi na određivanje površine na kojoj se javlja kvant magnetnog fluksa (34). Ovaj kvant magnetnog fluksa predstavlja po svojoj prirodi uzrok kvantu magnetnog fluksa predviđen BCS teorijom [2]. Naime, u kvantnoj mehanici [7] se kaže da je izvor kvanta magnetnog fluksa kod uparenih elektrona, poznati kao Kuperov par, magnetni moment jednog elektrona. I treći važan rezultat se odnosi na ukupnu energiju elektrona u stanju mirovanja. Ona je jednaka dvostruko vrednosti energije mirovanja elektrona i iznosi  $2 m_0 c^2$ . Na prvi pogled moglo bi se reći da ovaj paradoksalni rezultat nije tačan jer je eksperimentalna činjenica da je ona  $m_0 c^2$ . Međutim, kvantna mehanika predviđa zbog talasnih osobina čestica prema De Brojlu i Hajzenbertgove relacije neodređenosti, da svaka partikla, ako se ponaša kao linearni harmonijski oscilator, a elektron to jeste s obzirom na njegovu diferencijalnu jednačinu (8), mora da poseduje nultu energiju jednaku  $\hbar\omega/2$  (47). Prema tome, očigledno je da elektron, pored Ajnštajnovе vrednosti  $m_0 c^2$  poseduje i dodatni član nulte energije iztažen formulom (47), tako da energija elektrona u stanju mirovanja glasi :

$$E = m_0 c^2 + \frac{1}{2} \hbar \omega_0, \quad (\omega_0 = \frac{c}{x_0}).$$

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# ON THE INTERNAL MASS DISTRIBUTION OF THE ELECTRON

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**Abstract** - *In accordance with the equation of continuity inside the volume of the electron, an evidence of the claim validity was provided that the electron is a stable particle. Theoretical proceedings for computing the electron internal energy has been confirmed that its value coincides with the energy of the electron in the standstill. On the basis of the mass distribution in the cross sectional area of the electron has been pointed out that the whole mass of the electron could be of the electromagnetic nature.*

**Key words:** electron, Gauss's integral theorem, mass distribution

## Introduction

Enormous efforts have been made by the Theoretical Physicists in this century in order to solve some problems regarding the electron as an elementary particle. Nobody of them has been thought that the crucial idea is to bring up a question of the internal structure of the electron [2, 3].

## Methods

The decisive step for solving this problem by utilising theoretical methods confirmed by experimental results connected with the electron such as : the quantum of magnetic flux, Bohr's magneton, an evidence for the stability of the electron as an elementary particle, an evidence that the Ohm's law with the Maxwell's equations and the Poisson's theorem are in effect, is the idea of entering the electron by the Schrödinger equation [1].

## Results

In order to solve the problem of the mass distribution inside the volumes of an electron. we will start with the distribution of the vector of the probability of current density for one electron expressed by the following relation [1]

$$\vec{j} = \frac{i\hbar}{2m_0} (\Psi^* \text{grad} \Psi - \Psi \text{grad} \Psi^*) - 2 \frac{e\vec{A}}{m_0} \Psi^* \Psi \quad (1)$$

where the wave functions take the following forms:

$$\Psi(\rho, \varphi, z) = R(\rho) \Phi(\varphi) Z(z) \quad (2)$$

$$R(\rho) = \frac{1}{x_0} e^{-\frac{\rho^2}{4x_0^2}} \quad (3)$$

$$\Phi(\varphi) = \frac{1}{\sqrt{2\pi}} \quad (4)$$

$$Z(z) = \text{const.}, \quad \int_0^\infty R^2 \rho d\rho \int_0^{2\pi} \Phi^2 d\varphi \int Z^2 dz = 1 \quad (5)$$

$$\vec{A} = \frac{1}{2} (\vec{B} \times \vec{\rho}) = \frac{1}{2} B \rho \vec{e}_\varphi \quad (6)$$

where

$\vec{\rho}$  is a radius vector of the electron in cylindrical coordinate system.

For one electron the mathematical expectation is equal to the probability of the electron being in  $dV$ ,

$$\Psi^* \Psi dV$$

Taking into account that the wave function,  $\Psi$  is a real function, the equation (1) is reduced to

$$\begin{aligned} \vec{J}_e &= 2 \frac{e^2}{m_0} \vec{A} \Psi^* \Psi = 2 \frac{e^2}{m_0} \frac{1}{2} B \rho \Psi^* \Psi \vec{e}_\varphi = \frac{e^2}{m_0} \frac{m_0 c}{x_0 e} \rho \Psi^* \Psi \vec{e}_\varphi = \\ &= \frac{c}{x_0} \rho \Psi^* \Psi \vec{e}_\varphi = \omega_0 \rho \Psi^* \Psi \vec{e}_\varphi, \quad \left( B = \frac{m_0 c^2}{\mu_B} = \frac{m_0 c}{x_0 e} \right) \end{aligned} \quad (7)$$

The equation of continuity for this case reads :

$$\frac{\partial}{\partial t} (\Psi^* \Psi) + \text{div} (\omega_0 \rho \Psi^* \Psi) \vec{e}_\varphi = 0 \quad (8)$$

We point out that the electrical current density is [1]

$$\begin{aligned} \vec{J}_e &= 2 \frac{e^2}{m_0} \vec{A} \Psi^* \Psi = 2 \frac{e^2}{m_0} \frac{1}{2} B \rho \Psi^* \Psi \vec{e}_\varphi = \frac{e^2}{m_0} \frac{m_0 c}{x_0 e} \rho \Psi^* \Psi \vec{e}_\varphi = \\ &= e \frac{c}{x_0} \rho \Psi^* \Psi \vec{e}_\varphi \end{aligned} \quad (9)$$

The differential element of the electrical current intensity is

$$dI_e = \vec{J}_e d\vec{\sigma} = e \frac{c}{x_0} \rho \Psi^* \Psi d\rho dz \quad (10)$$

and the total current is

$$\begin{aligned}
 I_e &= \iint dI_e = e \frac{c}{x_o} \int_0^\infty \rho R^2 d\rho \int Z^2 dz \Phi^2 = \\
 &= e \frac{c}{x_o} \frac{1}{2\pi} = e \frac{\omega_o}{2\pi}
 \end{aligned} \tag{11}$$

Using the fact that the electrical current has been defined by the formula

$$I_e = e \frac{\omega_o}{2\pi} = e v_o \tag{12}$$

according to the equation (12) immediately following

$$\omega_o = \frac{c}{x_o} \tag{13}$$

Let us find the value of the second part of the equation (8)

$$\begin{aligned}
 \text{div } \vec{J}_e &= \text{div}(\omega_o \rho \Psi^* \Psi) \vec{e}_\varphi = \left( \frac{1}{\rho} \frac{\partial}{\partial \varphi} \vec{e}_\varphi \right) \left( \frac{\omega_o}{x_o^2} \rho e^{-\frac{\rho^2}{2x_o^2}} \Phi^2 Z^2 \right) \vec{e}_\varphi = \\
 &= \frac{1}{\rho} \frac{\partial}{\partial \varphi} \left( \omega_o \rho \frac{1}{(2r_o)\pi(\sqrt{2} x_o)^2} e^{-\frac{\rho^2}{2x_o^2}} \right) = 0
 \end{aligned} \tag{14}$$

From the equation (13) follows

$$\frac{\partial}{\partial t}(\Psi^* \Psi) = 0, \quad \frac{\partial}{\partial t} \left( \frac{1}{x_o^2} e^{-\frac{\rho^2}{2x_o^2}} \Phi^2 Z^2 \right) = 0 \tag{15}$$

Then the integral form of the equation of the continuity taking into account Gauss's integral theorem, will be :

$$\begin{aligned}
 \frac{\partial}{\partial t} \iiint_{V_o} \Psi^* \Psi dV &= -\omega_o \iiint_{V_o} \text{div}(\rho \Psi^* \Psi \vec{e}_\varphi) dV = -\oint_{\sigma_o} \vec{j} d\vec{S} = 0 \\
 -\frac{2\rho}{2x_o^2} \frac{d\rho}{dt} &= 0 \Rightarrow \frac{d\rho}{dt} = 0, \quad \dot{\rho} = 0
 \end{aligned} \tag{16}$$

where  $V_0$  is the volume of the electron and  $\sigma_0$  is a surface that encompasses  $V_0$ .

The equation (16) can be interpreted as a meaning that the change in the mathematical expectation of the matter of electron inside  $V_0$  is equal to the mathematical expectation of that matter of electron passing through  $\sigma_0$  in unit time. The fact that these quantities have equated to null means that the electron is a stable particle. The equality to null of divergency of the vector of the distribution of the probability current density means that its field has the solenoidal character. In other words there are neither the precipices nor the sources of the matter in the volume of the electron.

If we multiply the equation (7) by the electron mass in the standstill  $m_0$ , we will get the vector of the probability mass density

$$\vec{j}_m = \omega_0 m_0 \rho \Psi^* \Psi \vec{e}_\varphi \quad (17)$$

The differential element of the electron mass is equal to

$$\begin{aligned} dm &= \frac{2\pi\rho}{v} m_0 \left\{ \frac{i\hbar}{2m_0} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) - 2 \frac{e\vec{A}}{m_0} \Psi^* \Psi \right\} d\vec{\sigma} \\ m_0 \vec{j}_m d\vec{\sigma} &= dm \frac{\omega_0}{2\pi} = dm v_0 \end{aligned} \quad (18)$$

where  $v_0$  is linear frequency,  $v = \rho \omega_0$ ,  $d\vec{\sigma} = (d\rho dz) \vec{e}_\varphi$ , or

$$dm = \frac{2\pi\rho}{\rho\omega_0} m_0 \omega_0 \rho \Psi^* \Psi d\rho dz = m_0 \Psi^* \Psi 2\pi\rho d\rho dz = m_0 \Psi^* \Psi dV \quad (19)$$

The internal energy of the electron we can compute by relation:

$$\begin{aligned} E &= \frac{1}{2} \iiint v^2 dm = \frac{1}{2} \iiint (\rho \omega_0)^2 m_0 \Psi^* \Psi dV \\ &= \frac{1}{2} m_0 \omega_0^2 \int_0^\infty R^* \rho^2 R \rho d\rho \int_0^{2\pi} \Phi^2 d\varphi \int Z^2 dz \\ &= \frac{1}{2} m_0 \omega_0^2 2x_o^2 = m_0 \frac{c^2}{x_o^2} x_o^2 = m_0 c^2 \\ &\quad (x_o = \frac{\hbar}{2m_0 c}) \end{aligned} \quad (20)$$

Using the definition of the moment of inertia

$$J = \iiint \rho^2 dm \quad (21)$$

we can compute the moment of inertia of the electron

$$J_e = \iiint \rho^2 (m_o \Psi^* \Psi dV) = m_o \int_0^\infty \rho^2 R^2 \rho d\rho \int_0^{2\pi} \Phi^2 d\varphi \int Z^2 dz = m_o (\sqrt{2} x_o)^2 \quad (22)$$

If we use the relation that connects the energy with the moment of inertia

$$E = \frac{1}{2} J_e \omega_o^2 \quad (23)$$

one obtains

$$E = \frac{1}{2} m_o (\sqrt{2} x_o)^2 \omega_o^2 = \frac{1}{2} m_o 2 x_o^2 \frac{c^2}{x_o^2} = m_o c^2 \quad (24)$$

Using the relation (14) we can get the mass distribution inside the volume of the electron  $V_o$  [2]

$$dm = \frac{m_o}{(2r_o) \pi (\sqrt{2} x_o)^2} e^{-\frac{\rho^2}{2x_o^2}} dV = \frac{m_o}{V_o} e^{-\frac{\rho^2}{2x_o^2}} dV = \frac{m_o}{V_o} e^{-\frac{\rho^2}{2x_o^2}} \rho d\rho d\varphi dz \quad (25)$$

From the equation (19) we can see what is the volume density of the electron matter

$$dm = \rho_m dV \quad (26)$$

$$\rho_m = \frac{m_o}{V_o} e^{-\frac{\rho^2}{2x_o^2}} \quad (27)$$

The distribution of  $\rho_m$  has been drawn in Figure 1.

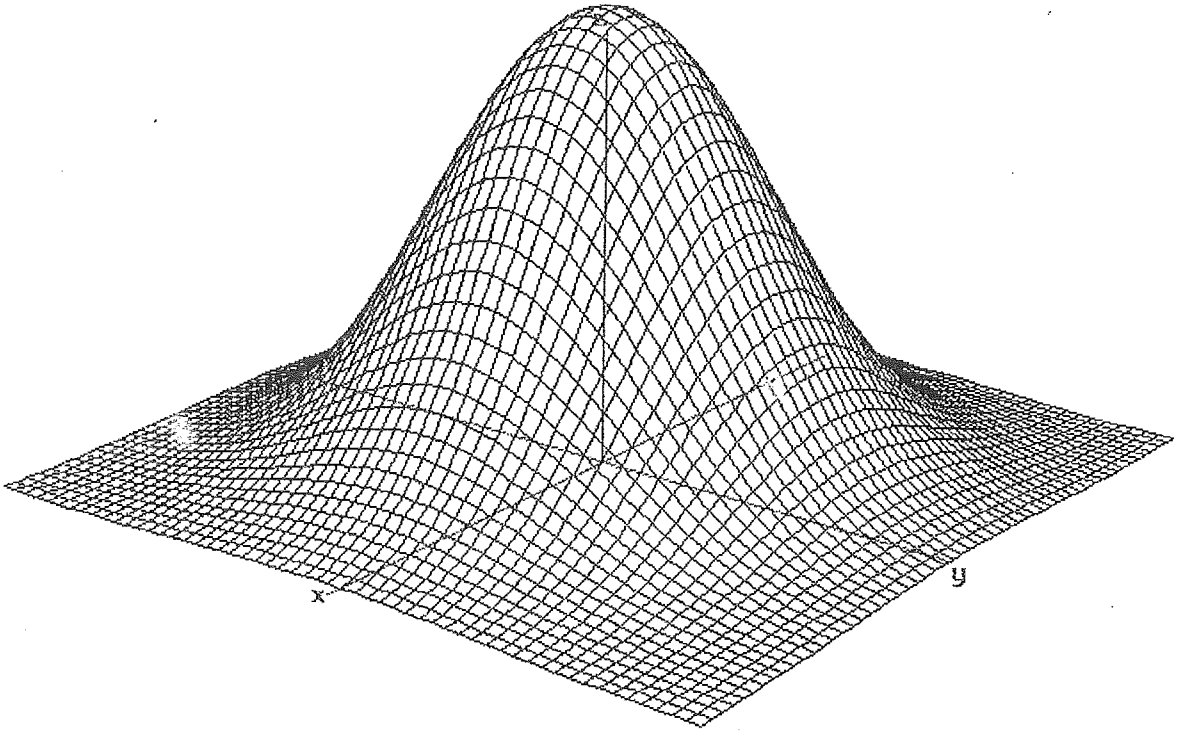


Fig.1.

The distribution of the electron matter accross the differential surface element  $d\sigma$  would be

$$\frac{dm}{d\sigma} = 2\pi \frac{m_o}{V_o} \rho e^{-\frac{\rho^2}{2x_o^2}} \quad (28)$$

The aspect of this distribution is shown in Fig.2.



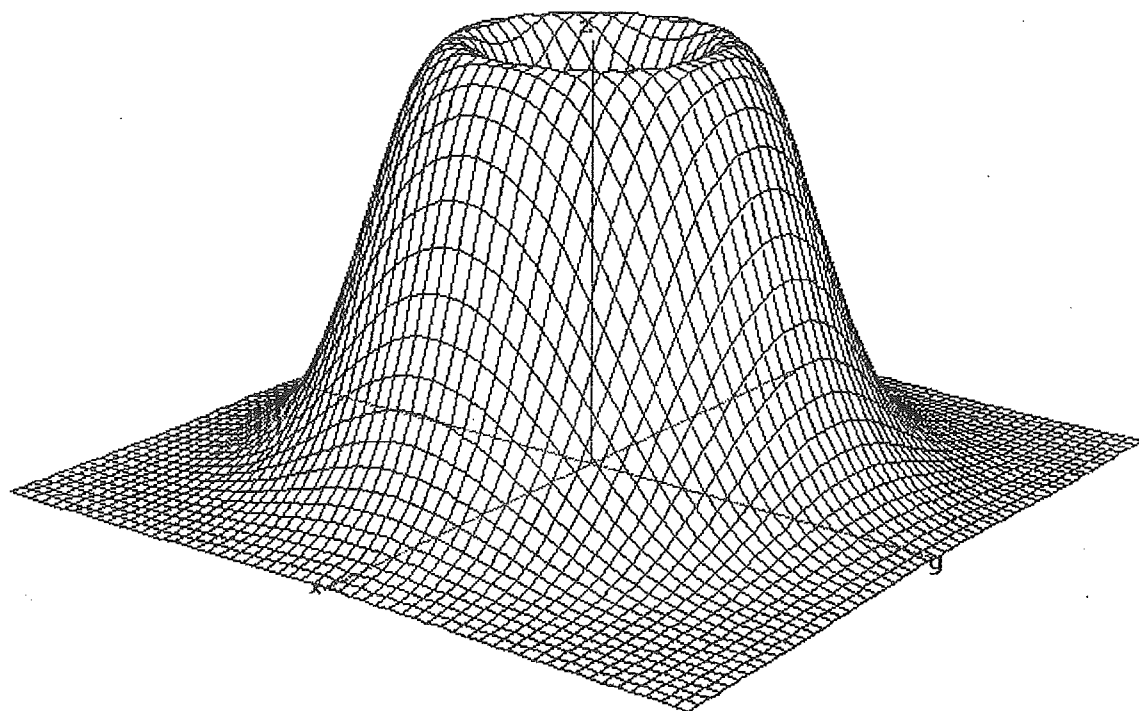


Fig.2.

The relation (15) points out that the electron mass could be purely of the electromagnetic nature. Also in favor of such claim the fact of the equality of the distribution functions inside the electron of the electrical current density [10] and the mass current density (24). In the other words : the flow of the electrical current density coincides with the flow of the mass current density.

### Acknowledgment

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## Rezime

## O UNUTRAŠNJOJ RASPODELI MASA U ELEKTRONU

Todorović, Z

Saglasno kvantnomehaničkim zakonima određena je raspodela mase u zapremini elektrona. Izračunat je moment inercije i unutrašnja energija elektrona i pokazano je da se ona poklapa sa Ajnštajnovom relacijom  $m_0 c^2$ . Korišćenjem Gausove integrale teoreme pokazano je da je divergencija vektorskog polja unutrašnje raspodele električne struje jednaka nuli. To znači da je to polje solenoidnog karaktera. Ova činjenica sa stanovišta matematičke formulacije solenoidnog polja znači da u tom polju nema niti izvora niti ponora. Drugim rečima, prostor u kome cirkulišu ove struje je konstantne zapremine pa samim tim i mase, što potvrđuje shvatanje u savremenoj fizici da je elektron stabilna čestica. S obzirom na poklapanje unutrašnjih raspodela elementarnih električnih struja i masa, zaključuje se da bi celokupna masa elektrona mogla biti elektromagnetne prirode.

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# Mechanical modeling of Theropoda/Tetanurae gait with the extension in frequency domain

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## ABSTARCT

In this paper it is proposed and disscussed a mechanical model of the Theropoda gait in which the whole 3D dynamics of the body during the gait process is reduced to a hybrid static system with variable continuous load defined by a set of functions, where its equivalent concentric load acts at the axle between hip joints. This opportunity makes a background for a simplified dynamic model of the walk system which kinematics is thoroughly analysed implying the principle of least action. Considering the hind limb as a five-element system detailed stride kinemat-

ics is restored, yielding the stride length and its dependance of the gait mode. In order to estimate the minimum gait speed a generalised version of the model is analysed, with the definition of stationary gravitational walk. To complete the analysis of the whole stride period it is proposed the detailed qalytitative kinematics of the hind limb on its return trajectory to the starting position. It is also suggested the use of spectral and correlation analysis for trajectories of particular leg joints that would bring additional indicators for the gait study.

**Key words:** mechanical model, spectral analysis, dinosauria, theropoda

## INTRODUCTION

The walking system of bipedal dinosauria and its study is of great importance, since in its evolutionary appearance (according to Craig, A. [1]) can be found one of chief reasons for their rapid development, proliferation and dominance during almost the whole Mesozoic Era. Also, the analysis of its functioning considerably facilitates reliable restoration of the life style of many theropods. Its indisputable effectiveness confirms the fact that all Theropoda possess namely such a gait system, where during the Jurassic and Cretaceous Periods, after it became evolutionary matured in Triassic Period, two distinctive and characteristic variants can be traced in the Infraorder Tetanurae, with specific representatives in Families Dromaeosauridae and Tyrannosauridae, where the weight of the taxon is a main distinguishable criterion for the structure of the gait system.

For Dromaeosauridae possibly the most typical representative is Deinonychus (Fig.1) [2], belonging to small-size predators (with the length 2.5-3m and the weight 50-60kg), which second toe claw is modified into a huge rotatable sickle enabling a rather broad predation range from quite a small pray to one of multiple Deinonychus' size, making this predator extremely effective. Its terminal predatory operations include bite and rip. The importance of such an evolutionary design is indicated by the fact that over 20% of already limited feet support of that animal, being also bipedal, is used for predatory purposes, without decrease of its manoeuvrability, as such a gait system easily operates weight of 60kg and accepts accompanied dynamic loads. Deinonychus' anatomy



Fig.1 *Deinonychus* as a typical small predator in the Suborder Theropoda, shown aside man's silhouette for size indication.

Sl.1 *Deinonychus* kao tipičan manji predator iz reda Theropoda, prikazan u odnosu na gabarite čoveka.

shows its abilities to accomplish very complex moving modes: gait with various speeds, running, jumps, instant direction changes, all that requested perfect coordination. Complete mechanical analysis of all moving modes would be extremely complicated.

The huge predators are best represented through *Tyrannosaurus rex* (Fig.2) [2]. Because of its weight he was oriented predominatly to big and hugest prey specimens. The only terminal predatory operation was the bite, and the hind limbs had the task to provide to the 6 tonnes' body reliable static and dynamic support, so its two legs and six toes had no reserve for additional predatory organs, and also there was no need for that. Even the function of forelimbs was minorised, since besides strong bite the prominent forelimbs with catching claws, as at *Deinonychus*, providing firm grasp of victim and strong support for the ripping hind limb, was quite unnecessary. Because of its great weight

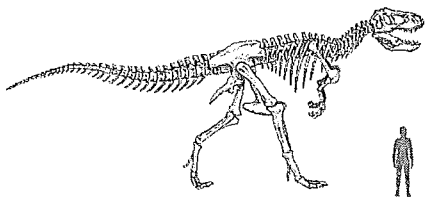


Fig.2 Tyrannosaurus rex - the biggest predator where bipedal system reached its top potentials

Sl.2 Tyrannosaurus rex, kod koga je bipedni koncept dostigao svoj vrhunac.

Tyrannosaurus rex covered rather narrow range of moving modes, including, as can be reliably stated, just walk at different speeds, i.e. without flight phase within the stride. Therefore, the modeling of Tyrannosaurus rex's gait appears to be much easier. Despite that fact there is a striking dispersion of its gait speed estimates and, consequently, moving modes throughout the literature: from 70km/h (Bakker, R.) over 35km/h (Robinson, J., Smith M.) to 7km/h (McNeill, A.) [3]. There we find the reason to propose another model.

### SPECIFIC ASPECTS OF THEROPODA'S MECHANICAL MODELS

Mechanical modeling of Theropoda's gait appears to be very complicated task, since one must deal with extremely complex system comprising numerous elements with different functionality, which relationships and functioning conditions is subject to permanent change, as a consequence of body flexibility. Above all, we lack the detailed knowledge of such a mechanical system, since the only reliable information is related to the remnants of the skeleton. Because of that fact certain simplifications must take place instead of searching for a precise model, which would be done in a way not to obscure and intolerably influence the essence of the problem and expected results.

### THE PROPOSED MECHANICAL MODEL

Basically, the general mechanical pattern of Tyrannosaurus rex, as in all Theropoda, includes the balance of the head and body at the hip joints by the large tail. A corresponding static scheme could be a rigid beam, shown on Fig.3, where the tail, the body with the head and forelimbs, without walk extremities, are represented by continuous load. Apparently, the model covers longitudinal behaviour of the animal, being permitted by its symmetry along that dimension. It would appear that the spine flexibility and lateral displacement of the tail make such a representation impossible, as the beam is a rigid 2D static system. Therefore, we propose a beam model that indirectly incorporates the 3rd dimension through the set of functions, where each one stands for a particular

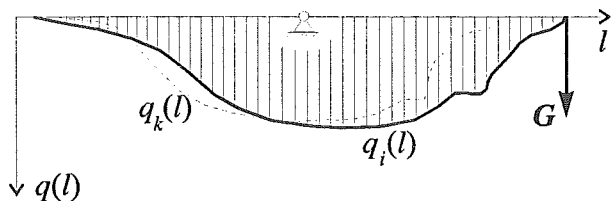


Fig.3 Beam with continuous load as a static model of theropod's posture.

Sl.3 Greda sa kontinualnim opterećenjem kao statički model bipeda.

continuous load related to equivalent mechanical state of the animal in different postures (including the degree of spine bending, lateral tail displacement, and even the quantity of consumed food). Although incorporates dynamic behaviour of the tyrannosaur this model is not completely dynamic but rather a hybrid one, since it doesn't involve typical dynamic loads (e.g. the impact with the prey at the bite, tearing the flesh pieces etc.). The corresponding family of functions can be represented as a collection

$$\{q(l)\} = \{q_1(l), q_2(l), \dots, q_k(l), \dots, q_N(l)\} \quad (1)$$

either discrete (convenient for simulation) or continuous. The head as a whole, with its permanent weight, could be represented by concentrated load (force  $G$  on Fig.3) and analytically by generalised Dirac's function  $\delta(l)$ , which location corresponds to particular head posture (on the side, up, down etc.) obviously always at the end of the continuous load function  $q(l)$ , so the modified family of load functions can be written as:

$$q_k^o(l) = q_k(l) + G \delta(l - L_k) \quad (2)$$

The full static load at the beam support, i.e. on the pelvis joint, directly follows from (2)

$$T_k = \int_0^{L_k} q_k(l) dl + G, \quad k = 1, 2, \dots, N \quad (3)$$

and the coordinates of the gravity center as well, by which we calculate the moment of the force at the joint:

$$I_k^c = \frac{\int_0^{L_k} l q_k^o(l) dl}{\int_0^{L_k} q_k^o(l) dl} = \frac{\int_0^{L_k} l q_k(l) dl + GL_k}{\int_0^{L_k} q_k(l) dl + G}, \quad (4a)$$

$$q_k^c = \frac{0.5 \int_0^{L_k} q_k^2(l) dl}{\int_0^{L_k} q_k(l) dl}, \quad k = 1, 2, \dots, N \quad (4b)$$

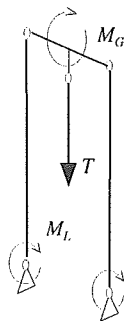


Fig. 4 Simplified model of gait apparatus as a frame with corresponding forces and moments at main joints.

Sl. 4 Hodni aparat bipeda predstavljen kao ram sa koncentrisanim opterećenjima.

The whole system can now be reduced to a simple frame, shown on Fig. 4, where the animal weight is a concentrated load at the axis of pelvis joints mounted on two pillars representing the legs. This elementary model describes standing posture of the animal, where the weight vector orientation passes through the nearhood of supports so that the tyrannosaur easily compensates fluctuations of the weight force moment by minimum muscular tonus. In that case moments indicated in Fig. 4 could take different orientations depending of instantaneous state of the balance.

Although this model of the tyrannosaur gait apparatus is pretty general, as it doesn't include limb elements and their joints, it is worth considering its kinematics which helps defining some principles in the background of bipedal tail-balanced gait.

Fig. 5 schematically shows walking process of the model from Fig. 4, presenting orthogonal projections of gravity center trajectory, from which general relations can be deduced. Only the active leg is shown. In such a case the gravity center traces complicated trajectory that is periodic in both orthogonal planes. While bringing ahead the passive leg to prepare the next step, a lateral moment of the weight force appears and the animal begins its side fall, where the force is increased

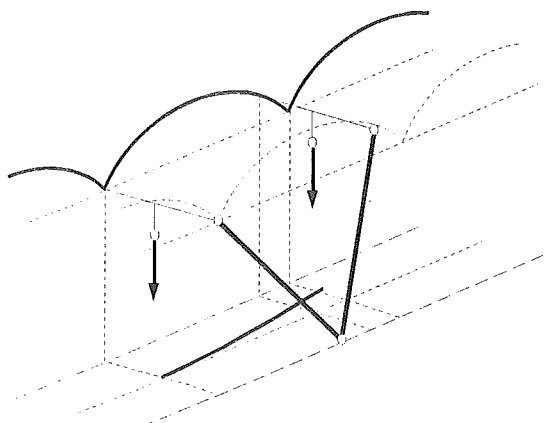


Fig. 5 Kinematics of the model from Fig. 4 shown by projections of gravity center trajectory on both orthogonal planes.

Sl. 5 Kinematička šema kretanja modela sa Sl. 4 sa pripadnim projekcijama trajektorije težišta.

exactly by the weight of the lifted leg and can be treated as a periodic time function  $T_k(t) = T_k + T_L$ . Here we do not take into account counter shift of the tail for lateral fall compensation.

The described walking style is intolerably energetic consumptive, because within each stride the whole body weight must be lifted for

$$\Delta H = D \left[ 1 - \cos \left( \arctg \frac{2D}{L} \right) \right] \quad (5)$$

where D and L stand for the leg and stride lengths, respectively, whereas the body must be also laterally returned. From a pure dynamical aspect that means a great moment of inertia of the body and the lifted leg relative to the temporary centre of rotation.

On the other hand, the kinematics of detailed structure of the gait system (Fig. 6) could be expressed mathematically in a conventional way through the set of equations relating outer coordinates of the terminal leg element-foot (given as variables  $x$  and  $y$  in the vertical plain of leg movement and the angle of toes  $\varphi$ ) with internal coordinates of other leg elements (given as a vector  $[q]$ ). Each internal variable  $q_i$  varies within corresponding variation range between  $q_{i\min}$  and  $q_{i\max}$  determined by anatomic and functional features of related element.

$$\begin{aligned} x &= f_1(q_1, q_2, \dots, q_n) \\ y &= f_2(q_1, q_2, \dots, q_n) \quad q_{i\min} \leq q_i \leq q_{i\max} \\ \varphi &= f_3(q_1, q_2, \dots, q_n) \end{aligned} \quad (6)$$

In a similar way a dynamic model can be defined. Computer simulation of such a model and even more complex one could yield an exact solution, but only when all parameters are precisely known. However, this case is specific in that the exactness of received results essentially depends on how accurate is the anatomic and functional restoration of the hind limb, particularly angle ranges of leg joints and potential power of muscle system, than how detailed and precise is the model itself. Hence, our objective is to define a simplified model but which is based on the most significant parameters of the gait apparatus, especially selected to give reliable results.

## THE FRAME OF THE MODEL

The results of the former considerations could serve as a basis for a hybri model, that comprises kinematics of the gait system but observed within global dynamic frame. It is well known that each mechanical system changes or tend to change its state in accordance with Hamilton's principle of least action, particularly that one which is the product of a long-lasting evolutionary refinements, that was not developed

itself in isolation but through interaction with a huge number of various factors, so the result of such an accordance is namely its mechanical i.e. energetic optimum. Mechanically optimal functioning of the walk system has its geometrical interpretation as the linear trajectory or translation of the weight center in both orthogonal coordinate planes.

Keeping the gravity center at the permanent height by adjusting the leg length during the stride period eliminates oscillations in the vertical plane. Coordinated lateral tail displacement decreases oscillations in the horizontal plane. The lateral stability is enhanced also by great span between the foot toes, that reduces equivalent distance between pelvis joints and, therefore, the lateral moment of the weight force. The effect is such that the tyrannosaur could firmly stand on just one leg if slightly inclined itself laterally. Clearly, from an energetic point of view, it is much more easier to keep the whole weight at the same height than to rise it with every stride and to strain the leg by excessive dynamic load when landing the foot ahead.

### THE MODEL STRUCTURE

The model is schematically represented on Fig.6 through its behaviour while performing the stride. The model involves the following elements with respective relative lengths: femur (0.324), tibia (fibula) (0.315), metatarsals (0.171) and two sets of phalanges (0.065 and 0.125), estimated from skeleton restoration [2] of the tyrannosaur, where  $0.324+0.315+0.171+0.065+0.125=1.00$  is the normalized length of the hind limb. The length of maximum forward step is deter-

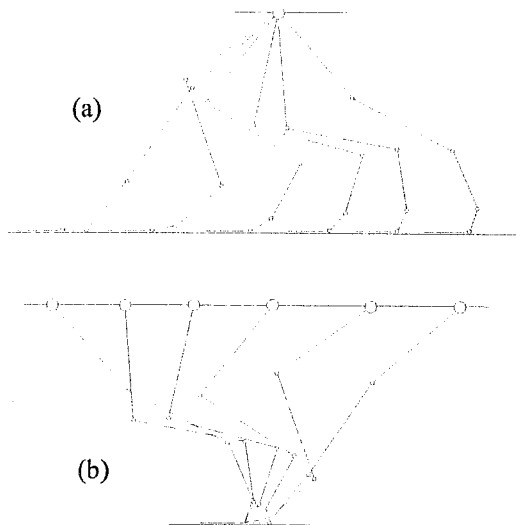


Fig.6 Restoration of stride kinematics of Theropoda's gait, with the leg comprising five elements: (a) center of rotation at hip joint; (b) center of rotation at toes.

Sl.6 Rekonstrukcija kinematike koraka kod Theropoda kada je noga modelirana sa pet elemenata: (a) centar rotacije u zglobu pelvisa; (b) centar rotacije na prstima.

mined by reliable foot landing, without slip, that is enhanced by claws and cushions on the toes. Here we do not incorporate the inertial force component of the moving animal at the contact foot-ground, as just a simplified model is considered. Through this parameter the floating height of the gravity center is defined. Taking into account estimated sequence of muscle activations, potential displacements of leg elements, and the fact that the forward stepping leg should be strengthened but the pushing-off leg should get the maximum length, it is synthesized a kinematic diagram as a restoration of the stride, shown on Fig.6a and Fig.6b for two centers of rotation.

### THE MODEL BEHAVIOUR

In the posture of standing rest the tyrannosaur's hind extremities are almost straight, and the animal consumes minimum energy to maintain such a posture, which is characterised by minimum moments at the joints. Because of that at the start of gait he lowers its gravity center at the needed height, that determines stride length and minimum speed. The change of kinematic disposition of leg elements is restored on Fig.6a. Another comparative diagram, related to the rotation around toes is given on Fig.6b.

From the former analysis we derive a simple dynamic interpretation of the gait process, that represents a dynamical model of so-called spontaneous or gravitational gait, performed just under the influence of gravitation, as body falls forward, without pushing activity of muscles, where the only role of the leg muscles is keeping the starting height of gravity center. That model is shown on Fig.7, where, for the sake of simplicity, we consider just one half of the stride. In the gravitational gait mode the movement is caused only by the force component  $F_w$  that increases with the step forward length:

$$F_w = m \frac{d^2 l}{dt^2} = T \sin \alpha = mg \sin \alpha \quad (7)$$

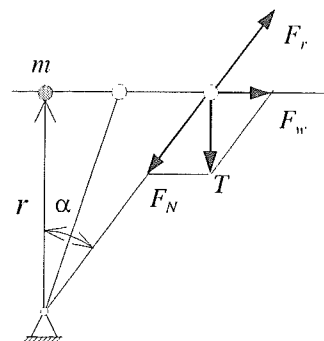


Fig.7 Simple dynamic model of gravitational gait with disposition of forces.

Sl.7 Dinamički model gravitacionog hoda bipeda sa prikazom dispozicije sila, pri čemu se posmatra samo jedna polovina koraka.

where  $m$  is the mass of the animal. The inertial force reaction causes appearance of excessive weight since the leg's radius-vector permanently increases its length:

$$\Delta T(\alpha) = T \left( \frac{1}{\cos \alpha} - \cos \alpha \right) \quad (8)$$

Now it is easy to calculate the timing of the stride as a function of the beginning height  $H$  from the differential equation of the model:

$$\frac{d^2 l}{dt^2} = \frac{dv}{dt} = g \sin \left( \arctg \frac{l}{H} \right) \quad (9)$$

Through such a procedure we get a quantitative base for an estimation of the gait minimum speed. Bearing on mind all simplifications done during the procedure of model creation and principles which it relies on, in the result of (9) certain corections should be included. Therefore, the estimated top cruising speed of the tyrannosaur, i.e. a stationary gait taken as a spontaneous and non-disturbed displacement over the ground, with just the necessary participation of propelling Caudifemoral muscle, is about 11 km/h. Obviously, the tyrannosaur could continually adjust the necessary speed in the range  $[0, 11]$  km/h, by choosing the stride length and decelerating in the midstep. On the other hand, when chasing the prey he couldn't run at the higher speed than 20-25 km/h. In such a mode the gait mechanics remains almost the same, where the only difference is the stride frequency.

## THE ANALYSIS OF RESULTS

The principles that stand in the background of the gait model had determined the relationship between the height of gravity center and the spontaneous step length, i.e. the acquired gait speed. However, despite anatomic potentials in that sense, we should point out the minimum height that allows reliable walk over obstacles and across rugged ground. Hence, we are inclined to distinguish cruise gait, as a mode of long-term and, naturally, minimum energy consuming walk, and fast gait, as a mode of short-term walk, when the gravity center trajectory becomes slightly periodical. In

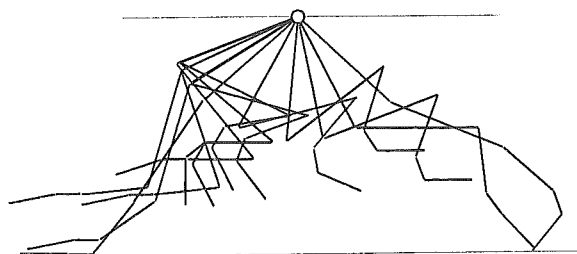


Fig.8 Restoration of leg return.  
Sl.8 Rekonstrukcija kinematike noge pri povratku.

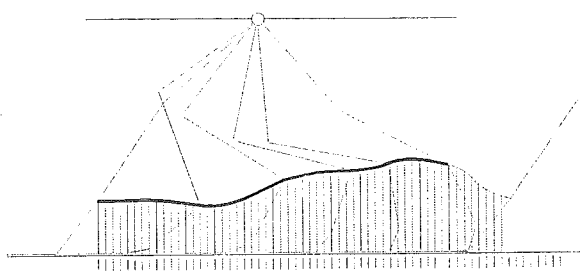


Fig.9 Trajectory of the ankle joint represented by continuous and sampled versions. The elements  $x_n$  of vector  $X$  are respective to the samples  $k$  as  $n=4k$ .

Sl.9 Trajektorija gležnja data u kontinualnoj i odmerenoj verziji. Elementi vektora  $X$  su respektivni svakom četvrtom odmerku.

that context, it is necessary to stress that we don't think the tyrannosaur showed any flight phase during the fast stride, which, in the opposite, was almost typical for small Theropoda.

To complete former considerations it is necessary to investigate kinematics of the leg during its return, i.e. its preparation for the next step. The analysis is completely compatible with the given model. The returning kinematic pattern of the leg is easy to restore with respect to some general principles. First of all, that pattern is not always the same but highly depends on the walk velocity, i.e. the speed of leg return. The leg's moment of inertia, related to the pelvis joint as the center of rotation, which should match that speed, is regulated by the value of the leg folding, that is by concentration of the leg mass around rotation center. It increases the angular speed of the leg and shortens the return time. Such a pattern is also influenced by the height of gravity center's translatory line, which in advance determines the room for the leg folding level. A complete kinematics of that process is restored on Fig.8. As can be seen, the leg in the rear position should be firstly folded, than swiftly rotated forward and stretched downward, first the foot and subsequently the other elements. Such a procedure is more prominent if the gait speed is higher, with its limit in the mode of fast running, typical for all smaller Theropoda.

## MODEL EXTENSION IN FREQUENCY DOMAIN

Each leg joint during the stride traces certain trajectory that can be expressed by corresponding periodic function  $f(l)$ . Applying the Fourier transform we get the counterpart of the function  $f(l)$  in frequency domain - its spectrum:

$$\{c_n\} = F[f(l)] = \left\{ \frac{1}{L} \int_0^L f(l) e^{-jn\Omega l} dl \right\} \quad (10)$$

where  $L$  represents the stride length and  $\Omega=2\pi/L$  is the basic spatial frequency.

Apart from the data concerning the walk process that is comprised in the function  $f(l)$  itself, its mapping to the frequency domain provides additional range for interpretation of its behaviour, and gives more complete picture of the whole process. In that we can recognise additional potentials for solving a very complex problem as it is Theropoda's gait. Moreover, the autocorrelation function of  $f(l)$  is an additional characteristic with high valuability in gait analysis, defined conventionally as:

$$R(\ell) = \frac{1}{T} \int_0^T f(l) f(l + \ell) dl \quad (11)$$

Applying discrete version of (11) to a sampled function  $f(n)$  of the ankle trajectory, as it is shown on Fig.9 and represented by the sample vector  $X(i/f)$

$X=[0.135 \ 0.137 \ 0.137 \ 0.13 \ 0.122 \ 0.14 \ 0.175 \ 0.2 \ 0.21 \ 0.213 \ 0.231 \ 0.238 \ 0.225 \ 0.175]$

we obtain the normalised autocorrelation function  $r(n)$ , given on Fig.10, which values are:

$r(n)=[3.27 \ 3.08 \ 2.80 \ 2.51 \ 2.23 \ 1.99 \ 1.74 \ 1.47 \ 1.23 \ 1.03 \ 0.84 \ 0.62 \ 0.39 \ 0.17]10^{-2}$ .

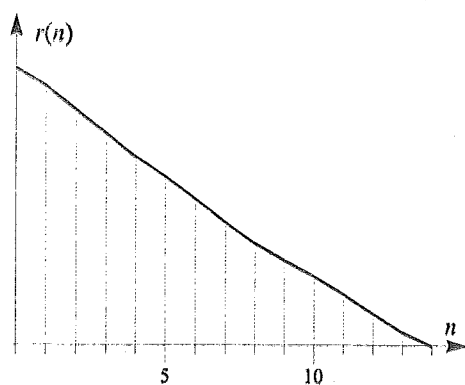


Fig.10 Normalised autocorrelation function of ankle joint trajectory.

Sl.10 Normalizovana autokorelaciona funkcija trajektorije gležnja

## CONCLUSION

Being the product of a long evolutionary process an animal can be considered as the optimal mechanical system relative to the environment it lives in. Therefore, the introduction of the minimum principle provides a reliable background for the model of theropoda's gait, that includes detailed kinematic restoration of the whole stride cycle - step forward and leg return. Such an approach enabled synthesis of the simple but effective model of gait dynamics, from which could be estimated values of stride parameters. Representing the obtained kinematic characteristics in the frequency domain, and calculating autocorrelation function it is enlarged the range for the interpretation of the whole problem.

The proposed model obviously does not cover all potentials of its theoretical background and inevitably needs further refinement, that should be accomplished through the improvement of the model itself, and incorporation of more precise values of the leg parameters, measured and estimated on existing mounted skeletons and today's models of the leg's muscle system, that would help more reliable evaluation of muscle power, which, in turn, would enable creation of improved models.

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## REZIME

MEHANIČKO MODELIRANJE HODA THEROPODA/TETANURAE SA PROŠIRENJE U FREKVENCIJSKOM DOMENU

BABIĆ Ranko

U radu je predložen mehanički model sistema za hodanje Theropoda, jednog od osnovnih faktora njihove dominacije tokom Mezozoika. Kvalitativno su upoređene specifičnosti mehanike hodnog aparata kod malih i velikih predatora. Model se odnosi na hodni deo pri čemu je uticaj čitavog tela izražen svedenim opterećenjem na osi zglobova pelvisa, tako što je trodimenzionalno pomeranje trupa i repa uprošćeno modelirano prostom balansiranom gredom sa kontinualnim opterećenjem izraženim familijom funkcija, gde svakoj odgovara određena dispozicija tela. Detaljna rekonstrukcija i analiza kinematike koraka se oslanja na primeni dinamičkog principa minimalnog dejstva. Takođe se rekonstruiše kinematika povratka noge. Uvođenjem koncepta minimalnog ili gravitacionog hoda sastavljen je dinamički model iz koga je, uz uvažavanje ostalih značajnih faktora iz procesa modeliranja, određena osnovna brzina hoda i dužina koraka. Kao proširenje prostora za analizu problema i interpretaciju rezultata predloženo je proučavanje pojedinih aspekata kinematike hoda u frekvencijskom domenu putem spektralne analize kao i primena korelacije što je ilustrovano konkretnim primerom autokorelacione funkcije putanje jednog zgloba noge.

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# ONE APPLICATION OF THE BEKLUND'S TRANSFORMATION

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## ABSTRACT

In this paper we shall use Beklund's transformation to determine some exact solutions of the observed partial equations of second order.

**Key words:** Beklund's transformation, Lambs diagram, soliton

## 1. Introduction

Exact solution have the essential role for developing approximation methods, whose scope of use is remarkable wider. Firstly, they represent a "standard" for approximate solutions, and give us possibilities to check accuracy approximations; secondly they lead to discovering of new approximate methods.

Using the transformation of Beklund we shall find a relation between four solutions of the given equation. After that, we are going to use this relation to determine some exact solutions of the observed equation.

## 2. Beklund's transformation

Let us know some exact solution of the equation

$$(1) \quad \frac{\partial^2 z}{\partial x \partial y} = f(z),$$

say,  $z_{n-1} = z_{n-1}(x, y)$ . In (1)  $f(z)$  is given function. It is necessary to find a second solution  $z_n = z_n(x, y)$  of the equation (1), connected, with the known solution  $z_{n-1}$ , by equations

$$(2) \quad \begin{cases} p_n = \Phi(z_{n-1}, z_n, p_{n-1}), \\ q_n = \Psi(z_{n-1}, z_n, q_{n-1}), \end{cases}$$

where  $p_i = \frac{\partial z_i}{\partial x}$ ,  $\frac{\partial z_i}{\partial y}$  ( $i = n-1, n$ ),  $\Phi$  and  $\Psi$  are functions which need determine.

We first determine the form of the functions  $\Phi$  and  $\Psi$ . Substituting these expressions for  $p_n$  and  $q_n$  in (1), we obtain the four systems [1, 2]. From these systems we find the next form for the functions  $\Phi$  and  $\Psi$

$$\begin{cases} \Phi = \varphi(z_n - K_2 z_{n-1}) + K_1 p_{n-1}, \\ \Psi = \psi(z_n - K_1 z_{n-1}) + K_2 q_{n-1}, \end{cases}$$

where  $K_1$  and  $K_2$  are arbitrary constants,  $\varphi$  and  $\psi$  still unknown functions, which satisfies the system

$$(3) \quad \begin{cases} \psi \frac{\partial \varphi}{\partial z_n} = f(z_n) - K_1 f(z_{n-1}), \\ \varphi \frac{\partial \psi}{\partial z_n} = f(z_n) - K_2 f(z_{n-1}). \end{cases}$$

We remark that the system (3) cannot be solved, in general.

Replacing the expressions for  $\Phi$  and  $\Psi$  in (2), we obtain the form of the Beklund's transformation for the equation (1)

$$(4) \quad \begin{cases} p_n = \varphi(z_n - K_2 z_{n-1}) + K_1 p_{n-1}, \\ q_n = \psi(z_n - K_1 z_{n-1}) + K_2 q_{n-1}. \end{cases}$$

### 3. Application of the Beklund's transformation

The constants  $K_1$  and  $K_2$  in (4), we shall determine, it is possible, so that the system (3) can be solvable. In this paper we shall consider three such cases.

$$1^0 \quad f(z) = az + b$$

In this case the equation (1) reduce to the equation

$$(1_1) \quad \frac{\partial^2 z}{\partial x \partial y} = az + b,$$

and system (3) becomes

$$(3_1) \quad \begin{cases} \psi(z_n - z_{n-1}) \frac{\partial \varphi}{\partial z_n} = az_n + b - K_1(az_{n-1} + b), \\ \varphi(z_n - z_{n-1}) \frac{\partial \psi}{\partial z_n} = az_n + b - K_2(az_{n-1} + b). \end{cases}$$

which one solution is

$$\begin{cases} \psi(z_n - z_{n-1}) = ak(z_n - z_{n-1}), \\ \varphi(z_n - z_{n-1}) = k^{-1}(z_n - z_{n-1}), \end{cases}$$

where  $k = \text{const}$ . Replacing these expressions in (4), we obtain the required Beklund's transformation for the equation (1<sub>1</sub>)

$$(4_1) \quad \begin{cases} p_n = p_{n-1} + k^{-1}(z_n - z_{n-1}), \\ q_n = q_{n-1} + ak(z_n - z_{n-1}). \end{cases}$$

We remark that the equations (4<sub>1</sub>) cannot be solved by quadratures, even if they are simpler than the equation (1<sub>1</sub>), and one can think that we done nothing by Beklund's transformation. But, it is not so. For any choce of  $k$  in (4<sub>1</sub>), we have

one solution of the equation (1<sub>1</sub>), say  $z_n$ . We may correspond, to this solution, Lamb's diagram with the coefficient of transformation  $k$

$$\boxed{z_{n-1}} \xrightarrow{k} \boxed{z_n}.$$

If the sequences of transformation with coefficients  $k_1, k_2$  and  $k_2, k_1$  are commutative, then four functions, which are determined by this coefficients, make the "block" of Lamb's diagram (Fig. 1),

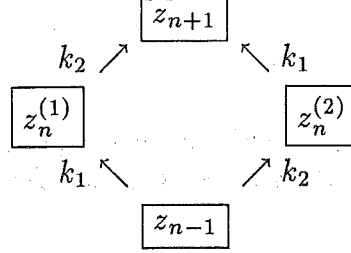


Fig. 1

and they are mutually connected only algebraically.

Indeed, for a sequence of the functions on Lamb's diagram

$$\boxed{z_{n-1}} \xrightarrow{k_1} \boxed{z_n^{(1)}} \xrightarrow{k_2} \boxed{z_{n+1}}$$

the corresponding first equations in (4<sub>1</sub>) are

$$\begin{cases} p_n^{(1)} = p_{n-1} + k_1^{-1} (z_n^{(1)} - z_{n-1}), \\ p_{n+1} = p_n^{(1)} + k_2^{-1} (z_{n+1} - z_n^{(1)}). \end{cases}$$

Eliminating  $p_n^{(1)}$ , from these equations, we have

$$p_{n+1} = p_{n-1} + k_2^{-1} z_{n+1} - k_1^{-1} z_{n-1} + (k_1^{-1} - k_2^{-1}) z_n^{(1)}$$

Similarly, for the sequence of functions on Lamb's diagram

$$\boxed{z_{n-1}} \xrightarrow{k_2} \boxed{z_n^{(2)}} \xrightarrow{k_1} \boxed{z_{n+1}}$$

we have

$$p_{n+1} = p_{n-1} + k_1^{-1} z_{n+1} - k_2^{-1} z_{n-1} + (k_2^{-1} - k_1^{-1}) z_n^{(2)}$$

Eliminating  $p_{n+1}$  and  $p_{n-1}$  from the two last equations, we obtain the desired connection between four solutions of the equation (1<sub>1</sub>):

$$(5) \quad z_{n+1} + z_{n-1} = z_n^{(1)} + z_n^{(2)}.$$

The simplicity of the formula (5), justifies the using of Beklund's transformation, which, although it does not lead to the solution of the equation (1<sub>1</sub>), gives

a possibility for finding fourth solutions, in the case when its three solutions are known. These three solutions must be connected by Beklund's formulas (4<sub>1</sub>) too.

Inded, the equation (1<sub>1</sub>) have evident solution  $z_{n-1} = -\frac{b}{a}$ . If, now, we replace  $z_{n-1}$  in (4<sub>1</sub>), we get from the system

$$p_n = \frac{1}{k} \left( z_n + \frac{b}{a} \right), \quad q_n = ak \left( z_n + \frac{b}{a} \right),$$

which solution is

$$z_n = -\frac{b}{a} + C \exp \left( \frac{x}{k} + ak y \right).$$

Hence, for  $k = k_i$ ,  $i = 1, 2$ , we have

$$z_n^{(i)} = -\frac{b}{a} + C_i \exp \left( \frac{x}{k_i} + ak_i y \right).$$

Replacing these three solutions  $z_{n-1}$ ,  $z_n^{(1)}$  and  $z_n^{(2)}$  in (5), we obtain the fourth solution of the equation (1<sub>1</sub>)

$$z_{n+1} = -\frac{b}{a} + C_1 \exp \left( \frac{x}{k_1} + ak_1 y \right) + C_2 \exp \left( \frac{x}{k_2} + ak_2 y \right).$$

$$2^0 f(z) = \sin x.$$

In this case the equation (1) reduce to the equation

$$(1_2) \quad \frac{\partial^2 z}{\partial x \partial y} = \sin z,$$

which call Sin-Gordon's equation, and it is known its connection by Josephson's effect in the quantum mechanics.

The system (3) takes the form [1]

$$(3_2) \quad \begin{cases} \psi \frac{\partial \varphi}{\partial z_n} = \sin z_n - K_1 \sin z_{n-1}, \\ \varphi \frac{\partial \psi}{\partial z_n} = \sin z_n - K_2 \sin z_{n-1}. \end{cases}$$

If we take  $K_1 = -K_2 = 1$ , than from (3<sub>2</sub>), we have the system which one solution is

$$\begin{cases} \varphi(z_n + z_{n-1}) = 2k \sin \frac{z_n + z_{n-1}}{2}, \\ \psi(z_n - z_{n-1}) = \frac{2}{k} \sin \frac{z_n - z_{n-1}}{2}, \end{cases}$$

where  $k = \text{const.}$

Substituting these expressions in (4) we obtain the Beklund's equation transformation, from the equation (1<sub>2</sub>), in the form

$$(4_2) \quad \begin{cases} p_n = p_{n-1} + 2k \sin \frac{z_n + z_{n-1}}{2}, \\ q_n = -q_{n-1} + \frac{2}{k} \sin \frac{z_n - z_{n-1}}{2}. \end{cases}$$

By analogy, with the procedure of 1<sup>0</sup>, we find the connection between the four solutions of the equation (1<sub>2</sub>), in the form [1]

$$(6) \quad z_{n+1} = z_{n-1} \pm 4 \operatorname{arctg} \left( \frac{k_1 + k_2}{k_1 - k_2} \operatorname{tg} \frac{z_n^{(2)} - z_n^{(1)}}{4} \right).$$

So, the clear algebraic, we can to determine every solution of the equation (1<sub>2</sub>), which may represent by pyramid of Lamb's diagram (Fig 2.)

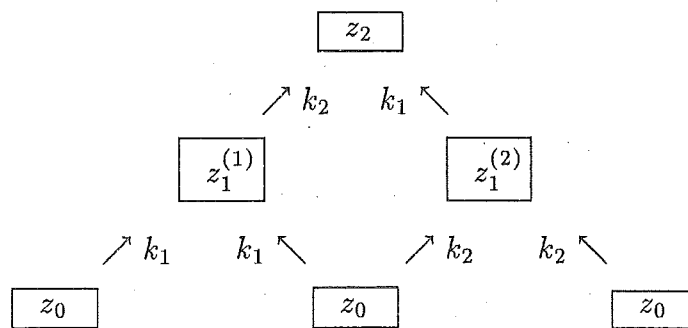


Fig.2

Indeed, the equation (1<sub>2</sub>) have evident solutions  $z_{n-1} = 0$  and  $z_{n-1} = \pi$ . Replacing in the equations (3<sub>2</sub>)  $z_{n-1} = 0$  we get from system

$$p_n = 2k \sin \frac{z_n}{2}, \quad q_n = \frac{2}{k} \sin \frac{z_n}{2},$$

which a solution is

$$z_n = \pm 4 \operatorname{arctg} \left[ \exp \left( kx + \frac{y}{k} + b \right) \right].$$

Hence, for  $k = k_i$ ,  $i = 1, 2$ , we have

$$z_n^{(i)} = \pm 4 \operatorname{arctg} \left[ \exp \left( k_i x + \frac{y}{k_i} + b_i \right) \right].$$

These solutions, for all  $k_i$  described the solitons. Replacing these three solutions  $z_{n-1}$ ,  $z_n^{(1)}$  and  $z_n^{(2)}$  in (6), we obtain the fourth solution of the equation (1<sub>2</sub>)

$$(7) \quad z_{n+1} = \pm 4 \operatorname{arctg} \left\{ \frac{k_1 + k_2}{k_1 - k_2} \frac{\exp \left( k_1 x + \frac{y}{k_1} + b_1 \right) - \exp \left( k_2 x + \frac{y}{k_2} + b_2 \right)}{1 + \exp \left[ (k_1 + k_2) x + \left( \frac{1}{k_1} + \frac{1}{k_2} \right) y + b_1 + b_2 \right]} \right\}.$$

For different  $k_1$  and  $k_2$  the solution (7) described mutually effect two solitons.

If we take that  $k_1$  and  $k_2$  are complex conjugate numbers  $k_{1,2} = a \pm ib$  and  $b_1 = b_2 = 0$ , than, from (7), we have a new solution of the equation (1<sub>2</sub>)

$$z = \pm 4 \operatorname{arctg} \left[ \frac{a \sin b \left( x - \frac{y}{a^2+b^2} \right)}{b \operatorname{ch} \left( x + \frac{y}{a^2+b^2} \right)} \right].$$

$$3^0 f(z) = \operatorname{sh} z.$$

In this case the equation (1) reduce to the equation

$$(1_3) \quad \frac{\partial^2 z}{\partial x \partial y} = \operatorname{sh} z,$$

and system (3) becomes [2]

$$(3_3) \quad \begin{cases} \psi \frac{\partial \varphi}{\partial z_n} = \operatorname{sh} z_n - K_1 \operatorname{sh} z_{n-1}, \\ \varphi \frac{\partial \psi}{\partial z_n} = \operatorname{sh} z_n - K_2 \operatorname{sh} z_{n-1}. \end{cases}$$

Taking  $K_1 = -K_2 = 1$  in (3<sub>3</sub>) we have the system, which a solution is

$$\begin{cases} \varphi(z_n + z_{n-1}) = 2k \operatorname{sh} \frac{z_n + z_{n-1}}{2}, \\ \psi(z_n - z_{n-1}) = \frac{2}{k} \operatorname{sh} \frac{z_n - z_{n-1}}{2}, \end{cases}$$

and Beklund equation, from the equation (1<sub>2</sub>), have the form

$$(4_3) \quad \begin{cases} p_n = p_{n-1} + 2k \operatorname{sh} \frac{z_n + z_{n-1}}{2}, \\ q_n = -q_{n-1} + \frac{2}{k} \operatorname{sh} \frac{z_n - z_{n-1}}{2}. \end{cases}$$

By analogy, with the procedure of 1<sup>0</sup>, we find the connection between the four solutions of the equation (1<sub>3</sub>), in the form [2]

$$(8) \quad z_{n+1} = z_{n-1} + 4 \operatorname{arth} \left( \frac{k_1 + k_2}{k_1 - k_2} \operatorname{th} \frac{z_n^{(1)} - z_n^{(2)}}{4} \right).$$

The equation (1<sub>3</sub>) have evident solution  $z_{n-1} = 0$ . Replacing in the equations (4<sub>3</sub>) this solution, we get from system

$$p_n = 2k \operatorname{sh} \frac{z_n}{2}, \quad q_n = \frac{1}{k} \operatorname{sh} \frac{z_n}{2},$$

which solution is

$$z_n = \pm 4 \operatorname{arth} \left[ \exp \left( kx + \frac{y}{k} + b \right) \right].$$

Hence, for  $k = k_i$ ,  $i = 1, 2$ , we have

$$z_n^{(i)} = \pm 4 \operatorname{arth} \left[ \exp \left( k_i x + \frac{y}{k_i} + b_i \right) \right].$$

Replacing these three solutions  $z_{n-1}$ ,  $z_n^{(1)}$  and  $z_n^{(2)}$  in (8), we obtain the fourth solution of the equation (1<sub>3</sub>)

$$(9) \quad z_{n+1} = \pm 4 \operatorname{arth} \left\{ \frac{k_1 + k_2}{k_1 - k_2} \frac{\exp \left( k_1 x + \frac{y}{k_1} + b_1 \right) - \exp \left( k_2 x + \frac{y}{k_2} + b_2 \right)}{1 - \exp \left[ (k_1 + k_2) x + \left( \frac{1}{k_1} + \frac{1}{k_2} \right) y + b_1 + b_2 \right]} \right\}.$$

If we take that  $k_1$  and  $k_2$  are complex conjugate numbers  $k_{1,2} = a \pm ib$  and  $b_1 = b_2 = 0$ , than, from (9), we have a new solution of the equation (1<sub>3</sub>)

$$z = \mp 4 \operatorname{arctg} \left[ \frac{a \sin b \left( x - \frac{y}{a^2 + b^2} \right)}{b \operatorname{ch} \left( x + \frac{y}{a^2 + b^2} \right)} \right].$$

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## О ЈЕДНОЈ ПРИМЕНИ БЕКЛУНДОВЕ ТРАНСФОРМАЦИЈЕ

Радован Николић

## ИЗВОД

Беклундова трансформација је је откривена још 1876. године и тада је коришћена за решавање проблема из диференцијалне геометрије а, историсјски гледано, је била прва помоћу које су одређена моносолитонска решења једначине

$$(1_2) \quad \frac{\partial z^2}{\partial x \partial y} = \sin z.$$

Основна идеја ове трансформације је да се дата парцијална једначина другог реда трансформише у систем парцијалних једначина првог реда.

У овом раду је напре одређена Беклундова трансформација за једначину

$$(1) \quad \frac{\partial z^2}{\partial x \partial y} = f(x),$$

а затим иста искоришћена за решавање једначине (1) у случајевима када је  $f(x) = ax + b$ ,  $f(x) = \sin x$  и  $f(x) = \operatorname{sh} x$ .

Осим тога је за једначину

$$(1_2) \quad \frac{\partial z^2}{\partial x \partial y} = \sin z$$

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# A Generalization of the Wilson and Leibniz Theorems

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## ABSTRACT

A theorem that is a generalization of the Wilson and Leibniz's theorems is given in the paper.

Also the method of factorials is defined.

**Key words:** Wilson theorem, left factorial !n.

## 1 Introduction

Two proofs of the theorem are given, where the first proof is defined by the method of factorials. An application of the method in solving a specific problem is also given. The method of determining whether a natural number is prime or composite is made easier by the method of factorials, as well as the conclusions that follow.

## 2 The Wilson and Leibniz's Theorems

In paper [1] the Wilson's theorem and a consequence of the theorem respectively are given and proved:

**Theorem 1:** For every prime number  $p$  the following holds

$$(p-1)!+1 \equiv 0 \pmod{p}.$$

**Theorem 2:** If  $n$  is a composite number, then

$$(n-1)!+1 \not\equiv 0 \pmod{n}.$$

In view of theorem 1 and theorem 2 we have:

**Theorem 3:** The necessary and sufficient condition for a number  $p>2$  to be a prime number is for

$$(1) \quad (p-1)!+1 \equiv 0 \pmod{p}$$

to hold.

Using theorem 3 it is possible to prove the Leibniz's theorem:

**Theorem 3.1:** The necessary and sufficient condition for a number  $p>2$  to be a prime number is for

$$(1') \quad (p-2)!-1 \equiv 0 \pmod{p}$$

to hold.

## 3 A Generalization

In view of theorem 3 for a number to be prime the necessary and sufficient condition is:

$$\begin{aligned} & (n-2)! n - (n-2)! + 1 \equiv 0 \pmod{n} \\ (2.1) \quad & \Leftrightarrow (n-2)! + (-1)^1 \equiv 0 \pmod{n} \\ & \Leftrightarrow (2-1)!(n-2)! + (-1)^{2-1} \equiv 0 \pmod{n} \\ & \Leftrightarrow (n-3)! n - 2(n-3)! + (-1)^{2-1} \equiv 0 \pmod{n} \end{aligned}$$

$$\begin{aligned}
 &\Leftrightarrow (n-3)! \cdot n-2(n-3)!+(-1)^{2-1} \equiv 0 \pmod{n} \\
 &\Leftrightarrow 1 \cdot 2(n-3)!+(-1)^2 \equiv 0 \pmod{n} \\
 (2.2) \quad &\Leftrightarrow (3-1)!(n-3)!+(-1)^{3-1} \equiv 0 \pmod{n}
 \end{aligned}$$

$$\begin{aligned}
 &\Leftrightarrow 2(n-4)! \cdot n-2 \cdot 3(n-4)!+(-1)^2 \equiv 0 \pmod{n} \\
 &\Leftrightarrow 2 \cdot 3(n-4)!+(-1)^3 \equiv 0 \pmod{n} \\
 (2.3) \quad &\Leftrightarrow (4-1)!(n-4)!+(-1)^{4-1} \equiv 0 \pmod{n}
 \end{aligned}$$

If we continue this procedure after  $m$  steps, analogonly to formulac (2.1), (2.2) and (2.3), we will get:

$$(2) \quad (m-1)!(n-m)!+(-1)^{m-1} \equiv 0 \pmod{n}.$$

In view of theorem 3 and formula (2) the following theorem holds:

**Theorem 4:** The necessary and sufficient condition for a number  $p > 2$  to be a prime number is that for every  $m \in \mathbb{N}$  the following holds

$$(3) \quad (m-1)!(n-m)!+(-1)^{m-1} \equiv 0 \pmod{n}.$$

If in formule (3) we take  $m=1$ , we get the Wilson's theorem, and for  $m=2$  the Leibniz's theorem. Theorem 4 can be proved in a different way. Firsstty we give the following lemma:

**Lemma 1:** For every two natural number  $a, b > 2$

$$ab-b-1 \geq a$$

holds.

**Proof:** Lemma 1 can be proved by induction with respect to  $b$ .

**Proof 2 of theorem 4:**

( $\Leftarrow$ ) Let formula (3) hold. Suppose not, that  $n$  is a composite number, is that there are natural number  $a, b > 2$  such that

$$n=ab$$

For case  $(a=2) \vee (b=2)$  the proof of theorem 4 is obvisions.

In view of formula (3) we have

$$(4) \quad \frac{(ab-b-1)!b!+(-1)^{m-1}}{ab} \in \mathbb{N}$$

If in formula (4) for  $m$  we take  $m=ab-b$ , and  $n=ab$  we get

$$\frac{(ab-b-1)!b!+(-1)^{ab-b-1}}{ab} \in \mathbb{N}$$

$$\Leftrightarrow \frac{(ab-b-1)!b!+(-1)^{ab-b-1}}{ab} \in \mathbb{N}$$

$$(5) \quad \Leftrightarrow \left( \frac{(ab-b-1)!(b-1)!}{a} + \frac{(-1)^{ab-b-1}}{ab} \right) \in \mathbb{N}$$

In view of lemma 1 we have  $ab-b-1 \geq a$ ,

is 
$$\frac{(ab - b - 1)!(b - 1)!}{a} \in \mathbb{N},$$

which contradicts formula (5). In view of that the assumption that  $n$  is a composite number is false. This proves that if formula (3) is satisfied,  $n$  is a prime number.

We still need to prove the sufficient condition of the theorem, is that formula (3) holds if  $n$  is a prime number.

( $\Rightarrow$ ) We will prove by induction with respect to  $m$ :

(i) For  $m=2$  formula (3) becomes

$$\Leftrightarrow (2-1)!(n-2)!+(-1)^{2-1} \equiv 0 \pmod{n}$$

$$\Leftrightarrow (n-2)!-1 \equiv 0 \pmod{n-1}$$

$$\Leftrightarrow (n-2)!(n-1)-n+1 \equiv 0 \pmod{n}$$

$$(6) \quad \Leftrightarrow (n-1)!+1 \equiv 0 \pmod{n}$$

Since  $n$  is a prime number in view of theorem 1 formula (6) holds.

(ii) Suppose that formula (3) holds for  $m$ .

(iii) We need to prove that formula (3) holds for  $m+1$ , is that

$$(7) \quad m!(n-m-1)!+(-1)^m \equiv 0 \pmod{n}$$

$$m!(n-m-1)!+(-1)^m \equiv 0 \pmod{n-1}$$

$$\Leftrightarrow -m!(n-m-1)!+(-1)^{m-1} \equiv 0 \pmod{n}$$

$$\Leftrightarrow (m-1)!(n-m-1)!-m(m-1)!(n-m-1)!+(-1)^{m-1} \equiv 0 \pmod{n}$$

$$\Leftrightarrow (m-1)!(n-m-1)!(n-m)+(-1)^{m-1} \equiv 0 \pmod{n}$$

$$(8) \quad \Leftrightarrow (m-1)!(n-m)!+(-1)^{m-1} \equiv 0 \pmod{n}$$

Since formula (8) is correct in view of the inductive hypothesis (ii), that proves formula (7), is (iii).

Theorem 4 is proved first time in [2].

**Corollary 1:** The necessary and sufficient condition for a number  $p>2$  to be a prime number is for

$$((n-1)/2)!^2+(-1)^{(n-1)/2} \equiv 0 \pmod{n}.$$

to hold.

**Proof:** In theorem 4 we take  $m = (n+1) / 2$ .

#### 4 An Application of the Method of Factorials

In paper [3], generalizing functions  $!n$  and  $n!$  amongst other things a sequence  $y_n$  is defined. Starting from function:

$$f(x)=e^{-x}/(1-x),$$

the sequence  $y_n$  is defined by  $y_n=f^{(n)}(0)$ .

In view of the properties of sequence  $y_n$  it is proved that for every prime number  $p$  that following holds

$$(9) \quad [(p-1)! / e] + 1 \equiv !p \pmod{p},$$

where  $[x]$  is a function defined by  $[x] \in \mathbb{Z}$  and  $[x] \leq x < [x] + 1$ .

I will prove claim (9) using the method of factorials:

$$\text{Let } p \text{ prime number and } !p \equiv t \pmod{p} \quad (10)$$

$$(10) \Leftrightarrow (p-1)! + (p-2)! + \dots + 2! + 1! + 0! \equiv t \pmod{p}$$

$$\Leftrightarrow (p-3)! + (p-4)! + \dots + 2! + 1! + 0! \equiv t \pmod{p} - 2!$$

$$\Leftrightarrow 2!((p-3)! + 1 + 2!((p-4)! + \dots + 2! + 1! + 0!)) - 1 \equiv 2!t \pmod{p}$$

$$\Leftrightarrow 2!((p-4)! + \dots + 2! + 1! + 0!) - (1! / 1!) \equiv 2!t \pmod{p} - 3$$

$$\Leftrightarrow 3!((p-4)! - 1 + 3!((p-5)! + \dots + 2! + 1! + 0!)) - 3 + 1 \equiv 3!t \pmod{p}$$

$$\Leftrightarrow 3!((p-5)! + \dots + 2! + 1! + 0!) - (3! / 2!) + (3! / 3!) \equiv 3!t \pmod{p} - 4$$

$$\Leftrightarrow 4!((p-5)! + 1 + 4!((p-6)! + \dots + 2! + 1! + 0!)) - (4! / 2!) + (4! / 3!) - (4! / 4!) \equiv 4!t \pmod{p}$$

$$\Leftrightarrow 4!((p-6)! + \dots + 2! + 1! + 0!) - (4! / 2!) + (4! / 3!) - (4! / 4!) \equiv 4!t \pmod{p}$$

after  $k$  steps we get

$$\Leftrightarrow k!((p-k-2)! + \dots + 0!) - (k! / 2!) + \dots + (-1)^{k+1} (k! / k!) \equiv k!t \pmod{p}. \quad (11)$$

If in expression (11) we take  $k=p-2$  we get:

$$(p-2)! (1 - 1/2! + 1/3! - \dots - 1/(p-3)! + 1/(p-2)!) \equiv (p-2)!t \pmod{p}$$

$$\Leftrightarrow (p-2)! (1 - 1/2! + 1/3! - \dots - 1/(p-3)! + 1/(p-2)!) \equiv (p-2)!t - t + t \pmod{p}$$

$$\Leftrightarrow (p-2)! (1 - 1/2! + 1/3! - \dots - 1/(p-3)! + 1/(p-2)!) \equiv t \pmod{p}$$

$$\Leftrightarrow (p-2)! (1 - 1/2! + 1/3! - \dots - 1/(p-3)! + 1/(p-2)!) \equiv !p \pmod{p} \quad (12)$$

From expansion of the function  $e^{-1}$  into series we have

$$(13) \quad 1 - 1/2! + 1/3! - \dots - 1/(p-3)! + 1/(p-2)! = 1 - e^{-1} + e^\alpha / (p-1)!$$

where  $e^\alpha < 1$ .

In we substitute expression (13) in (12) we have

$$(12) \Leftrightarrow (p-2)! (1 - e^{-1} + e^\alpha / (p-1)!) \equiv !p \pmod{p}$$

$$\Leftrightarrow (p-2)! - 1 + (p-2)!(-e^{-1} + e^\alpha / (p-1)!) + 1 \equiv !p \pmod{p}$$

$$\Leftrightarrow (p-2)!(e^\alpha / (p-1)!) - e^{-1} + 1 \equiv !p \pmod{p} - (p-1)$$

$$\begin{aligned}
&\Leftrightarrow -(p-1)!(e^\alpha/(p-1)! - e^{-1}) - p + 1 \equiv -!p(p-1) \pmod{p} \\
&\Leftrightarrow -(p-1)!e^\alpha/(p-1)! + (-(p-1)!(-e^{-1})) + 1 \equiv !p \pmod{p} \\
&\Leftrightarrow (p-1)!/e + 1 - e^\alpha \equiv !p \pmod{p} \tag{14}
\end{aligned}$$

since  $e^\alpha < 1$ , from expression (14) we have the correctness of claim (9).

The following claims quoted in [2] can be proved using the above method:

$$[p! / e] \equiv -1 \pmod{p},$$

$$[(p-1)! / e] + [(p-2)! / e] \equiv 0 \pmod{p}.$$

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### Rezime

#### UOPŠTENJE TEOREMA VILSONA I LAJBNICA

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U [2] je navedena teorema koja predstavlja uopštenje teoreme Vilsona . U radu su data dva dokaza pomenute teoreme i na osnovu nje definisan je metod faktorijela. Pomoću metoda faktorijela pokazano je kako je moguće dokazati većinu rezultata navedenih u [3].

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# Sequence $\{d_n\}$ and the Kurepa's Hypothesis for the Left Factorial

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## ABSTRACT

In the paper we define sequence  $\{d_n\}$ . In view of the properties of the sequence  $\{d_n\}$  the equivalent of the Kurepa's hypothesis for the left factorial is proved.

**Key words:** Kurepa's hypothesis, left factorial

## 1 Introduction

In [1], it is defined left factorial  $!n$  with  $!n=0!+1!+2!+\dots+(n-2)!+(n-1)!$ . Also, the hypothesis, which is called latter *Kurepa's hypothesis for left factorial (KH)*:

$$(1) \quad (!n, n!) = 2, n \in \mathbb{N}, n > 1,$$

where  $(!n, n!)$  is the greatest common divisor for  $!n$  and  $n!$ .

In [1], it is proved that the equivalent assertion for (1) is the assertion that for any prime numbers  $p$ ,  $p > 2$  it applies:

$$(2) \quad !p \not\equiv 0 \pmod{p}.$$

The stated problem is considered in [2] and [3].

## 2 Sequence $\{d_n\}$ of integers

**Definition 1** The sequence of integer  $\{d_n\}$  is defined by the following recurrent formula:

$$\begin{aligned} d_1 &= -1, \\ d_n &= -(n+1)d_{n-1} - 1, \end{aligned}$$

for every natural number  $n$ .

The terms of sequence  $\{d_n\}$  are  $d_1=-1$ ,  $d_2=2$ ,  $d_3=-9$ ,  $d_4=44$ ,  $d_5=-265$ ,...Sequence  $\{d_n\}$  is the union of two disjunctive sub-sequences, a sub-sequences whose terms are negative numbers  $\{d_n^-\}$  and a sub-sequences whose terms are positive numbers  $\{d_n^+\}$ .

**Definition 2**  $d_j \in \{d_n^-\} \Leftrightarrow (d_j \in \{d_n\} \wedge d_j < 0)$ .

**Definition 3**  $d_j \in \{d_n^+\} \Leftrightarrow (d_j \in \{d_n\} \wedge d_j > 0)$ .

**Consequence 1** Sequence  $\{d_n^-\}$  is given by the following recurrent formula:

$$\begin{aligned} d_1^- &= -1, \\ d_n^- &= (2n-1)(2n d_{n-1}^- + 1), \end{aligned}$$

for every natural numbers  $n$ .

**Proof**  $d_{2n-1} = -(2n-1+1)d_{2n-2} - 1 = -2n(-(2n-2+1)d_{2n-2-1} - 1) - 1$

$$(3) \Leftrightarrow d_{2n-1} = (2n-1)(2nd_{2n-3} + 1)$$

Since  $d_1 = d_1^-$ ,  $d_3 = d_2^-$ ,  $d_5 = d_3^-$ , ..., we have:

$$(4) \quad d_{2n-1} = d_n^- \quad i \quad d_{2n-3} = d_{n-1}^-$$

In view of (3) and (4) we have  $d_n^- = (2n-1)(2nd_{n-1}^- + 1)$ .

**Consequence 2** Sequence  $\{d_n^+\}$  is given by the following recurrent formula:

$$\begin{aligned} d_1^+ &= 2, \\ d_n^+ &= 2n((2n+1)d_{n-1}^+ + 1), \end{aligned}$$

for every natural numbers  $n$ .

**Proof**

$$(5) \quad d_{2n} = -(2n+1)d_{2n-1} - 1 = -(2n+1)(2nd_{2n-2} - 1) - 1 = 2n((2n+1)d_{2n-2} + 1)$$

Since  $d_2 = d_1^+$ ,  $d_4 = d_2^+$ ,  $d_6 = d_3^+$ , ..., we have:

$$(6) \quad d_{2n} = d_n^+ \quad i \quad d_{2n-2} = d_{n-1}^+$$

In view of (5) and (6) the theorem has been proved.

**Theorem 1** For every term of sequence  $\{d_n\}$ , the following holds:

$$a) \quad d_n + 1 \equiv 0 \pmod{n-1}$$

$$b) \quad d_n \equiv 0 \pmod{n}$$

$$c) \quad d_n + 1 \equiv 0 \pmod{n+1}$$

**Proof** In view of definition 1 we have:

$$(7) \quad d_{n-1} = -nd_{n-2} - 1 = -n(-(n-1)d_{n-3} - 1) - 1 = (n-1)(nd_{n-3} + 1)$$

$$(8) \quad d_n + 1 = -(n+1)d_{n-1} - 1 = -(n+1)d_{n-1}$$

In view of (7) and (8) we have:

$$(9) \quad d_n + 1 = -(n+1)(n-1)(nd_{n-3} + 1).$$

In view of equation (9) we have the corectness of claims a) and c).

In view of definition 1 we also have:

$$d_n = -(n+1)d_{n-1} - 1 = -(n+1)(-nd_{n-2} - 1) - 1 = (n+1)nd_{n-2} + n + 1 - 1$$

$$d_n = (n+1)nd_{n-2} + n = n((n+1)d_{n-2} + 1),$$

which proves claim b).



**Theorem 2** For every term of sequence  $\{d_n\}$ , the following holds:

$$a) d_i > 0 \Rightarrow d_{i+2} / (|d_i| + |d_{i+1}|) = i+2$$

$$b) d_i < 0 \Rightarrow d_{i+2} / (|d_i| + |d_{i+1}|) = -(i+2)$$

**Proof** In view of definition 1 we have:

$$(10) \quad d_{i+2} = -(i+3) d_{i+1} - 1 = -(i+3)(-(i+2) d_i - 1) - 1 = (i+2)((i+3) d_i + 1)$$

$$a) d_i > 0 \Rightarrow |d_i| + |d_{i+1}| = |d_i| + |-(i+2)d_i - 1| = d_i + (i+2)d_i + 1$$

$$(11) \quad = (i+3)d_i + 1$$

$$b) d_i < 0 \Rightarrow |d_i| + |d_{i+1}| = |d_i| + |-(i+2)d_i - 1| = -d_i + (i+2)(-d_i) - 1$$

$$(12) \quad = -((i+3)d_i + 1)$$

In view of (10), (11) and (12) the theorem has been proved.

**Consequence 3** For every term of sequence  $\{d_n\}$ , the following holds:

$$d_{i+2} \equiv 0 \pmod{|d_i| + |d_{i+1}|}.$$

### 3 The Equivalent of the Kurepa's Hypothesis for Left Faktorial

**Theorem 3** Let  $p$  be a prime number. Then:

$$!p \equiv 0 \pmod{p} \Leftrightarrow d_{p-2} \equiv 0 \pmod{p}$$

where  $d_{p-2} \in \{d_n\}$ .

**Proof** Let

$$!p \equiv 0 \pmod{p}$$

$$\Leftrightarrow (p-1)! + (p-2)! + (p-3)! + \dots + 2! + 1! + 0! \equiv 0 \pmod{p}$$

$$\Leftrightarrow (p-3)! + (p-4)! + \dots + 2! + 1! + 0! \equiv 0 \pmod{p} / (p-2) \equiv -2$$

$$\Leftrightarrow (p-2)! + (p-2)(p-4)! + \dots + (p-2)1! + (p-2)0! \equiv 0 \pmod{p}$$

$$\Leftrightarrow (p-2)(p-4)! + \dots + (p-2)1! + (p-2)0! \equiv -1 \pmod{p} / (p-3) \equiv -3$$

$$\Leftrightarrow (p-2)! + (p-2)(p-3)(p-5)! + \dots + (p-2)(p-3)1! + (p-2)(p-3)0! \equiv 3 \pmod{p}$$

$$\Leftrightarrow (p-2)(p-3)(p-5)! + \dots + (p-2)(p-3)1! + (p-2)(p-3)0! \equiv 2 \pmod{p} / (p-4) \equiv -4$$

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$$\Leftrightarrow (p-2) \dots (p-(p-4))2! + (p-2) \dots (p-(p-4))1! + (p-2) \dots (p-(p-4))0! \equiv d_{p-5} \pmod{p} / (p-(p-3)) \equiv -(p-3)$$

$$\Leftrightarrow (p-2)! + (p-2)! / 2 + (p-2)! / 2 \equiv -(p-3)d_{p-5} \pmod{p}$$

$$\Leftrightarrow (p-2)! / 2 + (p-2)! / 2 \equiv -(p-3)d_{p-5} - 1 \pmod{p}$$

$$\Leftrightarrow (p-2)! \equiv -(p-3)d_{p-5} - 1 \pmod{p}$$

$$\Leftrightarrow d_{p-4} \equiv (p-2)! \pmod{p}$$

$$\Leftrightarrow d_{p-4} - 1 \equiv 0 \pmod{p} / -(p-2)$$

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$$\Leftrightarrow d_{p-2} \equiv 0 \pmod{p}.$$

### References

- [1] Kurepa Đ., *On the left factorial function !n*, Math. Balcan. 1 (1971), 147-153.
- [2] Šami Z., *On the M-hypothesis of Đ. Kurepa*, Math. Balcan. 3 (1973), 530-532.
- [3] Šami Z., *On generalization of functions n! and !n*, Publ. Ins. Math. 56 (70), (1966), 5-14.

### Rezime

$\{d_n\}$  NIZ I KUREPINA HIPOTEZA ZA LEVI FAKTORIJEL

PETOJEVIĆ Aleksandar

U radu je definisan niz celih brojeva  $\{d_n\}$ . Na osnovu osobina niza  $\{d_n\}$  dokazan je ekvivalent Kurepinc hipoteze za levi faktorijel.

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## BOOK REVIEWS - PRIKAZI

**Marin P. i Tatić B., 1997.**

ETIMOLOŠKI REČNIK IMENA RODOVA I VRSTA  
VASKULARNE FLORE EVROPE. Izdavač: "Leksikon",  
Zemun - Beograd.

801.312 (038) ID = 52377868

Dugo vremena se osećala potreba za ovakvim rečnikom. Još od klasičnih botaničkih rečnika Gligića (1953), Simanovića (1959) i Šugara (1977 i 1990) kod nas nije bilo takvog dela. Istina, takva gradja je bila uključena u "Flori SR Srbije" ali je zasebno delo ovoga tipa nedostajalo. Sada su se istovremeno pojavila dva takva dela: Čuturilo S. i Janjić V. (1997) HERBOLOŠKI ENCIKLOPEDIJSKI REČNIK, i ETIMOLOŠKI REČNIK FLORE EVROPE, autora Marina i Tatića, kojeg ovde predstavljamo.

Rečnik autora Marina i Tatića je obima 220 štampanih stranica, od toga je korišćena literatura prikazana na dve stranice. Sama etimološka gradja je data u dva dela. U prvom delu, koji obuhvata 113 stranica,

data su imena rodova vaskularne flore Evrope (papratnjače, golosemenice i skrivenosemenice). U drugom delu, koji počinje 117. Stranicom, data su imena vrsta vaskularne flore Evrope.

Uobičajeno je da se imena rodova i vrsta označuju rečima imeničkog i pridevskog značenja, prihvaćenim iz latinskog i grčkog jezika. Takodje je poznato da su postojale mnoge proizvoljnosti u davanju naziva biljnim taksonima. U ovom delu autori su se držali utvrđenog postupka u imenovanju koje propisuje Međunarodni kodeks botaničke nomenklature (ICBN). Etimološki je obradjeno preko 1600 rodova i preko 3500 vrsta, pri čemu je svaki epitet dat samo jednom.

Rečnik će biti od koristi širokom krugu čitalaca pa ga svesrdno preporučujemo, naročito imajući u vidu njegovu popularnu cenu.

*P. Jakšić*

**Rašajski J., 1997.**

PTICE SRBIJE SA KARTAMA DISTRIBUCIJA.  
Izdavač: "Prometej", Novi Sad.

598.2 (497.11) ISBN 86-7639-266-8

Domaći i inostrani ornitolozi su konačno dobili delo za kojim se već duže vremena oseća potreba. Akademski slikar mr. Javor Rašajski je ovom knjigom objedinio vrsno slikarsko umeće i ljubav prema ornitologiji - ovakvo delo je jedino moglo i nastati iz spoja ljubavi i znanja. Obim knjige iznosi 273 stranice. Na preko 100 stranica - tabli predstavljene su crtežima u boji ptice Srbije a uz njih su date karte rasprostranjenja i osnovni biološki podaci za svaku vrstu. Crteži mužjaka, ženki i mladunaca su radjeni sigurnom rukom, sa jasno izraženim parametrima bitnim za determinaciju (poze, linije krila u letu i sl.) i odmerenim koloritom. Karte rasprostranjenja vrsta su radjene na UTM poljima 10 x 10 km. Možda je kod unošenja podataka trebalo selekcijom izostaviti one koji se odnose za vrste iščezle sa pojedinih staništa. Ali, te karte

pokazuju stepen istraženosti a ne stvarno stanje rasprostranjenja. Tako, mnoge vrste ptica kojih ima na području Kosova i Metohije nisu predstavljene na tim kartama. U svakom slučaju i takve karte su izuzetna osnova za dalja usmerena istraživanja, upravo na tim neistraženim područjima. Posle kartiranih dnevnih leptira prethodne Jugoslavije (Jakšić, 1988) i elemenata za kartiranje sitnih sisara (Petrov, 1992) ovim delom naša nacionalna fauna ima i treću grupu kartiranih organizama. Popis literature sadrži osnovne ornitološke reference, možemo samo zameriti autoru što je propustio da navede i Pančičevo (1867) delo o Pticama u Srbiji, sa kojim je i utemeljena domaća ornitologija.

U godinama izuzetne ekonomske krize autor može biti zadovoljan što je njegov višedecenijski trud ovako visoko ovekovečen. Biolozi pak, posebno ornitolozi, moraju biti zahvalni autoru na dragocenom poklonu nacionalnoj nauci. Uprkos dosta visokoj ceni delo bezrezervno preporučujemo, ne samo biolozima.

*P. Jakšić*

**Blaženčić Jelena (urednik), 1997.**

VLASINSKO JEZERO. HIDROBIOLOŠKA  
STUDIJA.

Izdavač: "Biološki fakultet Univerziteta u Beogradu"

**581.9 (285.2) (497.11) ID = 51207948**

Ovom reprezentativnom monografijom nastavljena je tradicija beogradske škole biologa, čiji je doajen naš znameniti biolog Siniša Stanković. U doba ekonomske krize i pogubnog dejstva sankcija međunarodne zajednice nastavnici Biološkog fakulteta u Beogradu su smogli snage da realizuju projekat kompleksnog multidisciplinarnog istraživanja ovog biološki izuzetno interesantnog područja i da nam rezultate rada prezentiraju u vidu ove monografije.

Vlasinsko jezero nije izabrano slučajno. Ono je i ranije privlačilo pažnju biologa, da pomenemo ovde Košanina (1908, 1910a, 1910b), Đorđevića (1910), Katića (1910) i drugih. U međuvremenu je sam objekat istraživanja pretrpeo značajnu promenu antropogenim uticajem - nekadašnja tresava je podizanjem brane pretvorena u jezero. Samim tim postavilo se kao značajno pitanje utvrđivanja promena nastalih kao posledica ovakve čovekove aktivnosti.

Radi odgovora na ova pitanja angažovan je impozantan krug stručnjaka čiji su rezultati rada ovde predstavljeni. Materija je izložena u 12 tematskih celina i na njima je radilo 18 autora. U prva tri poglavlja prikazani su opšti ciljevi studije, geološko-hidrogeološke i fizičko-hemijske karakteristike sliva Vlasinskog jezera, uključujući i samo jezero. U narednim

poglavljima obradjeni su sastav i zajednice mikroorganizama, fito- i zooplankton, bentosna makrofauna, ihtiofauna i njene parazitske cilijate. U poslednja dva poglavlja obradjena je hidrofilna vaskularna flora i vegetacija Vlasinskog jezera i alohtona mikopopulacija Vlasinskog jezera i njegovih pritoka.

Materija izložena u ovim poglavljima, koja predstavljaju zasebne celine, je sadržajno interesantna. Kooperativnom saradnjom autora pojedinih poglavlja omogućeno je da se floristički i faunistički podaci integralno tumače abiotičkim i biotičkim ekološkim faktorima. Ovde svakako značajna uloga pripada uredniku dr Jeleni Blaženčić koja je osmislila projekat i koordinirala rad učesnika. Na taj način monografija je dobila novi kvalitet jer deluje integralno, kao ekološka hidrobiološka studija, što je u njenom naslovu i istaknuto.

Uzimajući u obzir utvrđeni broj vrsta u svim grupama možemo videti da aktivnošću čoveka ovo područje nije uništeno, već naprotiv, biološki je obogaćeno. Primer Vlasinskog jezera može poslužiti kao obrazac kako se životna sredina može sačuvati, unaprediti i istovremeno staviti u funkciju višestruke ekonomske koristi.

Uvereni smo da ovako koncipirana monografija svojim brojnim kvalitetima može poslužiti kao model za slične hidrobiološke studije, pa je kao takvu iskreno preporučujemo.

*V. Urošević*