



UNIVERSITY OF PRISTINA-FACULTY OF SCIENCES  
KOSOVSKA MITROVICA-REPUBLIC OF SERBIA

THE  
UNIVERSITY  
THOUGHT  
PUBLICATION IN NATURAL SCIENCES

**VOL. 7, N° 2, 2017.**

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**ISSN 1450-7226 (Print)**

**ISSN 2560-3094 (Online)**

# **UNIVERSITY THOUGHT-PUBLICATION IN NATURAL SCIENCES**

## **Published by**

**University of Pristina-Faculty of Sciences**

**Kosovska Mitrovica-Republic of Serbia**

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The University Thought - Publication in Natural Sciences (Univ. thought, Publ. nat. sci.) is a scientific journal founded in 1994. by the University of Priština, and was published semi annually until 1998.

Today, the University Thought - Publication in Natural Sciences is an international, peer reviewed, Open Access journal, published semi annually in the online and print version by the University of Priština, temporarily settled in Kosovska Mitrovica, Serbia. The Journal publishes articles on all aspects of research in Biology, Chemistry, Geography, Information technologies, Mathematics and Physics in the form of original papers, short communications and reviews (invited) by authors from the country and abroad.

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# A CONTRIBUTION TO KNOWLEDGE OF THE BALKAN LEPIDOPTERA. SOME PYRALOIDEA (LEPIDOPTERA: CRAMBIDAE & PYRALIDAE) ENCOUNTERED RECENTLY IN SOUTHERN SERBIA, MONTENEGRO, THE REPUBLIC OF MACEDONIA AND ALBANIA

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## ABSTRACT

Pyraloidea (Lepidoptera: Crambidae & Pyralidae) were sampled in the territories of southern Serbia, Montenegro, the Former Yugoslav Republic of Macedonia and Albania on a total of 53 occasions during 2014, 2016 and 2017. A total of 173 species is reported here, comprising 97 Crambidae and 76 Pyralidae. Based upon published data, 29 species appear to be new to the fauna of Serbia, 5 species are new to the fauna of Macedonia and 37 are new to the fauna of Albania. The data are discussed.

**Keywords:** Faunistics, Serbia, Montenegro, Republic of Macedonia, Albania, Pyraloidea, Pyralidae, Crambidae.

## INTRODUCTION

Pyraloidea (Lepidoptera: Crambidae and Pyralidae) have been examined in detail in the neighbouring territory of the Republic of Bulgaria and the results have been published by one of us (Plant, 2016). That work presented data for the 386 species and 3 additional subspecies known from that country. The bulk of modern data used in that review originated from extensive field surveys undertaken by CWP during the period from 2002 to 2015, but also of importance was additional contemporary information obtained from ongoing Lepidoptera field surveys by SB and Dr Boyan Zlatkov and passed to CWP for examination. During the course of their field work, Beshkov and Zlatkov also made occasional forays across the border into Serbia, Montenegro, Macedonia and Albania and pyraloid material from those trips also found its way to CWP in England. Examination of the literature suggests that very little is published or known about the “pyrales” of this region of the southern Balkans and so the ongoing sampling by SB and AN was supplemented by an expedition during 2017 to target these moths in particular.

## METHODOLOGY

### *De facto sampling in 2014 and 2016*

During the years 2014 and 2016, several privately-funded expeditions were undertaken by SB and AN in order to sample Noctuoidea and other “macro” moths in the territory of southern Serbia, Montenegro, Macedonia and Albania, using various types

of light trap. Some sites were visited on more than one occasion; others were sampled once only.

As a by-product of this work, all remaining material from the traps was returned to Sofia where Dr Boyan Zlatkov was given the opportunity to extract the Tortricoidea. The remaining material was retained and sent by post to England after the end of each year, when any Pyraloidea were extracted, preserved and then named by CWP.

### *Targetted survey in 2017*

During 2017, CWP undertook self-funded, targetted collecting of Pyraloidea in Serbia on several nights in June in the company of PJ; the sites were pre-selected by PJ, in order to provide a representative cross-section of different habitats to be sampled. A total of 8 nights collecting had been proposed, with the intention of spending half these in the west of Serbia and the other half in the east of the country. Unfortunately, heavy rain reduced this to 7 nights – three in the west and four in the east.

Apart from captures made by a hand-held net, the pyraloid moths were sampled in 2017 using light trapping, as follows:

- Up to four “Robinson-pattern” traps were fitted with a 125 watt mercury-vapour bulb; these traps are in frequent use in Great Britain and are not only effective in attracting moths but are also efficient in retaining captured moths in the absence of anaesthetics. All traps were operated from the dusk until just beyond daybreak. These traps regularly catch very large numbers of moths making examination of the catch difficult; consequently, chloroform (trichloromethane) was introduced as an anaesthetic before opening the trap.
- A separate 125 watt mercury-vapour bulb was suspended between 1.5 and 2 metres above the ground

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in front of a white sheet, itself suspended vertically from the side of CWP's specially adapted Landrover vehicle. Moths were hand-collected from the sheet.

- Up to five “Hungarian” traps were also used. These consist of an 8 watt actinic “black light” tube suspended over a sealed, plastic bucket. The catch in these traps is significantly lower than that in the Robinson-pattern traps; the traps are charged with chloroform at set-up.

At each site, the traps used were positioned at a varying distances from each other – the primary consideration being that they were placed in differing habitat areas. No trap was set in a position from where it was possible to observe directly another light.

The combination of these two sampling techniques produced samples from a total of 40 separate localities on a total of 53 different dates as follows:

- Serbia: 17 sites on 23 dates
- Albania: 11 sites on 16 dates
- Montenegro: 2 sites on 2 dates
- Macedonia: 10 sites on 12 dates

The geographical spread of these sites is indicated in Fig. 1.



**Figure 1.** Collecting localities in 2014, 2016 and 2017 are indicated *approximately* by the solid black squares. Some squares represent more than one locality. The positions of the squares are not precise; refer to Tables 1, 2 and 3 for precise coordinates.

**Table 1.** Details of sampled sites within Serbia (17 sites on 23 dates – “sites 1 – 23”). Serbian place names are based on Jovanović (2010) and those from Albania on Friese (1967).

Site Ref	Place name	Coordinates	Altitude (metres)	Date sampled	Leg.
1	Jelašnička Klisura	43°16'36"N:22°04'09"E	345	18.iii.2014	SB & AN
2	Suva Planina, above Bojanine Vode	43°12'56"N:22°07'27"E	1000	2.vii.2014	SB & AN
3	Suva Planina, above Bojanine Vode	43°12'55"N:22°07'26"E	1030	19.vi.2017	CWP & PJ
4	Bela Palanka district, near Babin Kal village	43°19'09"N:22°23'23"E	760	3.vii.2014	SB & AN
5	Pirot, Mt. Vidlič, above Kamenolom Kitka to Crni Vrh Summit	43°10'49"N:22°38'35"E	870	4.vii.2014	SB & AN
6	Pirot Region, near Krupac village	43°06'15"N:22°42'55"E	688	5.vii.2014	SB & AN
7	Zvijezda, Savina Voda, near Jabuka Pass	43°22'02"N:19°33'07"E	1117	16.vii.2014	SB & AN
8	Pčinja River Valley, below Vražji kamen, near Trgovište	42°22'59"N:22°03'00"E	586	10.vii.2016	SB & AN
9	Bela Palanka district, Mt. Šljivoviki Vis, above Šljivovik village	43°08'29"N:22°23'12"E	925	30.v.2016	SB & AN
10	Bela Palanka district, Mt. Šljivoviki Vis, above Šljivovik village	43°08'29"N:22°23'12"E	925	3.x.2016	SB & AN
11	Preševo district, above Trnava village	42°16'18"N:21°36'47"E	800	31.v.2016	SB & AN
12	Preševo district, above Trnava village	42°16'18"N:21°36'47"E	800	9.vi.2016	SB & AN
13	Preševo district, above Trnava village	42°16'18"N:21°36'07"E	800	30.vi.2016	SB & AN
14	Preševo district, above Trnava village	42°16'08"N:21°36'47"E	800	9.vii.2016	SB & AN
15	Svrljiški Timok river gorge, near Niševac village	43°28'15"N:22°05'27"E	430	4.viii.2016	SB & AN

16	Vranje Region, Mt. Starac, Turski Grob,	42°20'10"N:19°52'26"E	840	5.viii.2016	SB & AN
17	Valjevo, Divčibare, Kaona	44°06'38"N:19°56'16"E	820	14.vi.2017	CWP & PJ
18	Ovčarsko-Kablarska Klisura (gorge), Planinarsko Društvo "Kablar"	43°54'44"N:20°12'26"E	294	15.vi.2017	CWP & PJ
19	Mt. Zlatibor, Tornik, near Ribnica village, below Bandera peak	43°39'36"N:19°37'45"E	1300	16.vi.2017	CWP & PJ
20	Mt. Tresibaba, Tresibaba pass	43°29'17"N:22°13'22"E	720	18.vi.2017	CWP & PJ
21	Pirot, Mt. Vidlič, Crni Vrh summit	43°11'12"N:22°39'02"E	1130	20.vi.2017	CWP & PJ
22	Pirot, Mt. Vidlič, Crni Vrh summit	43°10'51"N:22°38'52"E	1000	2.ix.2016	SB & AN
23	Bela Palanka district, Mt. Šljivoviki Vis, above Šljivovik village	43°08'36"N:22°23'15"E	950	21.vi.2017	CWP & PJ

**Table 2.** Details of sampled sites within Albania (11 sites on 16 dates – “sites 24 – 39”).

Site Ref	Place name	Coordinates	Altitude (metres)	Date sampled	Leg.
24	Korçë region, Devolli Gorge, near Strelçë (Maliq district)	40°43'18"N: 20°31'5"E	755	5.vi.2016	SB & AN
25	Ionian coast, Palasë, near Dhërmi	40°10'35"N:19°36'21"E	274	6.vi.2016	SB & AN
26	Delvinë region, Bistricë	39°55'24"N:20°07'53"E	116	7.vi.2016	SB & AN
27	Delvinë region, Bistricë	39°55'24"N:20°07'53"E	116	19.x.2016	SB & AN
28	Korçë region, above Kloçë, near the radio mast (= antenna)	40°41'02"N:20°41'40"E	1217	4.vii.2016	SB & AN
29	Korçë region, above Drenovë	40°53'18"N:20°48'23"E	1050	7.vii.2016	SB & AN
30	Korçë region, above Drenovë	40°53'18"N:20°48'23"E	1050	9.viii.2016	SB & AN
31	Korçë region, above Drenovë	40°53'18"N:20°48'23"E	1050	24.ix.2016	SB & AN
32	Korçë region, above Boboshticë and below Dardhë	40°32'26"N:20°47'31"E	1225	8.vii.2016	SB & AN
33	Berat region, Poliçan district, Vukopoles River Gorge between Ibrollara and Valë	40°33'36"N:20°05'38"E	217	10.viii.2016	SB & AN
34	Berat region, Poliçan district, Vukopoles River Gorge between Ibrollara and Valë	40°33'36"N:20°05'38"E	217	25.ix.2016	SB & AN
35	Berat region, Poliçan district, Vukopoles River Gorge between Ibrollara and Valë	40°33'36"N:20°05'38"E	217	16.x.2016	SB & AN
36	Lushnjë region, above Stan-Karbunarë	40°55'03"N:19°43'57"E	64	11.viii.2016	SB & AN
37	Korçë region, Mt. Ivan, above Zvezdë village	40°43'59"N: 20°52'49"E	1088	23.ix.2016	SB & AN
38	Ionian coast, Dhërmi district, below Ilias at St. Theodor Monastery	40°07'52"N:19°39'19"E	153	17.x.2016	SB & AN
39	Ionian coast, between Butrint and Sarandë, near Ksamil	39°48'31"N:20°00'27"E	56	18.x.2016	SB & AN

**Table 3.** Details of sampled sites within Montenegro (2 sites on 2 dates) and Macedonia (10 sites on 12 dates) – sites 40 – 53.

Site Ref	Place name	Coordinates	Altitude (metres)	Date sampled	Leg.
	<b>MONTENEGRO</b>				
40	Pivska Planina Mts, Trsa village, near Kulići	N43°10'42":E18°55'05"	1740	17.vii.2014	SB & AN
41	Durmitor, the road to Boban, between Lojanik and Šupljika	N43°06'27":E18°59'33"	1794	18.vii.2014	SB & AN
	<b>MACEDONIA</b>				
42	Demir Kapija Gorge	41°24'07"N:22°17'17"E	150	27.iv.2013	SB
43	Veles Region, above Sveti Ilija Monastery, below the antenna	41°24'14"N:21°48'27"E	458	11.v.2016	SB & AN
44	Mt. baba, Mountain hut "Široka"	41°00'17"N:21°10'17"E	1955	2.vi.2016	SB & AN
45	Prespa Lake near Konjsko village	40°54'51"N:20°58'48"E	940	3.vi.2016	SB & AN
46	Mt. Galičica, between Dva Javora and Bugarska Čuka summit	40°59'27"N:20°51'28"E	1587	4.vi.2016	SB & AN
47	Mt. Galičica, between Dva Javora and Bugarska Čuka summit	40°59'27"N:20°51'28"E	1587	3.vii.2016	SB & AN
48	Mt. Galičica, between Dva Javora and Bugarska Čuka summit	40°59'27"N:20°51'28"E	1587	17.viii.2013	SB & AN
49	Mt. Baba, on the road to Široka	41°01'52"N: 21°11'43"E	1493	2.vii.2016	SB & AN
50	Skopje – Treska Gorge, Gorna Matka	41°58'28"N:21°17'47"E	423	8.viii.2014	SB & AN
51	Vardar River Valley, Katlanovska Spa, Banjski rid	41°54'06"N:21°41'25"E	304	14.x.2016	SB & AN
52	Mt. Galičica, Petrina planina, between Ohrid and Veletovo	41°05'26"N:20°49'38"E	1005	15.x.2016	SB & AN
53	Vardar River Valley, Demir Kapija Gorge, road to Klisura village	41°23'21"N:22°17'41"E	155	20.x.2016	SB & AN

#### Laboratory aspects

Regrettably, most of the material in the trap debris retained by SB and AN was in very poor condition. The specimens that remained dry tended to be lacking in legs, antennae or palpi; other samples had become "greasy" so that external morphological features were obliterated. Consequently, almost without exception moths were named by genitalia examination. Where there was any ambiguity, the "problem" moths were degreased by overnight immersion in ethyl acetate so that characters of the wing patterns might become visible; a small proportion, mostly females, could not be named. The decomposed nature of most material rendered it unsuitable for any eventual molecular analysis and with rather few exceptions, these specimens were discarded.

Material from active sampling by CWP in 2017 tended to be in better condition and in some cases identification could be based on wing morphology alone. Voucher specimens from this expedition are retained in the CWP collection in England.

All identifications, including all genitalia examinations, were undertaken by CWP.

#### RESULTS

A grand total of 173 species of Pyraloidea is reported here, comprising 97 species of Crambidae and 76 taxa within the Pyralidae. The number recorded in each political unit is indicated in Table 4.

**Table 4.** Numbers of pyraloid taxa recorded in each country examined during the present survey.

Region	Number of		
	Crambidae	Pyralidae	all Pyraloidea
Serbia	63	57	120
Albania	62	49	111
Macedonia	25	31	56
Montenegro	11	8	19
All areas:	97	76	173

#### BIOLOGY

At each of this overall total of 40 sites, the catches from the several different traps used on the same night were pooled to provide a single date sample for that site. These 53 site/date sample points are summarised and allocated an identifying number in Tables 5 – 7. Lists for each of the 53 separate points were then collated.

In order to provide a permanent record that will be of value to future researchers into the fauna of this geographical region, the names of all species recorded on each of the 53 dates are presented in Table 5 (for Serbia), Table 6 (for Albania) and Table 7 (for Montenegro and Macedonia).

**Table 5.** Inventory of recorded species at sites in Serbia.

Site ref →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
taxon ↓																							
<b>Crambidae</b>																							
<b>Crambinae</b>																							
<i>Chrysoteuchia culmella</i> (Linnaeus, 1758)			+										+				+	+		+			
<i>Crambus ericella</i> (Hübner, 1813)																	+						
<i>Crambus lathoniellus</i> (Zincken, 1817)																	+	+	+		+		
<i>Crambus pascuella</i> (Linnaeus, 1758)																	+						
<i>Crambus pratella</i> (Linnaeus, 1758)																		+	+				
<i>Angustalius malacellus</i> (Duponchel, 1836)																	+						
<i>Agriphila tristella</i> ([Denis & Schiffermüller], 1775)																						+	
<i>Catoptria confusella</i> (Staudinger, 1881)															+							+	
<i>Catoptria falsella</i> ([Denis & Schiffermüller], 1775)					+		+							+									+
<i>Catoptria lythargyrella</i> (Hübner, 1796)										+												+	+
<i>Catoptria myella</i> (Hübner, 1796)			+																				
<i>Catoptria mytilella</i> (Hübner, 1805)							+																
<i>Catoptria osthelderi</i> (de Lattin, 1950)																	+						
<i>Catoptria pinella</i> (Linnaeus, 1758)					+		+	+						+	+								+

Site ref →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
taxon ↓																							
<i>Xanthocrambus saxonellus</i> (Zincken, 1821)					+			+					+	+							+		
<i>Chrysocrambus cassentiellus</i> (Herrich-Schäffer [1848])																	+						
<i>Chrysocrambus craterella</i> (Scopoli, 1763)		+		+	+		+					+	+				+	+		+			+
<i>Thisanotia chrysonuchella</i> (Scopoli, 1763)				+			+		+								+	+	+		+		+
<i>Platytes cerussella</i> ([Denis & Schiffermüller], 1775)																			+	+	+		
<b>Scopariinae</b>																							
<i>Scoparia ambigualis</i> (Treitschke, 1829)																							+
<i>Scoparia basistrigalis</i> Knaggs, 1866														+			+	+			+		
<i>Scoparia ingrattella</i> (Zeller, 1846)		+															+		+	+	+		
<i>Scoparia perplexella</i> (Zeller, 1839)							+												+				
<i>Scoparia pyralella</i> ([Denis & Schiffermüller], 1775)					+		+										+		+	+	+		+
<i>Scoparia subfusca</i> Haworth, 1811																	+						
<i>Eudonia lacustrata</i> Panzer, 1804							+														+		
<i>Eudonia mercurella</i> (Linnaeus, 1758)					+																		
<i>Eudonia sudetica</i> (Zeller, 1839)							+								+						+		+
<b>Cybalomiinae</b>																							
<i>Hyperlais dulcinalis</i> (Treitschke, 1835)							+					+	+				+						

Site ref →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
taxon ↓																							
<b>Odontiinae</b>																							
<i>Cynaeda dentalis</i> ([Denis & Schiffermüller], 1775)								+															
<i>Cynaeda pustulalis</i> (Hübner, 1823)					+																+		+
<i>Ephelis cruentalis</i> (Geyer, 1832)								+					+										
<i>Eurrhysis pollinalis</i> ([Denis & Schiffermüller], 1775)																							+
<b>Evergestinae</b>																							
<i>Evergestis aenealis</i> ([Denis & Schiffermüller], 1775)				+					+		+						+	+	+				+
<i>Evergestis limbata</i> (Linnaeus, 1767)																							+
<i>Evergestis mundalis</i> (Guenée, 1854)														+									
<i>Evergestis sophialis</i> (Fabricius, 1787)					+															+	+		+
<b>Pyraustinae</b>																							
<i>Loxostege aeruginalis</i> (Hübner, 1796)				+	+				+			+											+
<i>Paratalanta pandalis</i> (Hübner, 1825)																+							+
<i>Pyrausta aurata</i> (Scopoli, 1763)					+			+						+							+		+
<i>Pyrausta cingulata</i> (Linnaeus, 1758)												+											
<i>Pyrausta despicata</i> (Scopoli, 1763)										+	+						+						+
<i>Pyrausta nigrata</i> (Scopoli, 1763)																+							
<i>Pyrausta purpuralis</i> (Linnaeus, 1758)							+			+	+		+			+							
<i>Pyrausta sanguinalis</i> (Linnaeus, 1767)					+					+	+				+								+

Site ref →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
taxon ↓																							
<i>Sitochroa palealis</i> ([Denis & Schiffermüller], 1775)																+							
<i>Sitochroa verticalis</i> (Linnaeus, 1758)			+															+			+		+
<i>Ostrinia nubilalis</i> (Hübner, 1796)			+																				
<i>Psammotis pulveralis</i> (Hübner, 1796)			+														+						
<i>Anania coronata</i> (Hufnagel, 1767)																		+					
<i>Anania crocealis</i> (Hübner, 1796)			+																				
<i>Anania fuscalis</i> ([Denis & Schiffermüller], 1775)																			+				
<i>Anania hortulata</i> (Linnaeus, 1758)																		+					
<i>Anania verbascalis</i> ([Denis & Schiffermüller], 1775)																		+					
<b>Spilomelinae</b>																							
<i>Udea ferrugalis</i> (Hübner, 1796)					+									+		+				+		+	
<i>Pleuroptya ruralis</i> (Scopoli, 1763)			+					+									+	+		+	+		+
<i>Mecyna balcanica</i> Slamka & Plant, 2016					+		+	+						+							+		
<i>Mecyna trinalis</i> ([Denis & Schiffermüller], 1775)														+									
<i>Agrotera nemoralis</i> (Scopoli, 1763)																		+					+
<i>Dolicharthria punctalis</i> ([Denis & Schiffermüller], 1775)																							+
<i>Diaphania perspectalis</i> (Walker, 1859)																		+					
<i>Clasperia ophialis</i> (Treitschke, 1839)								+					+										

Site ref →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
taxon ↓																							
<i>Nomophila noctuella</i> ([Denis & Schiffermüller], 1775)																+						+	+
<b>Pyralidae</b>																							
<b>Phycitinae</b>																							
<i>Bradyrrhoa gilveolella</i> (Treitschke, 1832)													+	+									
<i>Oncocera semirubella</i> (Scopoli, 1763)							+					+		+		+	+	+					
<i>Laodamia faecella</i> (Zeller, 1839)					+	+	+																
<i>Rhodophaea formosa</i> (Haworth, 1811)																	+						
<i>Sciota fumella</i> (Eversmann, 1844)			+														+	+					
<i>Sciota hostilis</i> (Stephens, 1834)		+																					
<i>Denticera divisella</i> (Duponchel, 1842)					+																		
<i>Pempelia palumbella</i> ([Denis & Schiffermüller], 1775)			+	+	+		+		+	+	+		+		+		+					+	+
<i>Selagia spadicella</i> (Hübner, 1796)								+						+	+	+							
<i>Phycita coronatella</i> (Guenée, 1845)						+																+	
<i>Phycita cryptica</i> Plant & Slamka, 2016																						+	
<i>Phycita meliella</i> (Mann, 1864)								+					+		+	+							
<i>Phycita poteriella</i> (Zeller, 1846)														+									
<i>Phycita roborella</i> ([Denis & Schiffermüller], 1775)							+	+				+		+	+	+							+
<i>Phycita torrenti</i> Agenjo, 1963													+										
<i>Dioryctria simplicella</i> Heinemann, 1863																	+		+				

Site ref →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
taxon ↓																							
<i>Etiella zinckenella</i> (Treitschke, 1832)								+			+			+		+							
<i>Epischnia prodromella</i> (Hübner, 1799)											+				+	+			+				
<i>Catastia marginea</i> ([Denis & Schiffermüller], 1775)																					+		
<i>Elegia similella</i> (Zincken, 1818)		+			+									+			+		+		+		
<i>Khorassania compositella</i> (Treitschke, 1835)	+																						
<i>Trachonitis cristella</i> (Hübner, 1796)																		+					
<i>Delplanqueia dilutella</i> ([Denis & Schiffermüller], 1775) s. str.					+						+	+	+	+					+		+		+
<i>Delplanqueia inscriptella</i> (Duponchel, 1836)							+				+				+								
<i>Pempeliella bulgarica</i> Slamka & Plant, 2016								+															
<i>Pempeliella ornatella</i> ([Denis & Schiffermüller], 1775)			+	+	+	+	+		+		+	+							+	+	+		
<i>Pempeliella sororiella</i> Zeller, 1839					+																+		
<i>Hypochalcia ahenella</i> ([Denis & Schiffermüller], 1775)			+				+													+	+		+
<i>Acrobasis advenella</i> (Zincken, 1818)					+																		
<i>Acrobasis consociella</i> (Hübner, 1813)													+	+							+		
<i>Acrobasis fallouella</i> (Ragonot, 1871)								+				+		+									
<i>Acrobasis legatea</i> (Haworth, 1811)																							+

Site ref →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
taxon ↓																							
<i>Acrobasis marmorea</i> (Haworth, 1811)			+																	+	+		+
<i>Acrobasis sodalella</i> (Zeller, 1858)												+		+									
<i>Acrobasis suavella</i> (Zincken, 1818)																							+
<i>Acrobasis tumidana</i> ([Denis & Schiffermüller], 1775)							+							+		+							
<i>Episcythrasitis tabidella</i> (Mann, 1864)								+								+							
<i>Eurhodope cirrigerella</i> (Zincken, 1818)				+																			
<i>Myelois circumvoluta</i> (Fourcroy, 1785)								+					+	+									+
<i>Euzopherodes charlottae</i> (Rebel, 1914)																+							
<i>Ancylosis cinnamomella</i> (Duponchel, 1836)					+	+		+			+	+	+	+	+						+		+
<i>Homoeosoma nebulella</i> ([Denis & Schiffermüller], 1775)		+													+	+							
<i>Homoeosoma sinuella</i> (Fabricius, 1794)																+	+			+			
<i>Phycitodes benticella</i> Pierce, 1937							+																
<i>Ephestia kuehniella</i> (Zeller, 1879)														+									
<i>Ephestia woodiella</i> Richards & Thompson, 1932								+															
<i>Cadra furcatella</i> (Herrich-Schäffer, 1849)		+																					
<i>Ematheudes punctella</i> (Treitschke, 1833)								+															

Site ref →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
taxon ↓																							
<b>Pyralinae</b>																							
<i>Endotricha flammealis</i> ([Denis & Schiffermüller], 1775)		+					+	+						+	+	+							
<i>Synaphe punctalis</i> (Fabricius, 1775)																+					+		
<i>Pyralis farinalis</i> (Linnaeus, 1758)			+																				+
<i>Pyralis regalis</i> ([Denis & Schiffermüller], 1775)		+			+			+															+
<i>Stemmatophora brunnealis</i> (Treitschke, 1839)																+						+	+
<i>Stemmatophora honestalis</i> (Treitschke, 1829)					+																		
<i>Hypsopygia costalis</i> (Fabricius, 1775)		+															+	+			+		
<i>Hypsopygia glaucinalis</i> (Linnaeus, 1758)		+														+					+		
<i>Hypsopygia rubidalis</i> ([Denis & Schiffermüller], 1775)					+			+								+	+						
<b>Totals per site/date</b>	<b>1</b>	<b>10</b>	<b>13</b>	<b>7</b>	<b>25</b>	<b>4</b>	<b>22</b>	<b>22</b>	<b>5</b>	<b>5</b>	<b>11</b>	<b>11</b>	<b>15</b>	<b>25</b>	<b>13</b>	<b>22</b>	<b>27</b>	<b>18</b>	<b>14</b>	<b>12</b>	<b>27</b>	<b>9</b>	<b>35</b>

**Table 6.** Inventory of recorded species at sites in Albania (sites 24 to 39)

Site ref →	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
taxon ↓																
<b>Crambidae</b>																
<b>Crambinae</b>																
<i>Euchromius bellus</i> (Hübner, 1796)			+													
<i>Chrysoteuchia culmella</i> (Linnaeus, 1758)						+										
<i>Crambus pascuella</i> (Linnaeus, 1758)	+															
<i>Angustalius malacellus</i> (Duponchel, 1836)											+					
<i>Agriphila brioniella</i> (Zerny, 1914)														+		
<i>Agriphila tolli</i> (Bleszyński, 1952)											+					
<i>Catoptria falsella</i> ([Denis & Schiffermüller], 1775)						+								+		

Site ref →	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
taxon ↓																
<i>Catoptria mytilella</i> (Hübner, 1805)								+								
<i>Xanthocrambus saxonellus</i> (Zincken, 1821)						+										
<i>Ancylolomia palpella</i> ([Denis & Schiffermüller], 1775)														+		
<i>Pediasia aridella</i> (Thunberg, 1788)											+					
<b>Scopariinae</b>																
<i>Scoparia basistrigalis</i> Knaggs, 1866	+		+													
<i>Scoparia ingrata</i> (Zeller, 1846)						+										
<i>Scoparia manifestella</i> (Herrich-Schäffer, 1848)						+										
<i>Eudonia lacustrata</i> Panzer, 1804						+										
<i>Eudonia mercurella</i> (Linnaeus, 1758)										+						
<i>Eudonia phaeoleuca</i> (Zeller, 1846)						+										
<i>Eudonia sudetica</i> (Zeller, 1839)	+					+										
<b>Cybalomiinae</b>																
<i>Hyperlais dulcinalis</i> (Treitschke, 1835)			+			+										
<b>Acentropinae</b>																
<i>Cataclysta lemnata</i> (Linnaeus, 1758)				+												
<i>Elophila nymphaeata</i> (Linnaeus, 1758)				+												
<b>Odontiinae</b>																
<i>Aporodes floralis</i> (Hübner, 1809)													+			
<i>Cynaeda dentalis</i> ([Denis & Schiffermüller], 1775)	+	+							+	+	+		+			
<i>Cynaeda gigantea</i> Staudinger, 1880					+											
<i>Ephelis cruentalis</i> (Geyer, 1832)					+				+							
<b>Glaphyriinae</b>																
<i>Hellula undalis</i> (Fabricius, 1781)											+					
<i>Hydriris ornatalis</i> (Duponchel, 1832)											+					
<b>Evergestinae</b>																
<i>Evergestis aenealis</i> ([Denis & Schiffermüller], 1775)	+								+							
<i>Evergestis mundalis</i> (Guenée, 1854)						+										
<i>Evergestis politalis</i> ([Denis & Schiffermüller], 1775)			+													
<i>Evergestis serratalis</i> (Staudinger, 1870)								+						+		
<b>Pyraustinae</b>																
<i>Loxostege aeruginalis</i> (Hübner, 1796)					+	+										
<i>Ecpyrrhorrhoe diffusalis</i> Guenée, 1854							+									
<i>Ecpyrrhorrhoe rubiginalis</i> (Hübner, 1796)						+										
<i>Achyra nudalis</i> (Hübner, 1796)			+										+			
<i>Pyrausta aurata</i> (Scopoli, 1763)		+	+	+	+						+		+			+
<i>Pyrausta cingulata</i> (Linnaeus, 1758)					+											

Site ref →	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
taxon ↓																
<i>Pyrausta despicata</i> (Scopoli, 1763)	+										+			+		
<i>Pyrausta nigrata</i> (Scopoli, 1763)					+											
<i>Pyrausta purpuralis</i> (Linnaeus, 1758)			+							+			+			
<i>Pyrausta sanguinalis</i> (Linnaeus, 1767)		+	+							+			+			
<i>Pyrausta virginalis</i> (Duponchel, 1832)	+		+													+
<i>Uresiphita gilvata</i> (Fabricius, 1794)		+									+					
<i>Sitochroa verticalis</i> (Linnaeus, 1758)													+			
<i>Ostrinia nubilalis</i> (Hübner, 1796)	+		+						+	+			+			
<i>Anania hortulata</i> (Linnaeus, 1758)						+										
<b>Spilomelinae</b>																
<i>Udea ferrugalis</i> (Hübner, 1796)	+	+									+		+		+	+
<i>Udea numeralis</i> (Hübner, 1796)			+													
<i>Udea prunalis</i> ([Denis & Schiffermüller], 1775)					+											
<i>Pleuroptya balteata</i> (Fabricius, 1798)	+									+	+					
<i>Pleuroptya ruralis</i> (Scopoli, 1763)											+					
<i>Mecyna balcanica</i> Slamka & Plant, 2016					+				+							
<i>Mecyna flavalis</i> ([Denis & Schiffermüller], 1775)	+					+										
<i>Mecyna subsequalis</i> (Herrich-Schäffer, 1851)		+									+					
<i>Mecyna trinalis</i> ([Denis & Schiffermüller], 1775)						+										
<i>Diasemiopsis ramburialis</i> (Duponchel, 1834)															+	+
<i>Antigastra catalaunalis</i> (Duponchel, 1833)															+	+
<i>Spoladea recurvalis</i> (Fabricius, 1775)																+
<i>Palpita vitrealis</i> (Rossi, 1794)			+								+					
<i>Metasia gigantis</i> (Staudinger, 1871)																+
<i>Clasperia ophialis</i> (Treitschke, 1839)		+			+					+			+		+	
<i>Nomophila noctuella</i> ([Denis & Schiffermüller], 1775)		+		+	+		+			+	+	+	+	+	+	+
<b>Pyalidae</b>																
<b>Galleriinae</b>																
<i>Lamoria anella</i> ([Denis & Schiffermüller], 1775)		+	+													
<b>Phycitinae</b>																
<i>Isauria dilucidella</i> ([Denis & Schiffermüller], 1775)							+						+			+
<i>Oncocera semirubella</i> (Scopoli, 1763)									+	+			+			
<i>Laodamia faecella</i> (Zeller, 1839)													+			
<i>Denticera divisella</i> (Duponchel, 1842)							+				+				+	+
<i>Pempelia alpigella</i> (Duponchel, 1836)					+				+							
<i>Pempelia palumbella</i> ([Denis & Schiffermüller], 1775)		+	+						+	+	+				+	
<i>Selagia spadicella</i> (Hübner, 1796)					+				+	+						

Site ref →	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
taxon ↓																
<i>Phycita coronatella</i> (Guenée, 1845)					+											
<i>Phycita roborella</i> ([Denis & Schiffermüller], 1775)					+											
<i>Etiella zinckenella</i> (Treitschke, 1832)	+	+											+		+	
<i>Epischnia illotella</i> Zeller, 1839			+													
<i>Epischnia prodromella</i> (Hübner, 1799)		+														
<i>Elegia fallax</i> (Staudinger, 1881)	+															
<i>Elegia similella</i> (Zincken, 1818)	+															
<i>Matilella fusca</i> (Haworth, 1811)											+					
<i>Pterothrixidia rufella</i> (Duponchel, 1836)	+	+				+	+		+							
<i>Pima boisduvaliella</i> (Guenée, 1845)	+										+					
<i>Khorassania compositella</i> (Treitschke, 1835)									+				+			
<i>Delplanqueia dilutella</i> ([Denis & Schiffermüller], 1775) s. str.	+	+	+				+			+	+				+	
<i>Pempeliella bulgarica</i> Slamka & Plant, 2016													+			
<i>Pempeliella ornatella</i> ([Denis & Schiffermüller], 1775)	+								+							
<i>Pempeliella sororiella</i> Zeller, 1839	+		+			+										
<i>Oxybia transversella</i> (Duponchel, 1836)		+	+	+						+	+				+	
<i>Psorosa nucleolella</i> (Möschler, 1866)																+
<i>Acrobasis consociella</i> (Hübner, 1813)		+	+									+				
<i>Acrobasis fallouella</i> (Ragonot, 1871)		+													+	
<i>Acrobasis legatea</i> (Haworth, 1811)					+											
<i>Acrobasis marmorea</i> (Haworth, 1811)																
<i>Acrobasis repandana</i> (Fabricius, 1798)		+														
<i>Acrobasis sodalella</i> (Zeller, 1858)				+												
<i>Eurhodope rosella</i> (Scopoli, 1793)		+	+		+				+							
<i>Myelois circumvoluta</i> (Fourcroy, 1785)					+				+							
<i>Ancylosis cinnamomella</i> (Duponchel, 1836)	+						+									
<i>Phycitodes binaevella</i> (Hübner, 1813)									+				+			
<i>Phycitodes inquinatella</i> (Ragonot, 1887)			+										+			
<i>Ephestia welseriella</i> (Zeller, 1848)							+									
<i>Ephestia woodiella</i> Richards & Thompson, 1932			+												+	
<i>Cadra furcatella</i> (Herrich-Schäffer, 1849)	+															
<i>Ematheudes punctella</i> (Treitschke, 1833)		+				+							+			
<b>Pyralinae</b>																
<i>Endotricha flammealis</i> ([Denis & Schiffermüller], 1775)		+	+		+					+					+	
<i>Pyralis farinalis</i> (Linnaeus, 1758)												+				
<i>Pyralis regalis</i> ([Denis & Schiffermüller], 1775)		+	+		+	+			+		+				+	

Site ref →	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
taxon ↓																
<i>Stemmatophora brunnealis</i> (Treitschke, 1839)													+			
<i>Stemmatophora combustalis</i> (Fischer Von Röslerstamm, 1842)					+	+										
<i>Hypsopygia costalis</i> (Fabricius, 1775)	+			+						+		+	+			
<i>Hypsopygia glaucinalis</i> (Linnaeus, 1758)	+									+	+	+				
<i>Hypsopygia rubidalis</i> ([Denis & Schiffermüller], 1775)			+													
<b>total for each site/date</b>	<b>23</b>	<b>22</b>	<b>25</b>	<b>7</b>	<b>21</b>	<b>19</b>	<b>8</b>	<b>2</b>	<b>16</b>	<b>16</b>	<b>23</b>	<b>5</b>	<b>22</b>	<b>6</b>	<b>14</b>	<b>11</b>

**Table 7.** Inventory of recorded species at sites in Montenegro (40 & 41) and Macedonia (42 – 53).

Site ref →	40	41	42	43	44	45	46	47	48	49	50	51	52	53
taxon ↓														
<b>Crambidae</b>														
<b>Crambinae</b>														
<i>Euchromius bellus</i> (Hübner, 1796)	+													
<i>Chrysoteuchia culmella</i> (Linnaeus, 1758)								+						
<i>Crambus lathoniellus</i> (Zincken, 1817)		+												
<i>Catoptria confusella</i> (Staudinger, 1881)											+			
<i>Catoptria domaviella</i> (Rebel, 1904)		+												
<i>Catoptria falsella</i> ([Denis & Schiffermüller], 1775)														+
<i>Catoptria pinella</i> (Linnaeus, 1758)									+	+				
<b>Scopariinae</b>														
<i>Scoparia ambigualis</i> (Treitschke, 1829)	+													
<i>Scoparia ingratella</i> (Zeller, 1846)										+				
<i>Scoparia subfusca</i> Haworth, 1811		+												
<i>Eudonia mercurella</i> (Linnaeus, 1758)	+								+		+			
<i>Eudonia sudetica</i> (Zeller, 1839)								+						
<b>Cybalomiinae</b>														
<i>Hyperlais dulcinalis</i> (Treitschke, 1835)				+										
<b>Odontiinae</b>														
<i>Ephelis cruentalis</i> (Geyer, 1832)										+				
<b>Glaphyriinae</b>														
<i>Evergestis aenealis</i> ([Denis & Schiffermüller], 1775)							+							
<i>Evergestis serratalis</i> (Staudinger, 1870)												+		
<i>Evergestis sophialis</i> (Fabricius, 1787)				+				+						
<i>Hellula undalis</i> (Fabricius, 1781)														+
<b>Pyraustinae</b>														
<i>Ecpyrrhorhoe rubiginalis</i> (Hübner, 1796)	+													
<i>Paratalanta pandalis</i> (Hübner, 1825)			+											

Site ref →	40	41	42	43	44	45	46	47	48	49	50	51	52	53
taxon ↓														
<i>Pyrausta (Panstegia) aerealis</i> (Hübner, 1793)								+						
<i>Pyrausta castalis</i> (Treitschke, 1829)				+										
<i>Pyrausta aurata</i> (Scopoli, 1763)	+													
<i>Pyrausta cingulata</i> (Linnaeus, 1758)	+													
<i>Pyrausta despicata</i> (Scopoli, 1763)													+	
<i>Pyrausta purpuralis</i> (Linnaeus, 1758)			+											
<i>Pyrausta virginalis</i> (Duponchel, 1832)	+		+											
<b>Spilomelinae</b>														
<i>Udea ferrugalis</i> (Hübner, 1796)				+				+				+	+	+
<i>Mecyna balcanica</i> Slamka & Plant, 2016	+							+						
<i>Antigastra catalaunalis</i> (Duponchel, 1833)														+
<i>Metasia carnealis</i> (Treitschke, 1829)											+			
<i>Clasperia ophialis</i> (Treitschke, 1839)											+			
<i>Nomophila noctuella</i> ([Denis & Schiffermüller], 1775)											+	+	+	+
<b>Pyralidae</b>														
<b>Phycitinae</b>														
<i>Bradyrrhoa gilveolella</i> (Treitschke, 1832)				+										
<i>Alophia combustella</i> (Herrich-Schäffer, 1855)				+										
<i>Denticera divisella</i> (Duponchel, 1842)														+
<i>Pempelia alpigenella</i> (Duponchel, 1836)								+						
<i>Pempelia palumbella</i> ([Denis & Schiffermüller], 1775)								+						
<i>Selagia spadicella</i> (Hübner, 1796)									+					
<i>Phycita roborella</i> ([Denis & Schiffermüller], 1775)								+						
<i>Phycita torrenti</i> Agenjo, 1963											+			
<i>Etiella zinckenella</i> (Treitschke, 1832)	+		+											
<i>Epischnia prodromella</i> (Hübner, 1799)								+						
<i>Elegia fallax</i> (Staudinger, 1881)				+										
<i>Khorassania compositella</i> (Treitschke, 1835)				+			+	+					+	
<i>Delplanqueia dilutella</i> ([Denis & Schiffermüller], 1775) s. str.				+	+					+				+
<i>Pempeliella ornatella</i> ([Denis & Schiffermüller], 1775)	+	+		+			+	+		+				
<i>Pempeliella sororiella</i> Zeller, 1839							+				+			
<i>Psorosa dahliella</i> (Treitschke, 1832)												+		
<i>Hypochalcia ahenella</i> ([Denis & Schiffermüller], 1775)							+	+						
<i>Acrobasis consociella</i> (Hübner, 1813)											+			
<i>Acrobasis fallouella</i> (Ragonot, 1871)											+			+
<i>Acrobasis tumidana</i> ([Denis & Schiffermüller], 1775)											+			
<i>Eurhodope rosella</i> (Scopoli, 1793)								+						

Site ref →	40	41	42	43	44	45	46	47	48	49	50	51	52	53
taxon ↓														
<i>Eurhodope cirrigerella</i> (Zincken, 1818)	+													
<i>Metallosticha argyrogrammos</i> (Zeller, 1847)	+													
<i>Ancylosis cinnamomella</i> (Duponchel, 1836)	+		+						+			+		
<i>Homoeosoma sinuella</i> (Fabricius, 1794)								+						
<i>Phycitodes inquinatella</i> (Ragonot, 1887)											+			
<i>Ephestia woodiella</i> Richards & Thompson, 1932											+			+
<i>Cadra furcatella</i> (Herrich-Schäffer, 1849)								+						
<i>Ematheudes punctella</i> (Treitschke, 1833)	+													
<b>Pyalinae</b>														
<i>Endotricha flammealis</i> ([Denis & Schiffermüller], 1775)	+										+			
<i>Synaphe punctalis</i> (Fabricius, 1775)											+			
<i>Stemmatophora brunnealis</i> (Treitschke, 1839)									+					
<i>Stemmatophora combustalis</i> (Fischer Von Röslerstamm, 1842)	+													
<i>Hypsopygia costalis</i> (Fabricius, 1775)													+	
<i>Hypsopygia glaucinalis</i> (Linnaeus, 1758)														+

The taxonomy of the Pyraloidea is in a state of fluidity, with much current research being undertaken. In the presentation of our results we follow, in broad terms, the nomenclature adopted by Slamka (2006; 2008; 2013) and for groups not yet reviewed by him we follow Leraut (2009; 2014). However, we have updated names where more recent data are available. For simplicity, we have adopted the sequence of subfamilies and species presented by Slamka (*op. cit.*) for the Crambidae and Leraut (2014) for the Pyralidae.

## DISCUSSION

### General comments

The pyraloid fauna of the Balkan Peninsula, apart from that of Bulgaria, is rather poorly known. Plant (2016) lists 224 Crambidae and 165 Pyralidae for Bulgaria, a total of 389 taxa. Rákossy, Goia & Kovács (2003) recognised 391 species (217 Crambidae and 174 Pyralidae) within the Romanian fauna. Hungary, to the north, is not a part of the Balkan Peninsula, but it is of interest for comparative purposes to record that Pastorális (2010) lists 305 species (171 Crambidae and 134 Pyralidae). There seems to be no comprehensive inventory for Greece to the south; the only available list appears to be that within Karsholt & Razowski (1996).

A major problem affecting faunal listings for Macedonia, Montenegro and southern Serbia is one of historical politics; these areas have been incorporated under the label of “Yugoslavia” as a single entity. For the former Republic of

Yugoslavia, Karsholt & Razowski (1996) list 390 species (202 Crambidae and 188 Pyralidae), but in addition to the lack of distinction between the component territories, the area covered also includes Bosnia & Herzegovina, Croatia and Slovenia.

As far as we are aware, the earliest comprehensive inventory of Pyraloidea for any part of the southern Balkans within the present study area is that which can be found in the work by Rebel & Zerny ([1931] 1934). This important work addresses the fauna of Albania, Macedonia and Montenegro and lists 175 species in total, of which 113 belong in the present day Crambidae and 62 within Pyralidae *sensu stricto*. However, a measure of interpretation is required in transferring the data from this paper to a modern database, since it was published at a time when the national boundaries of countries within the Balkans followed different lines. For example, a significant number of records have, as their source locality, “Mazedonien: Alibotuš Pl.” (usually credited to Drenowski). This refers to the Alibotush Mountain area, now called Slavyanka Mountain and firmly within the boundary of the modern-day Republic of Bulgaria.

Thurner (1941) lists a total of 182 pyraloid taxa from the Lake Ohrid area of Macedonia. Careful examination of synonymy and a consideration of taxonomic changes that have taken place in the intervening 78 years reduce this list by ten, leaving 172 valid species recorded by Thurner for Macedonia. Just over twenty years later Moucha (1963) reported several new lepidopteran species for Albania, including 7 Pyraloidea, collected in 1959, but contemporary works on all of these areas are sparse. Misja (1987) reported 54 species of Tortricoidea and Pyraloidea of which 11 were tortricids and 43 were Pyraloidea

and of these, 16 pyrales were stated to be new to the Albanian fauna. Mihajlović *et al* (1991) listed 77 species for Durmitor in Montenegro. Most recently, Beshkov (1994; 1996) reported on several species of migratory Lepidoptera in Albania and Macedonia mentioning three species in each year (a total of five species altogether). The earlier of the two papers included *Palpita vitrealis* (Rossi, 1794) from three localities in Albania – a species claimed later, erroneously, as new to Albania by Plóciennik, Pawlikiewicz & Pabis (2009). Karsholt & Razowski (1996) summarised available published data at that date and listed 185 taxa (98 Crambidae and 87 Pyralidae) for Albania.

We are not aware of any existing formal checklist of Serbian Pyraloidea, although one is in long-term preparation (Dejan Stojanović, pers. comm., 2017). Stojanović & Radaković (2016) mention 45 Crambidae and 29 Pyralidae from the Đerdap National Park. Jakšić (2016a), reported on the monitoring of Lepidoptera at the Zvezdarska Forest Nature Monument in

Belgrade, listing 11 species of Pyraloidea; neither of these studies affect our current, casual definition of southern Serbia.

Leraut (2009, 2014) provided distribution maps for most species (and text for the remainder) in which the different Balkan countries were distinguished. However, some maps contain errors with regard to other parts of Europe and since original data sources are not cited it is not possible to know if all of the maps are accurate in relation to the Balkan Peninsula. Accurate distribution maps were provided by Slamka (2006; 2008; 2013), but this series of books is incomplete and so a total number of species is not available. Most recently, Jakšić (2016) provided a tentative list of Serbian microlepidoptera based on a search of published sources only and lists 146 species of Crambidae and 82 Pyralidae – a total of 228 taxa. This work provides the most up to date inventory, but it omits unpublished data so that the list is still incomplete. Table 8 summarises the numbers of pyraloid taxa already known (published) from the different territories of the Balkans (and Hungary).

**Table 8.** Summary of published data on recorded pyraloid taxa in the Balkans and Hungary.

Region	Number of		
	Crambidae	Pyralidae	all Pyraloidea
Hungary (Pastorális, 2010)	171	134	305
Romania (Rákossy, Goia & Kovács, 2003)	217	174	391
Bulgaria (Plant, 2016)	224	169	389
F. R. Yugoslavia (Karsholt & Razowski, 1996)	202	188	390
Greece (Karsholt & Razowski, 1996)	192	165	357
Albania (Karsholt & Razowski, 1996)	98	87	185
Serbia (Jakšić, 2016)	146	82	228

Plant (2016) used a free statistical program (EstimateS) to predict that the Bulgarian fauna might rise to 420 species, but concluded that the rate of recruitment would be so low that this process would take a further 150 years of survey: thus the Bulgarian list of pyrales can be regarded as complete. With that in mind, the close similarity between the total numbers for Romania, Bulgaria and Yugoslavia is interesting and suggests that a fauna of just below 400 species might reasonably be expected for a country the size of Serbia.

Albania, though doubtless much under-recorded, occupies a significantly smaller area of land and has a less varied range of biotopes within its boundary; it is perhaps less likely to achieve a comparable total.

This present work adds 29 species to the Serbian fauna and 37 to that of Albania, so that these lists rise to 257 and 222 respectively (Table 9). A further species (3 Crambidae and 2 Pyralidae) appear to be new to the published fauna of Macedonia.

**Table 8.** Summary of published data on recorded pyraloid taxa in the Balkans and Hungary.

Region	Number of species of								
	Crambidae			Pyralidae			all Pyraloidea		
	was	added here	now	was	added here	now	was	added here	now
<b>Serbia</b>	146	8	154	82	21	103	228	29	257
<b>Albania</b>	98	20	117	87	17	105	185	37	222
<b>Macedonia</b>	?	3	?	?	2	?	?	5	?

## Comments on selected species

### CRAMBIDAE

#### CRAMBINAE

##### *Crambus ericella* (Hübner, 1813)

According to the distribution shown at the Fauna Europaea website ([https://fauna-eu.org/cdm\\_dataportal/taxon/33bedfd9-5f66-4f05-8182-4b1ef2f9b348](https://fauna-eu.org/cdm_dataportal/taxon/33bedfd9-5f66-4f05-8182-4b1ef2f9b348)), accessed 3 September 2017, this moth is unrecorded from the entire of the Balkan Peninsula ("Yugoslavia", Greece and Bulgaria), although it is shown for Romania and for Hungary and Austria to the north. This might reflect the low level of recording effort here, although we can confirm the apparent absence from Bulgaria during the years 2002 to 2016 and so perhaps it is genuinely rare here. We found it only at Valjevo (site 17) in Serbia on 14 June 2017, where it was common in tall, open grassland growing between the edge of a *Pinus* woodland and an associated forest track that was dominated by a ruderal flora. Apparently new to the Serbian fauna.

##### *Angustalius malacellus* (Duponchel, 1836)

This is one of the smaller members of the Crambinae and appears to be extremely local, across the Mediterranean region from Spain to Greece and further east on Cyprus. Confusion is possible with *Mesocrambus candiellus* (Herrich-Schäffer, 1848) which is also present in the former Yugoslavia; the genitalia are distinctive in both sexes. Several females came to light at Valjevo (site 17) in Serbia on 14 June 2017 and there were 3 females in the sample from Albania in the Vukopoles River Gorge (site 34) on 25 September 2016. Apparently new to the Albanian fauna.

##### *Agriphila tolli* (Bleszyński, 1952)

This is a widespread and abundant "grass moth" present throughout eastern Europe, including the Balkans, as far north as Austria, Hungary and Romania. It appears to replace *Agriphila geniculea* (Haworth, 1811) in the south-east, although Slamka (2008) reports both species present in Austria, Hungary and Italy. All records of *A. geniculea* from any of the Balkan countries, at least in the south, are potentially misidentified examples of *A. tolli*; new material must be critically examined and voucher specimens that support older records should be located and checked.

##### *Pediasia aridella* (Thunberg, 1788)

This is a surprising record, but is confirmed by critical examination of the genitalia of specimens of both sexes from the Berat region of Albania in the Vukopoles River Gorge (site 34) on 25 September 2016. It is a species of central and northern parts of Europe and is extremely local everywhere south of Hungary. The moth also persists on the Black Sea coast in Bulgaria, at a site very close to the Romanian border (Plant, 2016). Apparently new to the Albanian fauna.

### SCOPARIINAE

##### *Scoparia basistrigalis* Knaggs, 1866

Members of the genera *Scoparia* Haworth and *Eudonia* Billberg are hopelessly under-recorded across the whole of Europe and the Balkans present no exception. With experience, fresh examples of some species can be named with confidence, but examination of genitalia is often necessary, especially with worn material. Although it occurs in both lowland and montane biotopes, *S. basistrigalis* is most often encountered in woodland areas. We record it now from sites 14, 17, 18 & 21 in Serbia and sites 24 & 26 in Albania. Apparently new to the Serbian fauna. Apparently new to the Albanian fauna.

##### *Scoparia perplexella* (Zeller, 1839)

This is a species that is apparently endemic to the Balkan Peninsula, with known reports from Bosnia & Herzegovina, Croatia, Macedonia and Greece, so it is not especially surprising that it is also present in Serbia. It is one of the larger "scops" and flies from the end of May to early July as single generation. We record it at sites 7 and 19. Apparently new to the Serbian fauna.

##### *Eudonia sudetica* (Zeller, 1839)

Most published distribution maps suggest that this species does not extend as far south, in Europe, as the Balkan and Italian Peninsulas. The maps at the *Fauna Europaea* website concur, but are not fully up to date; they show this species "absent" (as opposed to "no data"), from adjacent Bulgaria, in spite of it having been recorded here in recent years (Plant, 2016). The present survey showed that this species is also present throughout the southern Balkans, with records here from three sites in Serbia, two in Albania and one in Macedonia. Nuss (2005) informs that it is associated with native *Picea* forest in low montane sites and with *Pinus cembra* and/or *Juniperus* spp. in upper montane areas. (above 500 metres). Apparently new to the Serbian fauna. Apparently new to the Albanian fauna. Apparently new to the Macedonian fauna.

### CYBALOMIINAE

##### *Hyperlais dulcinalis* (Treitschke, 1835)

This rather attractive moth is known in Europe from Italy, Croatia, Hungary, Romania, Bulgaria, Macedonia and Greece, extending eastwards to the Russian territory of Sarepta. Its absence from the other countries of the Balkan Peninsula perhaps reflects under-recording. We found it in light traps at several sites in Serbia, in the Veles Region of Macedonia and in Albania near Bistricë (Delvina region) and above Drenovë (Korçë region). Apparently new to the Albanian fauna.

### ODONTIINAE

##### *Cynaeda gigantea* Staudinger, 1880

Two male examples were recognised by genitalia dissection of "greasy" specimens from the Korçë region of Albania, above Kloçë, near the radio mast (site 28) on 4 July 2016. In Europe, this moth occurs in the Balkan Peninsula and the Black seaboard zone. Apparently new to the Albanian fauna.

## GLAPHYRIINAE

*Hellula undalis* (Fabricius, 1781)

This species is resident in the African and Asian continents and European examples are regarded as primary immigrants. We caught examples in light traps in Albania (site 31) in late September and Macedonia (site 53) in mid October. Apparently new to the Albanian fauna. Apparently new to the Macedonian fauna.

*Hydriris ornatalis* (Duponchel, 1832)

As with the preceding species, this is a primary immigrant in Europe to the Mediterranean Basin only. As a migrant, it does not, apparently, extend as far north as *Hellula undalis*. It is also potential resident in the southern areas of Spain (Barry Goater, pers. Comm.) whilst Corley (2015) gives several localities in Portugal, in two broods. We recorded two males at light in the Berat region of Albania, in the Vukopoles River Gorge (site 34) on 25 September 2016. Apparently new to the Albanian fauna.

## EVERGESTINAE

*Evergestis mundalis* (Guenée, 1854)

Separation of this species from the rather more widespread *E. aenealis* (Denis & Schiffermüller, 1775) is usually based on the colour of the scales adorning the vertex of the head of the adult moth, but it is easy for a measure of complacency to cloud scientific judgement. Greasy specimens of *E. aenealis* frequently appear to have a vertex that is concolorous with the rest of the body whilst examination of the genitalia suggests that some examples of *E. mundalis* may include a few yellowish scales on the vertex. Examination of the genitalia is advised in all cases of doubt and in this present study *all* identifications reported have relied on that technique. We record males and females from Serbia in the Preševo district, above Trnava (site 14) at an altitude of 800 metres and from Albania in the Korçë region, above Drenovë (site 29) at 1050 metres, both in early July 2016. Apparently new to the Serbian fauna. Apparently new to the Albanian fauna.

*Evergestis politalis* ([Denis & Schiffermüller], 1775)

The absence of this species from most of the former Yugoslavia in published distribution maps is surprising. It could possibly be misidentified in flight for the broadly similar *E. limbata* (Linnaeus, 1767), although this too is lacking from most of the area under study. We recorded *E. politalis* from Bistricë, in the Delvina region of Albania (site 26) in early June. Apparently new to the Albanian fauna.

*Evergestis serratalis* (Staudinger, 1870)

Leraut (2012) refers to the distribution and status of this distinctive species as, simply, “Balkans, Crimea. Highly local”. We record it here from two sites in the Korçë region of Albania (sites 31 and 37), both in late September and at Vardar-Pčinja in Macedonia (site 51) in mid October. It is probably fairly widespread in low and high montane habitats throughout the area under study. Apparently new to the Albanian fauna.

## PYRAUSTINAE

*Paratalanta pandalis* (Hübner, 1825)

This is a species that is widespread and common in northern and central areas of Europe; it is far less frequent in the south. It is known from Albania and also from Bulgaria to the east. This present work records it now from Macedonia, in the Demir Kapija Gorge (site 42) and Serbia at Mt. Starac (site 16) and Mt. Šljivovički Vis (site 23). Apparently new to the Serbian fauna. Apparently new to the Macedonian fauna.

*Achyra nudalis* (Hübner, 1796)

Recorded at two sites in Albania, in the Delvinë region at Bistricë on 7 June (site 26) and above Stan-Karbunarë in the Lushnjë region on 11 August (site 36), suggesting that there are at least two generations of adults per year. Elsewhere in Europe, the larvae are reported on *Camphorosma* spp., *Echium* spp., *Chenopodium* spp., *Amaranthus* spp and *Beta* spp. The European distribution is circum-Mediterranean, but also extends inland away from any maritime influence in Bulgaria and then eastwards across Asiatic Turkey to the Caucasus and so there is potential for this moth to be found in Serbia.

*Ostrinia nubilalis* (Hübner, 1796)

The taxonomy of this apparently widespread and common European moth is confused – not least because different authors have applied conflicting names to various supposedly separate taxa. One taxon within this complex is known colloquially as the “European Corn Borer” and is a commercial pest species. However, another taxon, which some regard as a separate species and others only as an ecotype, appears to thrive on *Artemisia* as a larval pabulum. Consideration of this problem is beyond the scope of this present paper, but it should be noted that by reference to the male genitalia drawings presented in Slamka (2013), site 31, in Albania, sampled on 11 August 2016, produced both *nubilalis nubilalis* [= *maysalis* auctorum] (Slamka Fig. 78) and *nubilalis scapularis* (Slamka Fig. 79) [= *nubilalis nubilalis* auctorm]. All other sites in the results Tables refer to *nubilalis nubilalis* [= *maysalis* auctorum] (Slamka Fig. 78).

## SPILOMELINAE

*Udea numeralis* (Hübner, 1796)

Although apparently more frequent in western Europe, the range of this species extends along the Mediterranean coastline via Italy to Greece and Asiatic Turkey, so that it is unsurprising that we have now recorded it in Albania at Bistricë (site 26). Apparently new to the Albanian fauna.

*Udea prunalis* ([Denis & Schiffermüller], 1775)

A history of under-recording is likely to explain the apparent absence, until now, of this species in Albania. We recorded it above Klocë, near the radio mast, in the Korçë region (site 28). Apparently new to the Albanian fauna.

*Pleuroptya balteata* (Fabricius, 1798)

This is a species that is associated with the southern areas of Europe along the whole of the Mediterranean seaboard then extending eastwards across Asiatic Turkey and into Syria, Iran

and adjacent territories. It is previously known from Serbia and Macedonia and we now report it from Albania at the Devolli Gorge in the Korçë region (site 24) and the Vukopoles River Gorge in the Berat region on two occasions (sites 33 & 34). Apparently new to the Albanian fauna.

*Mecyna balcanica* Slamka & Plant, 2016

This is the undescribed species number 144 in the recent work by Slamka (2013). Separation from *M. lutealis* (Duponchel) requires examination of genitalia in both sexes and it is probable that older reports of “*lutealis*” from south-eastern Europe are referable to this taxon (see Slamka & Plant, 2016 for discussion and a recent distribution map). Consequently, old records of *M. lutealis* must be regarded as “unconfirmed” until such time as a voucher specimen has been located and dissected. The known distribution includes the Carpathian Mountains in northern Hungary and south-west Bulgaria as well as the mountainous regions of the former Yugoslavia; the type locality is Croatia, and the series of paratypes include material from both Montenegro and Macedonia. Further east, there are reports from Turkey and from the Caucasus. We record it here at numerous sites in Serbia, Albania, Macedonia and Montenegro between 586 and 1740 metres altitude where it is common and flies in July. It is likely to be widespread throughout the whole Balkan Peninsula. Apparently new to the Serbian fauna. Apparently new to the Albanian fauna.

*Mecyna flavalis* ([Denis & Schiffermüller], 1775)

Although not a rare species, this is far more often encountered in Central Europe, with populations in the Balkans appearing to be scattered and sometimes isolated. Although usually recognisable with ease, worn specimens may resemble *M. balcanica* whilst bright specimens of the latter may on occasion be misidentified as the present species. Examination of genitalia is urged in all cases until the true distribution pattern has become clear. Apparently new to the Albanian fauna.

*Mecyna subsequalis* (Herrich-Schäffer, 1851)

This is a species of Turkey eastwards to Transcaucasia, extending southwards to the Lebanon and northwards around the Black Sea before extending back, westwards, only as far west as the Crimean Peninsula. In Europe, it is known from one site in the extreme south-west of Bulgaria (Plant, 2016), Greece and Crete. The discovery of males, conforming to ssp. *subsequalis* (Herrich-Schäffer), at two sites in Albania, on the coast at Palasë, near Dhërmi, altitude 274 metres (site 25) and in the Berat region in the Vukopoles River Gorge at 217 metres altitude (site 34) is a surprise. New to the Albanian fauna.

*Mecyna trinalis* ([Denis & Schiffermüller], 1775)

A generally widespread and not uncommon species of xerothermic habitats in southern Europe. The apparent absence of past Serbian records for this somewhat distinctive species is likely to be a measure of the degree to which the country is under-recorded. We record it here at site 14 (Serbia) and site 29

(Albania), both in early July. Apparently new to the Serbian fauna.

*Antigastra catalaunalis* (Duponchel, 1833)

This is, essentially, a tropical species that is also established in the Mediterranean basin. Elsewhere in Europe it is recognised as a primary immigrant. It is unclear if populations along the Adriatic seaboard represent established residents or if they depend upon occasional immigration for their longer-term maintenance, but the examples reported in the present survey, from the Ionian coast of Albania on 17 October 2016 (site 38) and the Vardar River Valley of Macedonia on 20 October 2016 (site 53) were all caught in association with other species that are known to migrate, such as *Udea ferrugalis* (Hübner, 1796) and *Nomophila noctuella* ([Denis & Schiffermüller], 1775). Apparently new to the Albanian fauna.

*Spoladea recurvalis* (Fabricius, 1775)

A native of tropical and subtropical regions, migrating north in late summer and perhaps with temporary residency in the extreme south of the Iberian peninsula, this distinctive species was, until fairly recently, known to European lepidopterologists as an immigrant to the western half of the continent. In recent years however, it has been recorded in the east, including south-west Bulgaria in 2009. The discovery of presumably immigrant examples in the samples from the Albanian coast between Butrint and Sarandë (site 39) on 18 October 2016 supports the notion that the global movement pattern has been altered. Reasons for this are obscure, but climate change is an obvious candidate. Apparently new to the Albanian fauna.

*Metasia gigantalisis* (Staudinger, 1871)

The drawing of the male genitalia at Fig. 122a, page 480 in Leraut (2012), which purport to belong to *M. carnealis* are in fact wrongly identified and in reality depict *M. gigantalisis*. Those of the female at Fig. 123a on page 481 appear to be correct. More detailed drawings are available in Slamka (2013). The species is known from Sicily, Crete, Cyprus and Asiatic Turkey as well as from Greece, where it seems widespread and Macedonia, whence it is reported only from the Lake Ohrid area (Turner, 1938-1941). We record it here from the Ionian Coast of Albania near Ksamil, between Butrint and Sarandë in mid October 2016 (site 39). Apparently new to the Albanian fauna.

PYRALIDAE

GALLERIINAE

*Lamoria anella* ([Denis & Schiffermüller], 1775)

A large and, when fresh, distinctive species that is found in most of Europe apart from the far north. It is likely to be overlooked in many places. We recorded it on the coast near Dhërmi (site 25) and at Bistricë (site 26), both in Albania, in early June 2016. Apparently new to the Albanian fauna.

PHYCITINAE

*Bradyrrhoa gilveolella* (Treitschke, 1832)

This is very much an eastern species in Europe, from Sicily (but apparently not Italy) and then from the Balkan Peninsula

eastwards across the Middle East as far as Kyrgyzstan. Our records here from the Preševo district of Serbia, above Trnava on two dates (sites 13 and 14) must surely represent under-recording. We also note it from Macedonia in the Veles Region, above Sveti Ilija Monastery (site 43). Apparently new to the Serbian fauna.

*Isauria dilucidella* ([Denis & Schiffermüller], 1775)

A widespread species throughout southern Europe; the absence of earlier records from much of the Balkans doubtless reflects a poor recording effort. We light-trapped examples at above Drenovë (site 30) on 9 August 2016, above Stan-Karburnarë (site 36) on 11 August 2016 and on the coast between Butrint and Sarandë (site 39) on 18 October 2016, all in Albania. Apparently new to the Albanian fauna.

*Laodamia faecella* (Zeller, 1839)

Existing European distribution maps suggest that this species is absent from the whole of the Balkans apart from Romania. However, it is widespread, if local, in Bulgaria (Plant, 2016). Jakšić (2016) lists it for the Serbian fauna and the present survey records three sites in Serbia (sites 5, 6 and 7) in July 2014. We also report it in the Lushnjë region of Albania, above Stan-Karburnarë (site 36) on 11 August 2016. Apparently new to the Albanian fauna.

*Sciota hostilis* (Stephens, 1834)

Published distribution maps suggest that this species is absent from most of the Balkans. However, it is present in Greece and European Turkey in the south and then north in Hungary and other countries to the west and east, so may simply be under-recorded. That said, we have only a single record, suggesting that it might be rare here, from Suva Planina, above Bojanine Vode (site 2). Apparently new to the Serbian fauna.

*Denticera divisella* (Duponchel, 1842)

This is a fairly widespread and not uncommon moth of southern Europe, the range of which extends southwards to include Sub-Saharan Africa. The lack of records from Balkan countries until now is a reflection of poor recording coverage. We caught examples in Serbia in the Pirot region (site 5), at four localities in Albania (sites 30, 34, 38 and 39) and in Macedonia along the road to Klisura in the Vardar River Valley (site 53). Apparently new to the Serbian fauna. Apparently new to the Albanian fauna.

*Phycita coronatella* (Guenée, 1845)

The recent recognition of *Phycita cryptica* (see next species) has thrown past records of most *Phycita* species (other than *roborella* (D. & S., 1775)) into disarray and existing distribution maps may not be reliable for this and some other *Phycita* species. We recorded examples in the Pirot region of Serbia and the Korçë region of Albania (sites 6, 22 & 28). Apparently new to the Serbian fauna. Confirmed for the Albanian fauna.

*Phycita cryptica* Plant & Slamka, 2016

As the specific epithet suggests, this species was “hidden” amongst other members of the genus; past records of several other species might refer to this newly recognised taxon (*vide* Plant & Slamka, 2016, for a detailed discussion). During the present survey we found males only at Crni Vrh, near Pirot (site 22) on 2 September 2016. New to the Serbian fauna.

*Phycita meliella* (Mann, 1864)

The comments under *P. coronatella* apply. We found examples of both sexes at four sites in Serbia (sites 8, 13, 15 & 16) in the present study. Apparently new to the Serbian fauna.

*Phycita poteriella* (Zeller, 1846)

The comments under *P. coronatella* apply. We found female examples in the trap samples from above Trnava in the Preševo district of Serbia (site 14) on 9 July 2016. Apparently new to the Serbian fauna.

*Phycita torrenti* Agenjo, 1963

For many decades, this species was thought to be endemic to the Iberian Peninsula, but that is not so. Confusion within the genus has been mentioned above; *P. torrenti* is in fact widespread in southern Europe (*vide* Plant & Slamka, 2016). We record it here from above Trnava in Serbia (site 13) and the Treska Gorge, Macedonia (site 50) and expect it to be found in many other places. Apparently new to the Serbian fauna. New to the Macedonian fauna.

*Dioryctria simplicella* Heinemann, 1863

This is primarily a species of the northern parts of Europe and its appearance in the south therefore requires comment. It has been recorded in adjacent Bulgaria, but not prior to the year 2003 (Plant, 2016) and there is a possibility that it has extended its range southwards in recent years. Its presence in Serbia at Divčibare, Kaona (site 17) and Tornik Ribna (site 19) might, therefore, be a recent phenomenon. In the case of worn or damaged specimens correct identification requires examination of the male genitalia. Apparently new to the Serbian fauna.

*Elegia fallax* (Staudinger, 1881)

Separation of all worn examples of *E. fallax* from *E. similella* (Zincken, 1818) requires examination of genitalia (both sexes are distinct in that respect); the number of “surprises” encountered over several years where specimens named as one taxon by a supposed expert appear to have “evolved” into the other taxon during museum storage suggests that even freshly-emerged adults might benefit from such critical appraisal. Both species are likely to be widespread in the region. A third species, *E. fallaximima* Nel & Mazel, 2012 is known from the Iberian Peninsula; so far, it has not been found elsewhere, but without dissection it would be overlooked. We found *E. fallax* in Albania near Strelçë village in the Devolli Gorge (site 24) on 5 June 2016 and in Macedonia above Sveti Ilija Monastery, Veles Region (site 43) on 11 May 2016. Apparently new to the Albanian fauna.

*Pterothrixidia rufella* (Duponchel, 1836)

A widespread and common species in all areas. The very similar taxon *Pterothrixidia squalidella* (Eversmann, 1842), which was raised to full species rank by Leraut (2014), has not been found in the present study area.

*Pima boisduvaliella* (Guenée, 1845)

Although known from the Balkans, this is evidently a very localised species. We found it near Strelca in the Korçë region of Albania (site 24) in June and in the Berat region in the Vukopoles River Gorge between Ibrollara and Vale (site 34) in late September. Apparently new to the Albanian fauna.

*Delplanqueia dilutella* ([Denis & Schiffermüller], 1775) s. str.

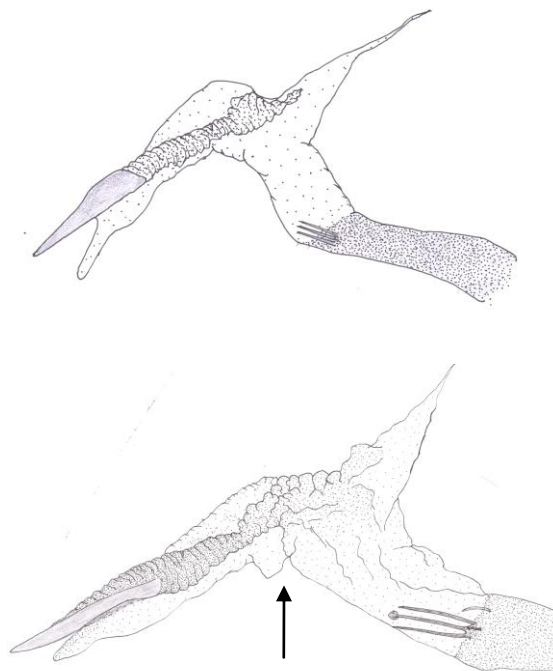
Leraut (2001), resurrected the split of *Pempeliella dilutella* into two segregate taxa – *inscriptella* (Duponchel, 1836) and *dilutella* D. & S. *sensu stricto*. The move remains controversial, although probably correct. Characters of wing morphology are treated by some workers as important identification features, but examination of material from a wide geographical range across Europe suggests that these features may not be quite so clear cut. The male genitalia usually show significant differences in the size and shape of the single cornutus of the aedeagus, although intermediate examples have been encountered in Bulgaria. A small difference in the fully everted vesica of the male has been reported (Plant, 2016) and that character (Fig. 2) coincides 100% with the form of the cornutus in specimens from Serbia, Macedonia and Albania. Old records of “*dilutella*” must, of course, be treated as unreliable unless voucher specimens are located and critically examined. Consequently: Apparently new to the Serbian fauna. Apparently new to the Albanian fauna. Apparently new to the Macedonian fauna.

*Delplanqueia inscriptella* (Duponchel, 1836)

This segregate of *D. dilutella* is discussed above. Male examples, identified on the shape of the cornutus of the aedeagus and on the presence of a small diverticulum positioned at the inner angle of the vesica (Fig. 2), were found at Zvijezda, Savina Voda, near the Jabuka Pass on 16 July 2014 (site 7), above Trnava in the Preševo district on 31 May 2016 (site 11) and in the Svrlijski Timok river gorge, near Niševacon 4 August 2016 (site 15) – all in Serbia. Apparently new to the Serbian fauna.

*Pempeliella bulgarica* Slamka & Plant, 2016

This species was recognised as distinct very recently, and is known at present only from Bulgaria, Asiatic Turkey and a single site in Hungary (an historic record). Its presence in other Balkan countries was predicted (Slamka & Plant, 2016a). It has been confused with *P. sororiella* Zeller, 1839, which is also common in the region; male and female genitalia are distinct and provide easy separation, but old reports of “*sororiella*” are to be treated as unconfirmed until voucher specimens are traced and dissected. The present survey records both sexes at the site near Trgovište, below Vražji Kamen in Serbia (site 8) and above Stan-Karbanarë in the Lushnjë region of Albania (site 36). New to the Serbian fauna. New to the Albanian fauna.



**Figure 2.** Everted vesica of the aedeagus of male *Delplanqueia* species: Top – *D. dilutella* sensu stricto; below – *D. inscriptella*. The arrow indicates the small additional diverticulum of *inscriptella* that is apparently absent from examples of *dilutella*. It is essential, in order to avoid false negatives, that the vesica is fully everted by careful use of a fine hypodermic syringe, or similar.

*Pempeliella sororiella* Zeller, 1839

We found this species at numerous sites in Serbia, Albania and Macedonia. Research undertaken by CWP on the distribution of *Pempeliella bulgarica* revealed, surprisingly, an absence of reports of *P. sororiella* from Serbia (*vide* map in Slamka & Plant, 2016). Apparently new to the Serbian fauna.

*Oxybia transversella* (Duponchel, 1836)

A fairly distinctive species when fresh, found throughout the southern part of Europe and eastwards across the Russian territories. We found it on three sites in Serbia and three more in Albania and expect that it will be added to the lists for most sites if they are properly surveyed. Apparently new to the Albanian fauna.

*Psorosa nucleolella* (Möschler, 1866)

Separation of this species from *P. mediterranea* is possible only through examination of the genitalia. Both taxa are present in Bulgaria, but whilst *P. nucleolella* was considered likely to be widespread there was only a single report of *P. mediterranea* (Plant, 2016). This latter species is prevalent in the western part of the Mediterranean Basin, but where the two meet is unclear and there is a high probability that they may overlap. Our single record of *P. nucleolella* in the present survey is from the coast, between Butrint & Saranda, near Ksamil (site 39) on 18 September 2016. Apparently new to the Albanian fauna.

*Acrobasis advenella* (Zincken, 1818)

A surprising omission from the Serbian fauna, until now where we report it from Pirot, above Kamenolom Kitka towards Crni Vrh at 870 metres altitude (site 5). Apparently new to the Serbian fauna.

*Acrobasis consociella* (Hübner, 1813)

Another species that is likely to prove to be widespread if properly looked for, since it is generally common across most of Europe. We found it at several sites in Serbia and Macedonia and at three localities in Albania (near Dhërmi on the coast, near Bistricë in the Delvina region and in the Vukopoles River Gorge between Ibrollara and Vale, Berat region. Apparently new to the Albanian fauna.

*Acrobasis fallouella* (Ragonot, 1871)

This species replaces *A. glaucella* Staudinger, 1859 in the east of Europe; males of the latter species are immediately recognised by the lack of a scale tuft at the base of the antenna (the “knot-horn”). This is present in *A. fallouella* males (Table 10). All existing records of *A. glaucella* are considered to be misidentifications of *A. fallouella* and the species *Acrobasis glaucella* is hereby formally deleted from the faunal lists for Serbia, Montenegro, Macedonia and Albania. *A. fallouella* is recorded by us at several sites in Serbia, Macedonia and Albania. Apparently new to the Serbian fauna. Apparently new to the Albanian fauna. Apparently new to the Macedonian fauna.

**Table 10.** Presence or absence of a “horn” of scales at the antennal base in European males of *Acrobasis* ZELLER, 1839.

Antennal “thorn” in males of species of <i>Acrobasis</i>		
Present	Absent	
<i>centunculella</i>	<i>advenella</i>	Present or probably present in the current study area
<i>consociella</i>	<i>dulcella</i>	
<i>fallouella</i>	<i>legatea</i>	
<i>Oblique</i>	<i>marmorea</i>	
<i>obtusella</i>	<i>sodalella</i>	
<i>repandana</i>	<i>suavella</i>	
<i>tumidana</i>	-	
<i>bithynella</i>	<i>getuliella</i>	Not yet found and probably
<i>porphyrella</i>	<i>glaucella</i>	not expected in the current study area

*Acrobasis legatea* (Haworth, 1811)

As with others of this genus, the lack of past reports is a probable consequence of under-recording. We found it in both Serbia and Albania – in the in the Korçë region of the latter

country, above Kloçë, near the radio mast, (site 28). Apparently new to the Albanian fauna.

*Acrobasis repandana* (Fabricius, 1798)

Another overlooked species, found by us on the Albanian coast at Palasë, near Dhërmi. Further survey is likely to show this species to be widespread and common in the region. Apparently new to the Albanian fauna.

*Episcythrastis tabidella* (Mann, 1864)

This somewhat nondescript, grey phycitine moth was found by us at two sites in Serbia, near Trgovište (site 8) and at Turski Grob (site 16), but is likely to be more or less widespread across the area under examination. It is a common member of the fauna in adjacent Bulgaria. Apparently new to the Serbian fauna.

*Eurhodope cirrigerella* (Zincken, 1818)

Listed as *Kyra cirrigerella* in many identification guides, this is a species of calcareous grassland habitats where the larval foodplants (*Knautia* and *Scabiosa*) are plentiful. Although both of the present records are from light traps, the adult insect can also be netted during the daytime. We found it near Babin Kal in the Bela Palanka district of Serbia (site 4) on 3 July 2014 and in the Pivska Planina Mountains of Montenegro at Trsa village, near Kulići, on 17 July 2014. Apparently new to the Serbian fauna.

*Euzopherodes charlottae* (Rebel, 1914)

This small moth is relatively distinctive, though has been confused with an extreme form of *E. vapidella* (Mann, 1857) in which the median fascia is atypically emphasised. In Europe, *E. charlottae* occurs in Austria, Hungary and throughout the Balkans, extending east to the Middle East. There are also a very few reports from the Mediterranean area of France. In the present survey we recorded it once only, at Turski Grob, above the Pčinja river valley (site 16) on 5 August 2016. Apparently new to the Serbian fauna.

*Phycitodes benticckella* Pierce, 1937

Leraut (2002) recognised this species, as *P. eliseannae*, as a cryptic sibling of *P. lacteella* (Rothschild, 1915), but later (Leraut, 2014) recognised it as synonymous with *P. benticckella*. Plant (2016) records a single example from Bulgaria, captured on 12 August 2011. The present survey records it near the Jabuka Pass at Savina Voda in Serbia (site 7) on 16 July 2014, but this is the only other Balkan record. New to the Serbian fauna.

*Ephestia woodiella* Richards & Thompson, 1932

Leraut (2002) revised the synonymy of this a widespread and common species, which is already reported throughout the area under review, but under a variety of synonyms including *E. parasitella* Staudinger, 1859 and *E. unicolorella* Staudinger, 1881. The true “*parasitella*” is found in Switzerland and Iberia as well as North Africa; the true “*unicorella*” is noted from Turkey and Syria. The European taxon is now regarded as “*woodiella*” and it is to this taxon which all material seen by us can be referred.

*Cadra furcatella* (Herrich-Schäffer, 1849)

This species extends from Portugal to Turkey, but is confined to the southern half of Europe. Under-recording in the region is doubtless the reason why it was considered to be absent from Serbia and Albania until now. We record it in Serbia at Suva Planina, above Bojanine Vode (site 2) on 2 July 2014 and in Albania in the Devolli Gorge, near Strelçë (site 24) on 5 June 2016. Apparently new to the Serbian fauna. Apparently new to the Albanian fauna.

#### PYRALINAE

*Stemmatophora brunnealis* (Treitschke, 1839)

A widespread and common species in most parts of Europe, particularly central and southern parts. We record it here from three sites in Serbia (sites, 16, 22 and 23) and one in Albania (site 36). Apparently new to the Albanian fauna.

*Stemmatophora honestalis* (Treitschke, 1829)

Central Europe and the Balkans are the main areas of population of this species in Europe and so absence from the existing Serbian list is surprising. We found it above Kamenolom Kitka towards Crni Vrh in the Pirot region on 4 July 2014 (site 5). Apparently new to the Serbian fauna.

#### ACKNOWLEDGMENTS

Permits for fieldwork in protected areas within Serbia were kindly provided by the Ministry of Environment, Mining and Spatial Planning, Republic of Serbia, No. 353-01-834/2017-17, dated 11 May 2017. SB and AN would like to thank Branko & Nikola Micevski (Republic of Macedonia, Skopje, Macedonian Entomological Society ENTOMAK) for collecting permits for the Republic of Macedonia and for financial support for two of their trips. All other aspects of this entire project were self-funded by the authors. Andrew Smith (Chelmsford, England) created the map at Figure 1 from initial artwork provided by CWP. Barry Goater (Chandlers Ford, England) read an early draft of this paper and made helpful comments which we have adopted.

#### REFERENCES

Beshkov, S. 1994. Migrant Lepidoptera species in Macedonia and Albania, 1993 (Lepidoptera). *Atalanta*, 25(3-4), pp. 461-468.

Beshkov, S. 1996. Migrant Lepidoptera species in Albania and Macedonia in 1995. *Atalanta*, 27(3/4), pp. 535-543.

Corley, M.F.V. 2015. *Lepidoptera of Continental Portugal*. Privately published, Corley. Oxford: Faringdon.

Friese, G. 1967. Ergebnisse der Albanien-Expedition 1961 des Deutschen Entomologischen Institutes. 61. Beitrag. Verzeichnis albanischer Fundorte. Beiträge zur Entomologie, 17(3-4), pp. 405-434.

Jakšić, P. 2016. Tentative checklist of Serbian microlepidoptera. *Ecologica Montenegrina*. Podgorica, 7, pp. 33-258.

Jakšić, P. 2016. Doprinos poznavanju faune noćnih leptira (Insecta: Lepidoptera) spomenika prirode „Zvezdarska šuma“ u Beogradu / A contribution to the knowledge of the moths fauna (Insecta: Lepidoptera) of the Zvezdara forest nature monument. *Zaštita prirode / Nature Conservation*, 66(2), pp. 35-40. Beograd, In Serbian, English summary.

Jovanović, G. 2010. Srbija, auto atlas 1: 200 000. Beograd.

Karsholt, O., & Razowski, J. 1996. *The Lepidoptera of Europe A Distributional Checklist*. Stenstrup: Apollo Books., pp. 1-380.

Leraut, P. 2001. Contribution à l'étude des Phycites Paléarctiques [Lepidoptera, Pyralidae, Phycitinae]. *Nouvelle Revue d'Entomologie*: Paris, 23(2), pp. 129-141.

Leraut, P. 2002. Contribution à l'étude des Phycites Paléarctiques [Lepidoptera, Pyralidae, Phycitinae]. *Nouvelle Revue d'Entomologie* (N. S.), Paris, 19(2), pp. 141-177.

Leraut, P. 2009. Zygaenids & pyralids. In *Moths of Europe*. NAP Editions. Volume 3.

Leraut, P. 2014. Pyralids 2. In *Moths of Europe*. NAP Editions. Volume 4.

Mihajlović, Lj., Zečević, M., & Jakšić, P. 1991. Pyralidae. In G. Nonveiller Ed., *Fauna Durmitora*. Titograd: Crnogorska akademija nauka i umjetnosti., pp. 243-275. In Serbian, English summary.

Misja, K. 1987. Të Dhëna Paraprake për Flatralusporët e Vegjël (Microlepidoptera) të Vendit Tonë / Some data about the Lepidoptera of our Country. *Bulletin I Shkëncave Natyrore*, 3, pp. 61-65.

Moucha, J. 1963. Eine Lepidopterenausbeute aus Albanien. *Entomologisches Nachrichtenblatt* (N.F.), 10(1), pp. 5-7. (2): 12-16.

Nuss, M. 2005. Scopariinae. In B. Goater, M. Nuss, & W. Speidel Eds., *Pyraloidea 1: Crambidae: Acentropinae, Evergestinae, Heliothelinae, Schoenobiinae, Scopariinae*. Microlepidoptera of Europe. Apollo Books. Volume 4.

Pastoralis, G. 2010. A checklist of microlepidoptera (Lepidoptera) occurring in the territory of Hungary (version 1. 4). *e-Acta Naturalia Pannonica*, 1(1), pp. 89-170.

Plant, C.W. 2016. An annotated, systematic, synonymic and distributional checklist of the Pyraloidea of Bulgaria. *Neue Entomologische Nachrichten*, 72, pp. 1-351.

Plant, C.W., & Slamka, F. 2016. Re-examination and revision of Zeller's original concept of *Phycita Metzneri* (Zeller, 1846) and description of *Phycita cryptica* sp. nov. (Lepidoptera, Pyraloidea, Pyralidae, Phycitinae). *Entomologist's Record and Journal of Variation*, 128, pp. 28-40.

Płóciennik, M., Pawlikiewicz, P., & Pabis, K. 2009. *Palpita vitrealis* (Rossi, 1794) and *Lygephila cracca* ([Denis & Schiffermüller], 1775) - Lepidoptera new for the fauna of Albania. *J. Ent. Res. Soc.*, 11(2), pp. 39-41.

Rákossy, L., Goia, M., & Kovács, Z. 2003. *Catolgul Lepidopterelor României - Verzeichnis der Schmetterlinge Rumäniens*. Cluj Napoca: Societatea Lepidopterologica Romana.

Rebel, H., & Zerny, H. 1931. 1934. *Die Lepidopterenfauna Albanien (mit Berücksichtigung der Nachbargebiete)*. Denkschriften / Akademie der Wissenschaften in Wien, Mathematisch-Naturwissenschaftliche. Wien: Hölder-Pichler-Tempsky, A.-G., pp. 1-21. Klasse 103: 37-161, text-figs 1-10, pl. 1, figs 1-21, 1 map.

- Slamka, F. 2006. Lepidoptera: Volume 1: Pyralinae, Galleriinae, Epipaschiinae. In *Pyraloidea of Europe*. Bratislava: Cathariinae & Odontiinae.
- Slamka, F. 2008. Lepidoptera. In *Pyraloidea of Europe*. Bratislava. Volume 2.
- Slamka, F. 2013. Lepidoptera: Pyraustinae and Spilomelinae. In *Pyraloidea of Europe*. Bratislava. Volume 3.
- Slamka, F., & Plant, C.W. 1839. 2016a. *Pempeliella bulgarica* sp. nov. a new species closely related to *Pempeliella sororiella* (Zeller, (Pyraloidea, Pyralidae, Phycitinae) and some new synonymies. *Entomologist's Record and Journal of Variation*, 128, pp. 99-111.
- Slamka, F., & Plant, C.W. 1833. 2016b. *Mecyna balcanica* sp. nov. a closely related species to *Mecyna flavalis* ([Denis & Schiffermüller], 1775) and *M. lutealis* (Duponchel, (Pyraloidea, Crambidae, Spilomelinae). *Entomologist's Record and Journal of Variation*, 128, pp. 137-145.
- Stojanović, D.V., & Radaković, N.Z. 2016. *Microlepidoptera* (Pyraloidea). In *Fauna Lepidoptera Nacionalnog Parka Đerdap*. Donji Milanovac - Novi Sad. In Serbian.
- Turner, J. 1938. Die Schmetterlinge der Ochrid-gegend in Mazedonien. *Mitteilungen aus den Königl. Naturwissenschaftlichen Instituten in Sofia*. *Bulletin des Institutions Royales d'Histoire Naturelle. A. Sofia – Bulgarie*, 14, pp. 9-35. 1938-1941.

# MONITORING BUTTERFLY BIODIVERSITY ON PRIME BUTTERFLY AREA Avala Mt. (Serbia) BY THE TRANSECT METHOD (Pollard Walks) IN THE YEAR 2017

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## ABSTRACT

An inventory of butterfly fauna was carried out in 2017 within the north-western part of the Prime Butterfly Area “Avala Mt.”. Butterfly fauna was surveyed with a transect method from March until October within three sectors with different land use and plant succession. We evaluated the sectors by butterfly species richness, abundance of individuals, species accumulation curves, Shannon index, ecological and zoogeographical characteristics. A total of 1147 individuals were recorded, representing 50 species. Three new species (*Pieris mannii* (Mayer, 1851), *Satyrrium acaciae* (Fabricius, 1787) and *Kirinia roxelana* (Cramer, 1777)) for the fauna of Avala Mt. were discovered that now counts 114 species. Abandoned dry grasslands in the process of secondary succession supported the highest number of species and individuals. Nevertheless, the observed butterfly species on Avala Mt. were almost equally characteristic for grasslands (38.4%) and forests (34.3%). In conclusion, butterfly biodiversity on Avala Mt. depends on habitat diversity, which is a combination of climax forest vegetation as well as formerly agricultural landscapes in different stages of secondary succession.

**Keywords:** Papilionoidea, Pollard Walks, Species Accumulation Curve, Secondary Succession, Serbia.

## INTRODUCTION

During the last 100 years, only four papers were published dealing with the butterfly fauna of Avala Mountain (Gušić, 1923; Gradojević, 1930-31; Moucha, 1966; Andus, 2008). A recent review paper with comprehensive data on 111 butterfly species of the Avala Mt. was published by Jakšić (2015).

Avala Mountain is situated 16 km south east from Belgrade, Serbia. The protected natural resource “Avala” (the landscape of outstanding features) is one of the Prime Butterfly Areas (PBA) in Serbia (Jakšić et al., 2008) with 6 species threatened on national or European level. It is located in the territory of the City of Belgrade. This low island-like mountain is made of serpentinite, flysch and loam deposits (Dimitrijević, 1931).

According to Fukarek & Jovanović (1986), natural potential vegetation of Avala is *Quercetum frainetto-ceris* s. lat. and *Querco-Carpinetum* s. lat. According to Bohn and Neuhausl (2000–2003) natural potential vegetation is *Pannonian–Danubian–Balkan lowland to submontane Balkan oak–bitter oak forests* (G19–G24).

Among 30 described habitat types (EUNIS habitat types), *Balkano–Anatolian thermophilous oak forests*, *Moesio–Danubian xerothermal oak forests* and *Sub–Pannonic steppes* (as fragments) were present as dominant types on the examined area.

The purpose of this study was to examine butterfly biodiversity of the Avala Mt. within areas with different land use and plant succession during one season with emphasis on species

richness, abundance of individuals, ecological and zoogeographical characteristics.

## MATERIALS AND METHODS

The investigated area on the Avala Mt., just above the village Beli Potok, was divided into three sectors A, B and C (Fig. 2). Originally, all sectors were covered by associations *Rusco aculeati-Carpinetum betuli* (Jovanović, 1979) and *Fraxino orni-Quercetum virgilianae* (Gajić, 1955). These associations were changed by human activities, mostly agriculture. Sector A was partly planted with *Robinia pseudoacacia* L. and sector C with Scots and black pines. Sector B was covered mostly by natural potential vegetation. In sector A, agriculture was abandoned 20 years ago and secondary succession was ongoing. In sector C, the land was arable, but agriculture was abandoned 3-4 years ago (Fig. 1).

The method of quantitative monitoring was a “Pollard Walk” transect counting method (Pollard, 1977). Routes of transects were laid out to sample representative habitats parallel to the slopes. Transect counts were conducted at a moderate walking pace in a time interval of approximately 14 days (first count on 21st March, last on 21st October 2017, Tab. 1). Counts were realized on mostly sunny days from 10:00 am to 13:00 pm. Only species and specimens observed within the five meter strip on each side of the route had been registered. Counts and determination of butterflies were done by the first author.

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**Figure 1.** Examined sectors on Avala Mt.: grassland in secondary succession – sector A (above), natural potential forest vegetation - B (middle), abandoned field in secondary succession - C (below) (All photos P. Jakšić).

The map of Avala Mt. was done by software packages QGIS and INKSCAPE (Fig. 2). Positions and coordinates, at which Lepidoptera were caught, were determined using a Garmin e-Trex 10 Vista GPS device. According to the length of the area, longitude and latitude coordinates were between: 41' 54.5" N; 31' 27.3" E (altitude 324 m) and 42' 00.0" N; 31' 03.0" E (altitude 267 m). Surface area was approximately 800 x 200 m.

The in situ photos of specimens were taken using Nikon Camera with AF-S Micro Nikkor Lens.



**Figure 2.** Study area map above Beli Potok village. Dotted line - represents a border of the examined area on the Avala Mt. Continuous line represents transect routes. A, B and C indicate transect sectors.

The climate of Avala Mt. is mild-continental with a great continental influence. There is a short period of semi drought during July and August. In January and February, average temperatures are below 0 °C and frost can appear from late October to early April (Anonymous, 2017). The period from May to September, with average temperatures over 15 °C, is characterized by full activity of Lepidoptera.

Taxonomic classification was done according to Nieukerken et al., (2011) and nomenclature according to Kudrna et al., (2015). Butterfly species were categorized according to habitat temperature preference (Mihut & Dincă, 2004), trophic specialisation of caterpillars (Stefanescu et al., 2011), habitat types (van Swaay et al., 2006) and zoogeographical characteristics or faunal elements (Kudrna et al., 2011).

Butterfly fauna was statistically evaluated for the three sectors by species richness and abundance of individuals per sampling unit, number of singletons and Shannon index. Species accumulation curves were drawn with the EstimateS program (Colwell, 2013). The numbers of species and individuals were correlated to temperature with a nonlinear regression model Lowess (Statistica, 2004) and (Karadžić & Marinković, 2009).

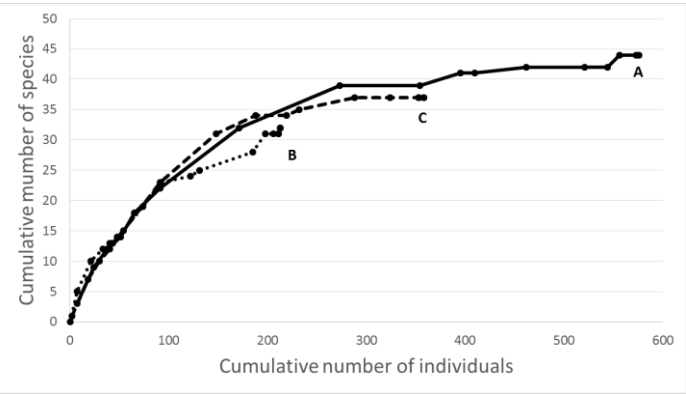
Fieldwork in a protected area was done on the basis of permits provided by the Ministry of Environment, Mining and Spatial Planning, Republic of Serbia, No. 353-01-834/2017-17, dated from 11. 05. 2017.

**Table 1.** Overview of realized transects showing the number of species identified by sectors and the total number of species observed.

Date of sampling units	Transect sectors and number of species			The total number of species
	A	B	C	
21 March 2017	3	0	1	3
2 April 2017	9	4	7	10
11 April 2017	7	7	5	13
26 April 2017	6	7	7	14
14 May 2017	8	6	4	13
28 May 2017	7	2	3	9
11 June 2017	20	10	10	25
23 June 2017	25	12	20	29
7 July 2017	16	9	8	23
17 July 2017	13	5	11	17
2 August 2017	6	12	5	18
15 August 2017	11	9	12	20
29 August 2017	9	4	5	12
11 September 2017	7	5	4	8
29 September 2017	6	0	0	6
12 October 2017	5	2	2	5
21 October 2017	2	0	0	2
$\Sigma$ total species	44	32	37	50

## RESULTS

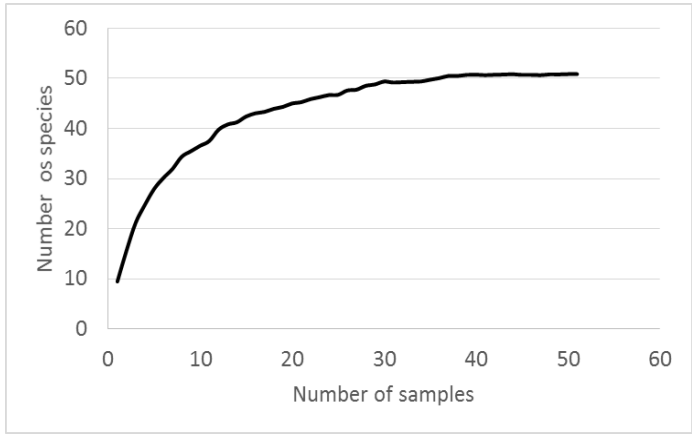
A total of 1147 individuals were recorded on Avala Mt., representing 50 species (Tab. 2, Fig. 7). The highest number (44) of butterfly species was recorded in sector A and the lowest (32) in sector B (Tab. 1). The sector A had the highest average number of species, abundance of individuals, with the lowest proportion of singletons, per sampling unit (Tab. 3). In contrast, sector B had the lowest average number of species and abundance of individuals per unit but a similar proportion of singletons as the sector C.



**Figure 3.** Cumulative number of butterfly species plotted against the cumulative number of individuals observed in each sector.

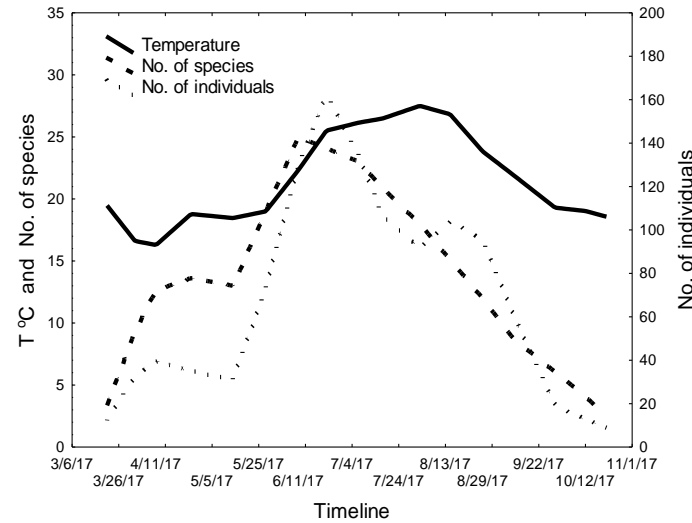
According to species accumulation curves in relation to the number of individuals (Fig. 3), sector B had the largest potential

for collection of undetected species. The species accumulation curve in sector A, on the other hand, has reached an asymptote. A sample-biased species accumulation curve (Fig. 4) implied that the number of species stabilized at about 30 sampling units and that enough sampling effort had been invested to estimate the butterfly species diversity at selected sectors in one season.



**Figure 4.** Species accumulation curve of abundance-based estimator Chao1 against the sampling effort for the complete dataset (51 samples).

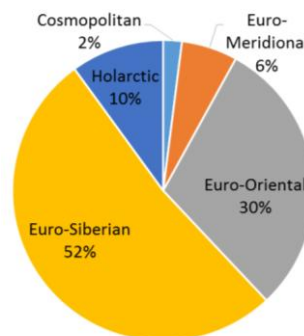
The highest butterfly species richness (29) and abundance of individuals (177) was recorded on 23<sup>rd</sup> June 2017 (Tab. 1, Fig. 5). Butterfly phenology was highly correlated to temperature. Correlation between the temperature on sampling days and the number of species was 0.56 ( $p=0.020$ ) and the number of individuals 0.65 ( $p=0.004$ ). In August, when temperatures peaked, the species number and abundance of individuals dropped (Fig. 5).



**Figure 5.** Day temperatures, predicted values of the observed species richness and abundance of individuals on the Avala Mt.

On Avala Mt., trophic specialisations of caterpillars of the observed species were as follows: monophagous (36%), oligophagous (42%), polyphagous (22%) (Tab. 2). Most of the butterfly species were mesophilic (76.5%) and mesothermophilic (13.7%) (Tab. 2). Nevertheless, a thermophilic species *Iphiclides podalirius* was one of the most abundant species at sector A. According to habitat types, the observed species were characteristic mostly for grasslands (38.4%) and forests (34.3%).

Two forest species (*Neptis sappho* and *Pararge aegeria*, Tab. 2) were most abundant in sector B. 52% of the observed species belonged to the Euro-Siberian faunal group, followed by the Euro-Oriental (30%) and Holarctic (10%) group (Fig. 6).



**Figure 6.** Zoogeographic composition of 50 examined species on Avala Mt. transect area.

**Table 2.** Abundance of observed species of butterflies during 2017 for three sectors on Avala Mt., with butterfly biological characteristics. **i)** Species abundance (Quantitative distribution in sectors A, B and C; total number of specimens). **ii)** Ecological characteristics: TS = Trophic specialisation of caterpillars – monophagous (m), oligophagous (o), polyphagous (p); T = Preferences according to habitats temperature – Xerothermophile (Xt), Thermophile (T), Moesothermophile (Mt), Mesophile (M); H = Characteristics according to habitat types: HS – Heath and Scrub, Gr – Grassland, Fo – Forest, We – Wetland, Un – Unvegetable, Ag – Agriculture, Ur – Urban. **iii)** FE = Zoogeographical characteristics – “faunal elements”: Cosmopolitan (4), Euro-Meridional (5), Euro-Oriental (6), Euro-Siberian (7), Holarctic (8).

Nb.	Scientific names	Species abundance				Ecological characteristics		FE
		A	B	C	TOTAL (A+B+C)	TS/T	H	
	<b>LEPIDOPTERA Linnaeus, 1758</b>							
	<b>PAPILIONOIDEA Latreille, 1802</b>							
	<b>Fam. Papilionidae Latreille, 1802</b>							
1	<i>Iphiclides podalirius</i> (Linnaeus, 1758)	50	0	18	68	o/T	HS/Gr/Ag	6
2	<i>Papilio machaon</i> Linnaeus, 1758	3	1	8	12	p/M	Gr/Ur	7
	<b>Fam. Hesperidae Latreille, 1809</b>							
3	<i>Erynnis tages</i> (Linnaeus, 1758)	10	0	6	16	o/M	Gr	7
4	<i>Pyrgus malvae</i> (Linnaeus, 1758)	19	4	15	38	p/M	Gr	7
5	<i>Thymelicus lineola</i> (Ochsenheimer, 1806)	24	4	13	41	o/M	Gr/Ur	8
6	<i>Thymelicus sylvestris</i> (Poda, 1761)	6	1	3	10	o/M	Gr/Fo	6
7	<i>Ochlodes sylvanus</i> (Esper, 1777)	14	3	7	24	o/M	Gr/Fo	7
	<b>Fam. Pieridae Swainson, 1820</b>							
8	<i>Leptidea sinapis</i> (Linnaeus, 1758) complex	11	8	5	24	o/M	Gr/Fo	7
9	<i>Anthocharis cardamines</i> (Linnaeus, 1758)	6	4	4	14	o/M	Gr/Fo	7
10	<i>Pieris brassicae</i> (Linnaeus, 1758)	16	13	3	32	p/M	Gr/Ag/Ur	7
11	<i>Pieris mannii</i> (Mayer, 1851)	0	4	4	8	o/Mt	HS/Fo	6
12	<i>Pieris rapae</i> (Linnaeus, 1758)	40	16	19	75	p/M	Ag/Ur	8
13	<i>Pieris napi</i> (Linnaeus, 1758)	26	7	5	38	o/M	Gr/Fo/Ag	7
14	<i>Colias croceus</i> (Fourcroy, 1785)	1	0	1	2	o/Mt	Gr/Ag	6
15	<i>Gonepteryx rhamni</i> (Linnaeus, 1758)	1	0	1	2	m/M	Fo/HS	7
	<b>Fam. Nymphalidae Rafinesque, 1815</b>							
16	<i>Issoria lathonia</i> (Linnaeus, 1758)	0	3	1	4	m/M	Gr/Ag/HS	7
17	<i>Brenthis daphne</i> (Bergsträsser, 1780)	7	0	0	7	m/M	Gr/Fo	7
18	<i>Brenthis hecate</i> (Denis & Schiffermüller, 1775)	3	0	0	3	m/M	Gr/Fo	7
19	<i>Argynnis paphia</i> (Linnaeus, 1758)	20	13	27	60	m/M	Fo	7
20	<i>Argynnis aglaja</i> (Linnaeus, 1758)	1	0	0	1	m/M	Gr/Fo	7
21	<i>Araschnia levana</i> (Linnaeus, 1758)	1	9	0	10	m/M	Fo/Gr	7
22	<i>Vanessa atalanta</i> (Linnaeus, 1758)	7	1	0	8	o/M	Gr/Fo	8
23	<i>Vanessa cardui</i> (Linnaeus, 1758)	1	0	0	1	p/M	Gr/Fo/Ur	4
24	<i>Aglais io</i> (Linnaeus, 1758)	5	0	8	13	m/M	Gr/Fo/Ur	7

Nb.	Scientific names	Species abundance				Ecological characteristics		FE
		A	B	C	TOTAL (A+B+C)	TS/T	H	
25	<i>Aglais urticae</i> (Linnaeus, 1758)	2	0	0	2	m/M	Gr/Fo/Ur	7
26	<i>Polygonia c-album</i> (Linnaeus, 1758)	2	10	1	13	p/M	Fo/Ag	7
27	<i>Melitaea athalia</i> (Rottemburg, 1775)	26	0	14	40	o/M	Gr/Fo	5
28	<i>Limnitis reducta</i> (Staudinger, 1901)	0	1	0	1	m/Xt	Fo/HS	6
29	<i>Neptis sappho</i> (Pallas, 1771)	2	30	0	32	m/M	Fo	7
30	<i>Kirinia roxelana</i> (Cramer, 1777)	6	0	0	6	p/Xt	Fo/HS	6
31	<i>Pararge aegeria</i> (Linnaeus, 1758)	0	22	0	22	o/M	Fo	6
32	<i>Lasiommata megera</i> (Linnaeus, 1767)	15	7	1	23	o/M	Gr/Fo	6
33	<i>Coenonympha arcania</i> (Linnaeus, 1760)	2	3	5	10	o/M	Gr/Fo	5
34	<i>Coenonympha pamphilus</i> (Linnaeus, 1758)	18	1	19	38	o/M	Gr	6
35	<i>Maniola jurtina</i> (Linnaeus, 1758)	52	23	46	121	o/M	Gr/Fo/Ag	6
36	<i>Melanargia galathea</i> (Linnaeus, 1758)	25	1	30	56	p/M	Gr/Fo	6
37	<i>Brintesia circe</i> (Fabricius, 1775)	10	1	11	22	o/Mt	Fo/Gr	6
38	<i>Minois dryas</i> (Scopoli, 1763)	2	0	3	5	o/Mt	Gr/Fo	7
	<b>Fam. Riodinidae Grote, 1895</b>							
39	<i>Hamearis lucina</i> (Linnaeus, 1758)	0	3	0	3	m/M	Gr/Fo	5
	<b>Fam. Lycaenidae Leach, 1815</b>							
40	<i>Cupido argiades</i> (Pallas, 1771)	10	6	7	23	o/M	Gr/Fo	8
41	<i>Celastrina argiolus</i> (Linnaeus, 1758)	1	9	2	12	p/M	Fo/Ag	7
42	<i>Plebeius argus</i> (Linnaeus, 1758)	23	0	8	31	p/M	HS/Gr/Fo	7
43	<i>Plebeius idas</i> (Linnaeus, 1760)	6	0	2	8	p/M	Gr/Fo	8
44	<i>Aricia agestis</i> (Denis & Schiffermüller, 1775)	7	0	3	10	m/M	Gr	7
45	<i>Polyommatus icarus</i> (Rottemburg, 1775)	73	4	50	127	o/M	Gr/Ur	7
46	<i>Lycaena dispar</i> (Haworth, 1802)	1	0	1	2	m/Hg	Gr/We	7
47	<i>Lycaena tityrus</i> (Poda, 1761)	19	0	5	24	m/M	Gr	7
48	<i>Lycaena thersamon</i> (Esper, 1784)	1	0	1	2	m/Mt	Gr/Ur	6
49	<i>Satyrrium acaciae</i> (Fabricius, 1787)	1	0	1	2	m/Mt	HS	6
50	<i>Favonius quercus</i> (Linnaeus, 1758)	0	1	0	1	m/Mt-Xt	Fo	6
	<b>Total</b>	<b>576</b>	<b>213</b>	<b>358</b>	<b>1147</b>			

**Table 3.** Descriptive statistics of species richness and abundance of individuals per sampling unit; Mean±Standard deviation, Median in the three sectors surveyed (A, B, C), with numbers of singletons, Shannon diversity index and the most common species.

Sector	A	B	C	Total
Species Average ± StD	9.4 ± 6.1	5.5 ± 4.0	6.1 ± 5.1	13.4 ± 7.8
Median	7	5	5	13
Abundance Average ± StD	33.9 ± 30.1	12.5 ± 14.2	21.1 ± 18.9	67.7 ± 54.6
Median	17	9	13	40
Singleton / percent of total species	9 / 8.0%	8 / 20.5%	8 / 21.6%	4 / 8.0%
Shannon index H	3.22	2.96	3.06	3.36
Most common species	<i>Polyommatus icarus</i> , <i>Maniola jurtina</i> , <i>Iphiclides podalirius</i>	<i>Neptis sappho</i> , <i>Maniola jurtina</i> , <i>Pararge aegeria</i>	<i>Polyommatus icarus</i> , <i>Melanargia galathea</i> , <i>Maniola jurtina</i> , <i>Argynnis paphia</i>	<i>Polyommatus icarus</i> , <i>Maniola jurtina</i> , <i>Pieris rapae</i> , <i>Iphiclides podalirius</i> , <i>Argynnis paphia</i>



**Figure 7.** Some of the observed species: a) *Pieris mannii* (Mayer, 1851), new species for Avala Mt.; b) *Cupido argiades* (Pallas, 1771), species found in all three sectors; c & d) *Iphiclide podalirius* Linnaeus, 1758, a female depositing eggs on *Prunus spinosa*, deposited egg; e). *Polyommatus icarus* Rottemburg, 1775, a complete gynandromorph; f). *Polygonia c-album* Linnaeus, 1758, third generation (All photos P. Jakšić).

## DISCUSSION

A total of 50 species of butterflies were recorded in the three sectors on Avala Mt. in 2017, representing 45% of the total number of butterfly species previously known to the Avala Mt. (111 species; Jakšić, 2015).

Three new species for the fauna of Avala Mt. were found: *Pieris mannii* (Mayer, 1851), *Satyrium acaciae* (Fabricius, 1787) and *Kirinia roxelana* (Cramer, 1777).

Euro-Siberian and Euro-Oriental butterfly species represented 82% of the basic faunal features of the Avala Mt. Most of the observed butterfly species were mesophilic (76.5%).

Sector A – dry grassland with ongoing secondary succession after 20 years of agricultural abandonment – was the area with the highest butterfly diversity and abundance. Among the three sectors, the highest Shannon's index for sector A suggested not only the highest species diversity but also evenness of distribution with the lowest number of singletons.

Sector B covered with natural forest vegetation supported the lowest number of species and individuals.

Sector A and C, which are both former agricultural landscapes, proved important for the biodiversity of butterflies on the Avala Mt. Abandoned dry grasslands with ongoing secondary succession have the highest plant biodiversity (Paušić et al., 2017). According to van Swaay & Warren (2006) abandonment of traditional land use is one of the major threats to Prime Butterfly Areas (PBAs). Grassland abandonment has a strong effect on butterfly diversity, especially on grassland specialists which are substituted by common butterflies, less important for conservation purposes (Stefanescu et al., 2009). Our results show that the observed butterfly species on Avala Mt., which is a PBA area, were almost equally characteristic for grasslands (38.4%) and forests (34.3%). This means that butterfly biodiversity on Avala Mt. depends on habitat diversity, which is a combination of climax forest vegetation as well as formerly agricultural landscapes in different stages of succession.

## ACKNOWLEDGMENTS

We are grateful to Dr Ana Savić, University of Niš, Faculty of Sciences and Miloš Jović, Natural History Museum, Belgrade for their valuable assistance and useful advices.

## REFERENCES

Anđus, Lj. 2008. Butterflies (Lepidoptera: Hesperioidea & Papilionoidea) from the collection of the Natural History Museum in Belgrade. Belgrade: Natural History Museum. Special issue 40: 1–94, 6 figs. [In English, Serbian summary].

Anonymous, 2017. Weather in Serbia. Daily & Monthly climate characteristics for the territory of Serbia. Republic: Hydrometeorological Service of Serbia. Web: <http://www.hidmet.gov.rs/eng/download/index.php>.

Bohn, U., Neuhausl, R., Gollub, G., Hettwer, C., Neuhauslová, Z., Raus, Th., Schlüter, H. & Weber, H. 2000. Karte der natürlichen Vegetation Europas Maßstab 1: 2500000 / Map of the Natural Vegetation of Europe. Scale 1:2500000. Münster: Landwirtschaftsverlag. 200/2003.

Colwell, R.K. 2013. EstimateS: Statistical estimation of species richness and shared species from samples. Version 9. User's Guide and application at <http://purl.oclc.org/estimates>.

Dimitrijević, B. 1931. Avala, petrografsko-mineraloška studija, sa geološkom kartom u razmeri 1: 50. 000. In Prirodnjački i matematički spisi. Beograd: Srpska kraljevska akademija. Posebna izdanja, Knjiga LXXXV, Knjiga 23: 1–150. [In Serbian].

Fukarek, P., Jovanović, B., & eds., 1986. Karta Prirodne Potencijalne Vegetacije SFR Jugoslavije, 1: 1 000 000 / Natural potential vegetation of the SFR Yugoslavia, 1: 1 000 000. In B. Jovanović, R. Jovanović, & M. Zupančič Eds., Prirodna Potencijalna Vegetacija Jugoslavije. Ljubljana.

Gradojević, M. 1930. Leptirovi Srbije I - Diurna. (Les papillons de Serbie I. Diurna). Glasnik Jugoslovenskog entomološkog društva, Beograd, 1930-31; V-VI (1-2): 133–158, [In Serbian].

Gušić, B. 1923. Ein Beitrag zur Rophaloceren-Fauna Serbiens. Verhandlungen der K. K. Zoologisch-botanischen Gesellschaft in Wien, 72: (12)-(13).

Jaksic, P. 2015. Aspects of butterfly zoogeography of some Pannonian island mountains. Matica Srpska Journal for Natural Sciences, 128, pp. 7-19. doi:10.2298/zmspn1528007j

Jakšić, P., Verovnik, R., & Dodok, I. 2008. Overview of Prime Butterfly Areas in Serbia / Pregled Odabranih područja za dnevne leptire u Srbiji. In P. Jakšić Ed., Odabrana područja za dnevne leptire: Put za ostvarenje zaštite prirode u Srbiji / Prime Butterfly Areas in Serbia: A tool for nature conservation in Serbia. Beograd: Habiprot Ed.. 43–203. [In Serbian and English].

Karadžić, B., & Marinković, S. 2009. Kvantitativna ekologija. Beograd: Institut za biološka istraživanja "Siniša Stanković". pp. 489 [In Serbian]

Kudrna, O., Pennerstorfer, J., & Lux, K. 2015. Distribution atlas of European butterflies and skippers. Schwanfeld, Germany: istribution atlas of European butterflies and skippers. Wissenschaftlicher Verlag Peks i. K.

Mihut, S., & Dincă, V. 2004. Fluturii de zi din România. Biodiv. Cluj., 1: 18-31. [In Romanian]

Moucha, J. 1966. Zur Kenntnis der Schmetterlingsfauna Jugoslawiens (Lepidoptera). Entomologische Nachrichten, 10(4): 49–53.

Nieukerken, V., & et al., 2011. Order Lepidoptera Linnaeus, 1758. In: Zhang, Z. Q. (Ed.) Animal biodiversity: An outline of higher-level classification and survey of taxonomic richness. Zootaxa, 3148(1), pp. 211-221.

Paušić, I., Ivajnsić, D., Kaligarić, M., & Pipenbaher, N. 2017. Relation between plant species diversity and landscape

- variables in Central-European dry grassland fragments and their successional derivatives. *Acta Botanica Croatica*, 76(2). doi:10.1515/botcro-2017-0001
- Pollard, E. 1977. A method for assessing changes in the abundance of butterflies. *Biological Conservation*, 12(2), pp. 115-134. doi:10.1016/0006-3207(77)90065-9
- Statistica. 2004. Statistica 7.0. Stat Soft Inc.
- Stefanescu, C., Carnicer, J., & Peñuelas, J. 2010. Determinants of species richness in generalist and specialist Mediterranean butterflies: the negative synergistic forces of climate and habitat change. *Ecography*, 34(3), pp. 353-363. doi:10.1111/j.1600-0587.2010.06264.x
- Stefanescu, C., Peñuelas, J., & Filella, I. 2009. Rapid changes in butterfly communities following the abandonment of grasslands: a case study. *Insect Conservation and Diversity*, 2(4), pp. 261-269. doi:10.1111/j.1752-4598.2009.00063.x
- van Swaay, C.A.M.A., & Warren, M.S. 2006. Prime Butterfly Areas of Europe: An Initial Selection of Priority Sites for Conservation. *Journal of Insect Conservation*, 10(1), pp. 5-11. doi:10.1007/s10841-005-7548-1
- van Swaay, C., Warren, M., & Loïs, G. 2006. Biotope Use and Trends of European Butterflies. *Journal of Insect Conservation*, 10(2), pp. 189-209. doi:10.1007/s10841-006-6293-4

# PREPARATION OF SILVER AND COPPER NANOPARTICLES IN PRESENCE OF ASCORBIC ACID AND INVESTIGATION OF THEIR ANTIBACTERIAL ACTIVITY

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## ABSTRACT

In this study, we present a synthesis of silver and copper nanoparticles (NPs) using ascorbic acid as stabilizing and sodium borohydride as reducing agents, respectively. Four colloidal dispersions were obtained, two of them additionally stabilized by gelatin. They were characterized by UV-Vis, AFM, DLS and zeta potential measurements. The size of both silver and copper NPs, determined by AFM measurements, was 10 nm before, and 15 nm after stabilization with gelatin. Antibacterial activity of synthesized NPs was tested using series of gram positive and gram negative bacteria. It was found that Ag and Cu NPs showed antibacterial activity in all cases.

**Keywords:** Silver nanoparticles, copper nanoparticles, antibacterial activity.

## INTRODUCTION

Metal nanoparticles (NPs) are the subject of interest for many researchers, among all because of their unique optical properties (Mulvaney, 1996; Daniel & Astruc, 2004; Kelly et al., 2003; Sosa et al., 2003.; Austin et al., 2014). Because of their electronic properties and large surface area, they possess different physical and chemical properties in comparison to bulk materials. It is well known that metal NPs, due to the excitation of localized surface plasmon resonance, possess unique optical properties in the visible range (Mulvaney, 1996). They have found applications in many different areas such as catalysis (Yacamán et al., 2001; Zhang et al., 2013), electronics (Li et al., 2005), optical sensing (Wang et al., 2012; Li et al., 2015) and biosensing (Walcarius et al., 2013). Numerous publications for synthesis of silver (Maria et al., 2013; Dong et al., 2009; Hu et al., 2004; Zhang et al., 2011; Mahmoud et al., 2012; Qin et al., 2010; Chekin & Ghasemi, 2014) and copper (Salavati-Niasari & Davar, 2009; Youngil et al., 2008; Zhu et al., 2005; Ramyadevi et al., 2012; Zain et al., 2014; Valodkar et al., 2011) colloids have been reported, and the most frequently used method is a chemical reduction of their ions, whereas various reducing and stabilizing agents were used. So far it has been reported the use of ascorbic acid as a reducing agent for metal NPs synthesis by using microwave irradiation (Qin et al., 2010; Chekin and Ghasemi, 2014).

Silver NPs are of interest because of their antibacterial activity and localized surface plasmon resonance properties (Song et al., 2013; Panáček et al., 2006; Amendola et al., 2010;

Mogensen & Kneipp, 2014). Developing the route of synthesis enabled the control of desired size, shape, and capping agents.

Copper NPs are attractive because of the high natural abundance of Cu and low cost, also there are multiple ways of preparing Cu-based nanomaterials. The interest for Cu NPs is due to their possible electrocatalytic and antimicrobial application as well as in biosensing (Zain et al., 2014; Valodkar et al., 2011; Ehsani et al., 2014; Lu et al., 2012; Wang et al., 2004).

In this study, we report the preparation of silver and copper NPs in aqueous solutions by chemical reduction method, and their antibacterial activity. In order to obtain the small size particles, we used sodium borohydride as a reducing agent and ascorbic acid as a stabilizing agent. Since ascorbic acid is not a strong stabilizer agent, the colloidal dispersion was additionally stabilized by gelatin. UV-Vis spectroscopy and zeta potential measurements were used for characterization of Ag and Cu colloids, while microscopic method Atomic Force Microscopy (AFM) and optical method Dynamic Light Scattering (DLS) were used in particle size characterization (Chicea, 2014; Klapetek et al., 2011; Hoo et al., 2008).

Lately, the interest in the application of NPs as antimicrobial agents increased because of the growing bacterial resistance to antibiotics (Zain et al., 2014; Valodkar et al., 2011). Synthesized silver and copper NPs were used to examine their antibacterial activity against gram-negative bacteria *Acinetobacter*, *Escherichia Coli*, *Pseudomonas aeruginosa* and *Klebsiella pneumoniae*, and against gram-positive bacteria *Enterococcus faecium*.

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## EXPERIMENTAL

### Materials

Silver nitrate ( $\text{AgNO}_3$ ), Copper (II) nitrate ( $\text{Cu}(\text{NO}_3)_2$ ), ascorbic acid ( $\text{C}_6\text{H}_8\text{O}_6$ ), sodium borohydride ( $\text{NaBH}_4$ ) and gelatin from bovine skin were of analytical grade and purchased from Aldrich or Merck. Purified Millipore Mili-Q water with a resistivity of  $18 \text{ M}\Omega$  was used in all cases.

### Synthesis

#### Synthesis of silver nanoparticles

For the synthesis of silver nanoparticles, silver nitrate was used as a precursor and ascorbic acid as a stabilizing agent. 100 ml of  $10^{-4} \text{ M}$   $\text{AgNO}_3$  was containing  $2.5 \times 10^{-5} \text{ M}$  of ascorbic acid, and then 10 mg of  $\text{NaBH}_4$  was added at room temperature. The transparent colorless solution becomes pale yellow indicating the formation of silver nanoparticles. The colloidal dispersion prepared in this way remains stable for few hours at room temperature and up to 24 h in the fridge at  $5^\circ\text{C}$ . In order to keep the silver colloidal dispersion stable for a longer period of time, 1 ml of 1 % gelatin was added. That way dispersion remains stable for few months.

#### Synthesis of copper nanoparticles

The copper colloidal dispersion was prepared using copper (II) nitrate as a precursor and ascorbic acid as a stabilising agent. In 100 ml of a solution containing  $5 \times 10^{-4} \text{ M}$   $\text{Cu}(\text{NO}_3)_2$  and  $1.25 \times 10^{-4} \text{ M}$  ascorbic acid, 50 mg of  $\text{NaBH}_4$  was added at room temperature. The Color became brownish-red, indicating the formation of copper nanoparticles, and the dispersion remained stable 4-5 h at room temperature. By the addition of 1 % gelatin, the copper colloidal dispersion can be stabilized up to 48h.

### Microbiological method for determining the antibacterial activity of colloidal dispersions

Antibacterial activity of the silver and copper colloids were examined with both gram-positive and gram-negative bacteria as per the below-mentioned procedure. The bacterial strains were isolated from patients and streaked onto Tryptone Soy Agar and incubated overnight at  $37^\circ\text{C}$ . That way prepared bacterial suspension was applied on the surface of a nutrient agar plate. Then, colloidal dispersion of silver and copper was injected, separately, along with control and incubated at  $37^\circ\text{C}$  for 24 h.

### Apparatus

Absorption spectra of colloidal dispersions were measured by Perkin Elmer Lambda 35 UV – Vis spectrophotometer using the quartz cuvette with 1cm path length.

Particle size determination by DLS and zeta potential (ZP), mobility and conductivity measurements by laser Doppler electrophoresis (LDE) were performed using a Zeta-sizer Nano ZS with 633 nm He-Ne laser (Malvern Instruments, UK). Data

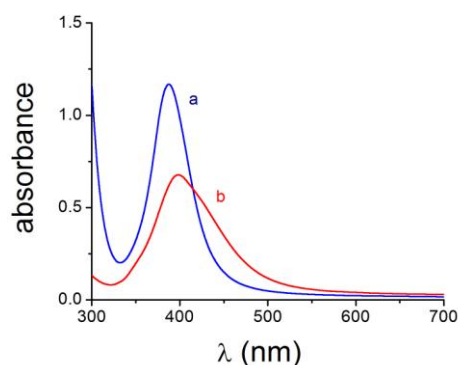
were analyzed by the Zetasizer Software Version 6.20 (Malvern Instruments, UK).

Atomic Force Microscopy (AFM) measurements were performed in air (at room temperature), providing spatially resolved chemical information of the sample along with its surface topography at the same place. The experiments were performed using commercial NTegra Spectra system from NT-MDT.

## RESULTS

### Absorption spectra

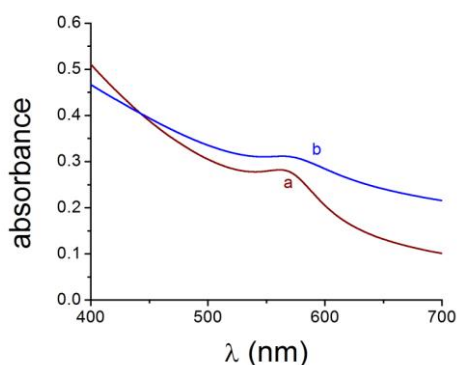
The absorption spectra of Ag colloidal dispersion containing  $1 \times 10^{-4} \text{ M}$  Ag ions in the absence and presence of 1 % gelatin are presented in Fig. 1. As can be seen from Fig. 1, silver colloidal dispersion has intense absorption band, due to the surface plasmon resonance, with a maximum at 387 nm. The shape of the absorption band suggests that Ag NPs are spherical (Laban et al., 2013; Laban et al., 2014; Laban et al., 2015; Laban et al., 2016). The presence of gelatin (Fig. 1 b) induced the broadening of the absorption band and red shifting of the maximum for 10 nm (absorption maximum at 397 nm), due to the change of surrounding media as well as a possible agglomeration of the certain number of NPs (Laban et al., 2014).



**Figure 1.** Absorption spectra of silver colloid ( $1 \times 10^{-4} \text{ M}$  Ag) before (a) and after stabilization with 1 % gelatin (b).

The obtained results are in accordance with the literature data (Laban et al., 2014), which show that small spherical Ag NPs possess an intense surface plasmon resonance peak. It is well known that due to collective oscillations of the conduction electrons, known as surface plasmon resonance, metal NPs possess characteristic band in the absorption spectra (Wiley et al., 2006). Position, width, and the number of the absorption bands depend on the nature of the metal, as well as the size and shape of the metal NPs (Liz-Marzán, 2006). Thus, spherical NPs possess one absorption band, while anisotropic NPs possess more than one band in the absorption spectrum (Wiley et al., 2006; Liz-Marzán, 2006).

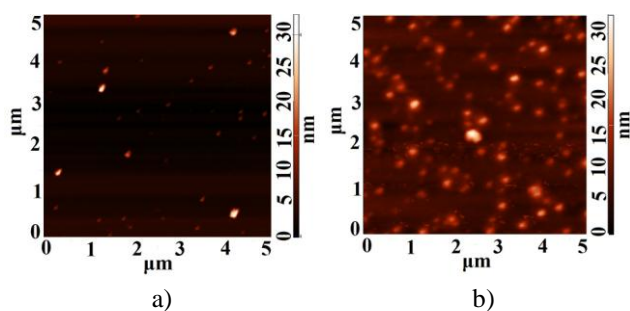
The absorption spectra of Cu NPs are given in Fig. 2. Copper colloid has absorption band with a maximum at 565 nm, which is in accordance with the previously reported data for spherical copper NPs (Valodkar et al., 2011). However, the presence of gelatin does not have a significant influence on the absorption spectra of Cu NPs.



**Figure 2.** Absorption spectra of copper colloid ( $4 \times 10^{-4}$  M Cu) before (a) and after stabilization with 1 % gelatin (b).

#### AFM measurements

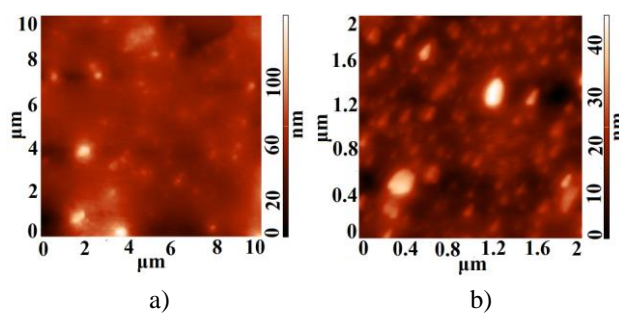
AFM topography obtained for Ag colloidal dispersion in the absence and presence of gelatin is given in Fig. 3. The samples were prepared by depositing a drop of colloidal solutions on a freshly cleaved mica substrate and left to dry for 1 h.



**Figure 3.** AFM topography of silver nanoparticles without (a) and in presence of gelatin (b).

From AFM topography it can be seen that the particles are of nanometer-size and nearly spherical in shape. The diameter of Ag NPs was determined by measuring the heights of particles above the substrate. The obtained values were  $10 \pm 2$  nm for particles from a colloidal dispersion prepared in the presence of ascorbic acid, and  $15 \pm 5$  nm for particles from a colloidal dispersion which is additionally stabilized with gelatin. Higher values in size of Ag NPs in presence of gelatin is due to additional shell formed by adsorbed gelatin molecules on the surface of NPs.

AFM topography for Cu colloidal dispersion is given in Fig. 4. The size of Cu NPs was determined in the same way as for Ag NPs, and the obtained values were  $10 \pm 3$  nm for particles which are not stabilized with gelatin, and  $15 \pm 3$  nm for particles additionally stabilized by gelatin. It can be seen that Cu NPs are nearly spherical, as well as Ag NPs, and also have higher values in size in presence of gelatin.



**Figure 4.** AFM topography of copper nanoparticles without (a) and in presence of gelatin (b).

#### DLS and zeta potential measurements

The size of nanoparticles measured by DLS method represents the hydrodynamic diameter of the nanoparticle (Tomaszewska et al., 2013) and the thickness of its electrical double layer has a significant effect on the measured size (Tomaszewska et al., 2013). The obtained values for mean particle size (z-average- $d_{av}$ ) of Ag NPs and Cu NPs before and after gelatin stabilization, together with data obtained from zeta potential, conductivity and mobility measurements are given in Table 1.

**Table 1.** Mean particle size ( $d_{av}$ ), zeta potential, conductivity and electrophoretic mobility for Ag and Cu colloids in the absence and presence of gelatin.

	PdI	$d_{av}$ (nm)	Zeta potential (mV)	Conductivity ( $\mu\text{S cm}^{-1}$ )	Mobility
Ag NPs	0.480	$56.16 \pm 1.9$	$-29.5 \pm 6.1$	$0.087 \pm 0.001$	$-2.32 \pm 0.47$
Ag NPs-g	0.358	$121.5 \pm 3.0$	$-21.7 \pm 2.9$	$0.090 \pm 0.001$	$-1.70 \pm 0.22$
Cu NPs	0.856	$49.97 \pm 3.6$	$-45.0 \pm 1.8$	$0.812 \pm 0.026$	$-3.53 \pm 0.14$
Cu NPs-g	0.767	$81.67 \pm 3.2$	$-43.5 \pm 3.4$	$0.981 \pm 0.028$	$-3.41 \pm 0.27$

Compared to AFM measurements, the values of  $d_{av}$  obtained by DLS measurements were higher and included the added solvent and stabilizer moving with the particle. Besides, the addition of gelatin to the colloid dispersions increased the diameter of the particles, as the additional shell of adsorbed gelatin molecules on the Ag and Cu NPs surface is present. Although high polydispersity index values, Pdi, ( $> 0.5$ ), were obtained in the case of Cu NPs, indicating a broad particle size distribution, the mean particle size values (obtained by DLS measurements) were used as relevant for particle size characterization. The zeta potential measurements indicated that Ag and Cu NPs are negatively charged, both in the absence and presence of gelatin. As can be seen from Table 1, the conductivity of the colloid dispersions can be changed due to the replacement of capping ions. Also, the electrophoretic mobility of Ag and Cu NPs is negative, indicating that the particles acquired a net negative charge.

#### Antibacterial properties

Antibacterial activity of all colloidal dispersions was tested against gram-negative bacteria *Acinetobacter*, *Escherichia Coli*, *Pseudomonas aeruginosa* and *Klebsiella pneumoniae*, and gram-positive bacteria *Enterococcus faecium*. It was found that both in the absence and presence of gelatin, Ag and Cu NPs exhibited antibacterial activity against all tested bacteria. This could be explained by the interaction between the outer membrane of bacteria with the released Au and Cu ions from a colloidal dispersion or from the surface of NPs. However, a further detailed study is required to clarify the antibacterial performance against tested bacteria, including the investigations on the concentration dependent manner and releasing mechanism of Ag and Cu NPs.

## CONCLUSION

In summary, we have synthesized silver and copper nanoparticles at room temperature by chemical reduction method in the presence of ascorbic acid and subsequently stabilized by gelatin. The size of Ag and Cu NPs determined by AFM measurements was around 10 nm in the presence of ascorbic acid and around 15 nm in presence of gelatin. The zeta potential measurements indicated that these nanoparticles are negatively charged, both in the absence and presence of gelatin. The colloidal dispersion of Ag, as well as of Cu nanoparticles, exhibited antibacterial activity against gram-negative bacteria *Acinetobacter*, *Escherichia Coli*, *Pseudomonas aeruginosa* and *Klebsiella pneumoniae* and against gram-positive bacteria *Enterococcus faecium*.

## ACKNOWLEDGMENTS

Authors would like to thank the Ministry of Education and Science of the Republic of Serbia (Project No. 172023) for their

financial support. Thank to Dr. Verica Simić, a specialist in microbiology and parasitology, from Public Health Institute of Kosovska Mitrovica, for help in the antibacterial studies.

## REFERENCES

- Amendola, V., Bakr, O.M., & Stellacci, F. 2010. A Study of the Surface Plasmon Resonance of Silver Nanoparticles by the Discrete Dipole Approximation Method: Effect of Shape, Size, Structure, and Assembly. *Plasmonics*, 5, pp. 85-97.
- Austin, L.A., Mackey, M.A., Dreaden, E.C., & El-Sayed, M.A. 2014. The optical, photothermal, and facile surface chemical properties of gold and silver nanoparticles in bionanomedicine, therapy, and drug delivery. *Arch. Toxicol.*, 88(7), pp. 1391-417. pmid:24894431
- Chekin, F., & Ghasemi, S. 2014. Silver nanoparticles prepared in presence of ascorbic acid and gelatin, and their electrocatalytic application. *Bulletin of Materials Science*, 37, pp. 1433-1437.
- Chicea, D. 2014. Using AFM Topography Measurements In Nanoparticle Sizing. *Romanian Reports in Physics*, 66, pp. 778-787.
- Daniel, M., & Astruc, D. 2004. Gold nanoparticles: Assembly, supramolecular chemistry, quantum-size-related properties, and applications toward biology, catalysis, and nanotechnology. *Chem. Rev.*, 104(1), pp. 293-346. pmid:14719978
- Dong, X., Ji, X., Wu, H., Zhao, L., Li, J., & Yang, W. 2009. Shape Control of Silver Nanoparticles by Stepwise Citrate Reduction. *The Journal of Physical Chemistry C*, 113, pp. 6573-6576.
- Ehsani, A., Jaleh, B., & Nasrollahzadeh, M. 2014. Electrochemical properties and electrocatalytic activity of conducting polymer/copper nanoparticles supported on reduced graphene oxide composite. *Journal of Power Sources*, 257, pp. 300-307.
- Hoo, C.M., Starostin, N., West, P., & McCartney, M.L. 2008. A comparison of atomic force microscopy (AFM) and dynamic light scattering (DLS) methods to characterize nanoparticle size distributions. *Journal of Nanoparticle Research*, 10(S1), pp. 89-96. doi:10.1007/s11051-008-9435-7
- Hu, J.-., Chen, Q., Xie, Z.-., Han, G.-., Wang, R.-., Ren, B., . . . Tian, Z.-. 2004. A Simple and Effective Route for the Synthesis of Crystalline Silver Nanorods and Nanowires. *Advanced Functional Materials*, 14(2), pp. 183-189. doi:10.1002/adfm.200304421
- Kelly, K.L., Coronado, E., Zhao, L.L., & Schatz, G.C. 2003. The Optical Properties of Metal Nanoparticles: The Influence of Size, Shape, and Dielectric Environment. *The Journal of Physical Chemistry B*, 107, pp. 668-677.
- Klapetek, P., Valtr, M., Nečas, D., Salyk, O., & Dzik, P. 2011. Atomic force microscopy analysis of nanoparticles in non-ideal conditions. *Nanoscale Research Letters*, 6(514).
- Laban, B., Vodnik, V., Vujačić, A., Sovilj, S.P., Jokić, A.B., & Vasić, V. 2013. Spectroscopic and Fluorescence Properties of Silver-Dye Composite Nanoparticles. *Russian Journal of Physical Chemistry A*, 87, pp. 2219-2224.
- Laban, B., Vodnik, V., Dramićanin, M., Novaković, M., Bibić, N., Sovilj, S.P., & Vasić, V.M. 2014. Mechanism and Kinetics of J-Aggregation of Thiocyanine Dye in the

- Presence of Silver Nanoparticles. *The Journal of Physical Chemistry C*, 118, pp. 23393-23401.
- Laban, B., Vodnik, V., & Vasić, V. 2015. Spectrophotometric observations of thiocyanine dye J-aggregation on citrate capped silver nanoparticles. *Nanospectroscopy*, 1, pp. 54-60.
- Laban, B., Zeković, I., Vasić Aničijević, D., Marković, M., Vodnik, V., Luce, M., . . . Vasić, V. 2016. Mechanism of 3, 3'-Disulfopropyl-5, 5'-Dichlorothiocyanine Anion Interaction With Citrate-Capped Silver Nanoparticles: Adsorption and J-Aggregation. *The Journal of Physical Chemistry C*, 120, pp. 18066-18074.
- Lee, Y., Choi, J., Lee, K.J., Stott, N.E., & Kim, D. 2008. Large-scale synthesis of copper nanoparticles by chemically controlled reduction for applications of inkjet-printed electronics. *Nanotechnology*, 19(41), p. 415604. pmid:21832649. doi:10.1088/0957-4484/19/41/415604
- Li, M., Cushing, S.K., & Wu, N. 2015. Plasmon-enhanced optical sensors: A review. *Analyst*, 140(2), pp. 386-406. pmid:25365823
- Li, Y., Wu, Y., & Ong, B.S. 2005. Facile synthesis of silver nanoparticles useful for fabrication of high-conductivity elements for printed electronics. *J. Am. Chem. Soc.*, 127(10), pp. 3266-7. pmid:15755129
- Liz-Marzán, L.M. 2006. Tailoring surface plasmons through the morphology and assembly of metal nanoparticles. *Langmuir*, 22(1), pp. 32-41. pmid:16378396
- Lu, L.M., Zhang, X.B., Shen, G.L., & Yu, R.Q. 2012. Seed-mediated synthesis of copper nanoparticles on carbon nanotubes and their application in nonenzymatic glucose biosensors. *Analytica Chimica Acta*, 715, pp. 99-104.
- Mahmoud, M.A., Chamanzar, M., Adibi, A., & El-Sayed, M.A. 2012. Effect of the dielectric constant of the surrounding medium and the substrate on the surface plasmon resonance spectrum and sensitivity factors of highly symmetric systems: Silver nanocubes. *J. Am. Chem. Soc.*, 134(14), pp. 6434-42. pmid:22420824
- Maria, K., Susmit, K., Rosaria, B., Simona, P., la Carola, T., Giovanni, B., . . . Athanassia, A. 2013. Electrical response from nanocomposite PDMS-Ag NPs generated by in situ laser ablation in solution. *Nanotechnology*, 24, p. 35707.
- Mogensen, K.B., & Kneipp, K. 2014. Size-Dependent Shifts of Plasmon Resonance in Silver Nanoparticle Films Using Controlled Dissolution: Monitoring the Onset of Surface Screening Effects. *The Journal of Physical Chemistry C*, 118, pp. 28075-28083.
- Mulvaney, P. 1996. Surface Plasmon Spectroscopy of Nanosized Metal Particles. *Langmuir*, 12, pp. 788-800.
- Panacek, A., Kvítek, L., Pucek, R., Kolar, M., Vecerova, R., Pizúrova, N., . . . Zboril, R. 2006. Silver colloid nanoparticles: Synthesis, characterization, and their antibacterial activity. *J Phys Chem B*, 110(33), pp. 16248-53. pmid:16913750
- Qin, Y., Ji, X., Jing, J., Liu, H., Wu, H., & Yang, W. 2010. Size control over spherical silver nanoparticles by ascorbic acid reduction. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, 372, pp. 172-176.
- Ramyadevi, J., Jeyasubramanian, K., Marikani, A., Rajakumar, G., & Rahuman, A.A. 2012. Synthesis and antimicrobial activity of copper nanoparticles. *Materials Letters*, 71, pp. 114-116.
- Salavati-Niasari, M., & Davar, F. 2009. Synthesis of copper and copper(I) oxide nanoparticles by thermal decomposition of a new precursor. *Materials Letters*, 63, pp. 441-443.
- Song, J., Kim, H., Jang, Y., & Jang, J. 2013. Enhanced Antibacterial Activity of Silver/Polyrhodanine-Composite-Decorated Silica Nanoparticles. *ACS Applied Materials & Interfaces*, 5, pp. 11563-11568.
- Sosa, I.O., Noguez, C., & Barrera, R.G. 2003. Optical Properties of Metal Nanoparticles with Arbitrary Shapes. *The Journal of Physical Chemistry B*, 107, pp. 6269-6275.
- Tomaszewska, E., Soliwoda, K., Kadziola, K., Tkacz-Szczesna, B., Celichowski, G., Cichomski, M., Szmaja, W. & Grobelny, J. 2013. Detection Limits of DLS and UV-Vis Spectroscopy in Characterization of Polydisperse Nanoparticles Colloids. *Journal of Nanomaterials*, pp. 10. doi: 10.1155/2013/313081
- Valodkar, M., Modi, S., Pal, A., & Thakore, S. 2011. Synthesis and anti-bacterial activity of Cu, Ag and Cu-Ag alloy nanoparticles: A green approach. *Materials Research Bulletin*, 46, pp. 384-389.
- Walcarius, A., Minter, S.D., Wang, J., Lin, Y., & Merkoçi, A. 2013. Nanomaterials for bio-functionalized electrodes: Recent trends. *Journal of Materials Chemistry B*, 1, pp. 4878-4908.
- Wang, F., Widejko, R.G., Yang, Z., Nguyen, K.T., Chen, H., Fernando, L.P., . . . Anker, J.N. 2012. Surface-enhanced raman scattering detection of pH with silica-encapsulated 4-mercaptobenzoic acid-functionalized silver nanoparticles. *Anal. Chem.*, 84(18), pp. 8013-9. pmid:22881392
- Wang, H., Huang, Y., Tan, Z., & Hu, X. 2004. Fabrication and characterization of copper nanoparticle thin-films and the electrocatalytic behavior. *Analytica Chimica Acta*, 526, pp. 13-17.
- Wiley, B.J., Im, S.H., Li, Z., McLellan, J., Siekkinen, A., & Xia, Y. 2006. Maneuvering the surface plasmon resonance of silver nanostructures through shape-controlled synthesis. *J Phys Chem B*, 110(32), pp. 15666-75. pmid:16898709
- Yacamán, M., Ascencio, J.A., Liu, H.B., & Gardea-Torresdey, J. 2001. Structure shape and stability of nanometric sized particles. *Journal of Vacuum Science & Technology B: Microelectronics and Nanometer Structures*, 19(4), p. 1091. doi:10.1116/1.1387089
- Zain, N.M., Stapley, A.G.F., & Shama, G. 2014. Green synthesis of silver and copper nanoparticles using ascorbic acid and chitosan for antimicrobial applications. *Carbohydrate Polymers*, 112, pp. 195-202.
- Zhang, Q., Li, N., Goebel, J., Lu, Z., & Yin, Y. 2011. A systematic study of the synthesis of silver nanoplates: Is citrate a "magic" reagent?. *J. Am. Chem. Soc.*, 133(46), pp. 18931-9. pmid:21999679
- Zhang, Q., Uchaker, E., Candelaria, S.L., & Cao, G. 2013. Nanomaterials for energy conversion and storage. *Chem Soc Rev*, 42(7), pp. 3127-71. pmid:23455759
- Zhu, H., Zhang, C., & Yin, Y. 2005. Novel synthesis of copper nanoparticles: Influence of the synthesis conditions on the particle size. *Nanotechnology*, 16(12), pp. 3079-3083. doi:10.1088/0957-4484/16/12/059

# REMOTE SENSING MACHINE LEARNING ALGORITHMS IN ENVIRONMENTAL STRESS DETECTION - CASE STUDY OF PAN-EUROPEAN SOUTH SECTION OF CORRIDOR 10 IN SERBIA

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## ABSTRACT

The construction of the Pan-European Corridor 10 is one of the major projects in the Republic of Serbia, and it enters the final phase. A vast natural area suffered a significant change to complete the project and therefore is the existence of a need to monitor those changes. Nature requires adequate and accurate detection of environmental stresses which inevitably arise after implementation of such large construction projects. Conversely to traditional field monitoring of the environment, this paper will present the remote sensing method which includes usage of European Space Agency's Sentinel 2A optical satellite data processed with different Machine Learning algorithms. An accuracy assessment is performed on land cover map results, and change detection carried out with best resulting data.

**Keywords:** Environment Monitoring, Gaussian Mixture Model, Random Forest, K-Nearest Neighbors, Confusion Matrix.

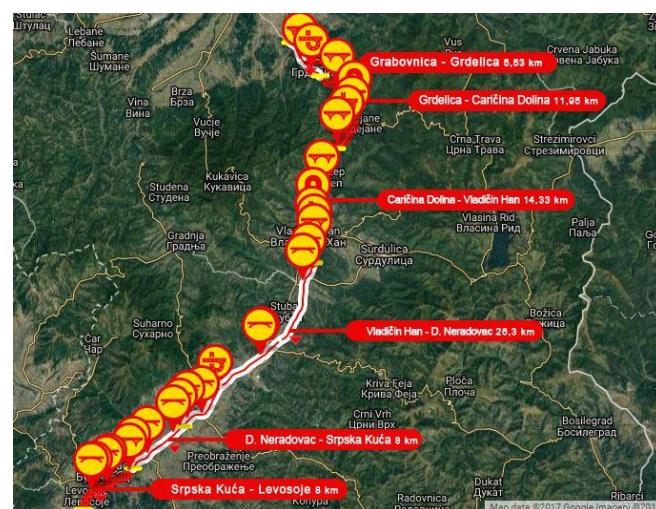
## INTRODUCTION

One of the major projects of the Republic of Serbia funded by the World Bank (WB), European Investments Bank (EIB), Hellenic Plan for the Economic Reconstruction of the Balkans (HiPERB) and the Republic of Serbia, is the construction of the main branch of Pan-European Corridor 10. The corridor connects Salzburg in Austria and Thessaloniki in Greece through Ljubljana in Slovenia, Zagreb in Croatia, Belgrade, and Niš in Serbia, Skopje, and Veles in Macedonia (Figure 1). In Serbia, the south part of Corridor 10 is called the "Highway E75 – project SOUTH" and it is presented and constructed as the motor road at this point (Koridori Srbije, 2017).



**Figure 1.** Pan-European corridors in Serbia.  
Source: belgradenet.com

The Highway E75 – project SOUTH extends for 74 km, from Grabovnica to Levosoje (Figure 2). There are five sections to complete in this area: Grabovnica – Grdelica (L=5.6 km), Grdelica – Caričina Dolina (L= 11.8 km), Caričina Dolina – Vladičin Han (L= 14.3 km), Vladičin Han – Donji Neradovac (L= 26.3 km), and Donji Neradovac - Levosoje (L= 16 km) (Figure 2) (Koridori Srbije, 2017).

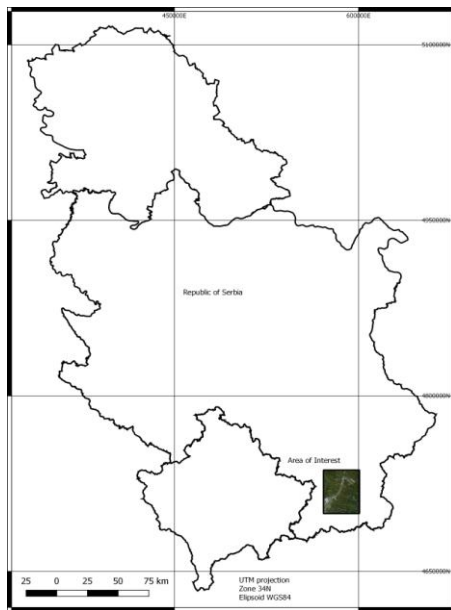


**Figure 2.** Corridor 10 South project in Serbia.  
Source: www.koridor10.rs printscreen

The construction zone of this scale indubitably has a significant impact on the environment. A proper monitoring is crucial to conserve the nature and mitigate the environmental stress. Considering that technology has advanced, we are going to use the achievements of remote sensing and its methods to monitor the changes that have occurred during the construction of Corridor 10. Further, the change detection of the land cover

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will be performed to present the changes for the monitored period. Area of interest is selected within the area that is under active construction and covers 1.095,4 sq. km (Figure 3).



**Figure 3.** Area of Interest - part of Corridor 10 working zone.

## EXPERIMENTAL

### Materials and methods

Remote sensing technology is employed to achieve the goal of this paper with the contemporary methodology that employs the *Machine Learning* (ML) algorithms (Canziani et al., 2008; Mas & Flores 2008; Jensen et al., 2009; Duro et al., 2012; Lary et al., 2016).

Sentinel 2 satellite imagery was obtained using Copernicus Sci Hub (Copernicus Open Access Hub, 2017) as starting data for the analysis. Sentinel 2 product consists of the granules that represent the particular region. The granule comes with 13 different bands where three different ground resolution bands are present: 10 m, 20 m, and 60 m. 10 m bands are: visible Blue (B), Green (G), Red (R), and Near InfraRed (NIR). 20 m bands are three Vegetation Red Edge bands, Narrow NIR and two Short Wave InfraRed (SWIR) bands. 60 m bands are Coastal Aerosol, Water, Vapour and SWIR Cirrus band (Sentinel 2 MSI, 2017).

Two different Sentinel 2 products Level-2A were downloaded for 2017. Since there were cloudy parts in the research area, the mosaic was made using two different granules T34TEN date from 01.07. – 31.07.2017. Remote sensing/ raster processing plugin for QGIS was applied to perform the mosaicking tasks.

To perform the change detection for the research area, the same images from August 2016 were downloaded from the Copernicus Sci Hub, and sub-scene created. The image was cloud-free, and there was no need for mosaicking. The product was Level-1C, so the data was processed to Level-2A using

SNAP (Sentinel Application Platform) toolbox software (ESA STEP, 2017), which took more than 13 hours to complete. Sentinel 2 products have multiple processing phases:

- *Level-0* and *Level-1A&B* products are in preprocessing phase and not available to users;
- *Level-1C* processing uses the *Level-1B* product and applies radiometric and geometric corrections (including orthorectification and spatial registration);
- Atmospheric correction is applied to Top-Of-Atmosphere (TOA) *Level-1C* orthoimage products, and a scene classification is presented as the *Level-2A* product. Bottom-Of-Atmosphere (BOA) corrected reflectance product is Level-2A with main output as an orthoimage. Additional outputs are Aerosol Optical Thickness (AOT) map, a Water Vapour (WV) map and a Scene Classification Map (SCM) together with Quality Indicators (QI) for cloud and snow probabilities at 60 m resolution (Sentinel 2 MSI, 2017).

Sentinel 2 bands used to complete the analysis are Red, Green, Blue and Near Infra-Red bands with 10m ground resolution.

Pixel-based *Machine Learning* (ML) algorithms were used to produce the land cover map of the area of interest. The most common three ML tasks are *Regression*, *Classification*, and *Clustering*.

*Regression* is employed as supervised learning task for modeling and predicting variables, where we have numeric *true* ground values for the research area. There are different regression algorithms, such as:

- *Linear Regression* (works when there are linear relationships between dataset variables);
- *Regression Tree* or *Decision Trees* repeatedly splits the dataset into separate branches and maximize the information gain. This allows the algorithm to learn nonlinear relationships;
- *Deep Learning* algorithm applies to multi-layer neural networks to learn extremely complex patterns using convulsions and drop-out mechanisms, and others;
- *Honorable Mention (Nearest Neighbors)* save each training observation. Further, they make predictions for new observations as they search for similar training observations and join the values (Elite Data Science, 2017).

*Classification*, as supervised learning task, is used in this paper to model and predict land cover categories as the ML algorithms can predict a class. Different classifications were used in this article to obtain the best possible accuracy of the data:

- *Classification Trees* is employed in Random Forest;
- *Gaussian Mixture Model* (GMM) take on that data points are generated from a mixture of a limited number of Gaussian distributions with unfamiliar parameters (Scikit learn, 2014).

*K-Neighbors Classifier* where the learning is based on the **k** nearest neighbors of each query point. **k** is an integer value specified by the user (Scikit learn, 2014).

The creation of a land cover map from BOA processed Sentinel 2 data required a ground training samples. To obtain such areas and create necessary vector file as training material, historical google maps were employed using different sources and plugins for QGIS. Seven different classes recognized for both 2016 and 2017 and consist of 175 and 164 polygons respectively. Two attributes created, as integer and text. Further, prepared subscene for each year was processed using dzetsaka ML plugin for QGIS.

The accuracy assessment was performed using training sample polygons in dzetsaka and SCP plugin for QGIS.

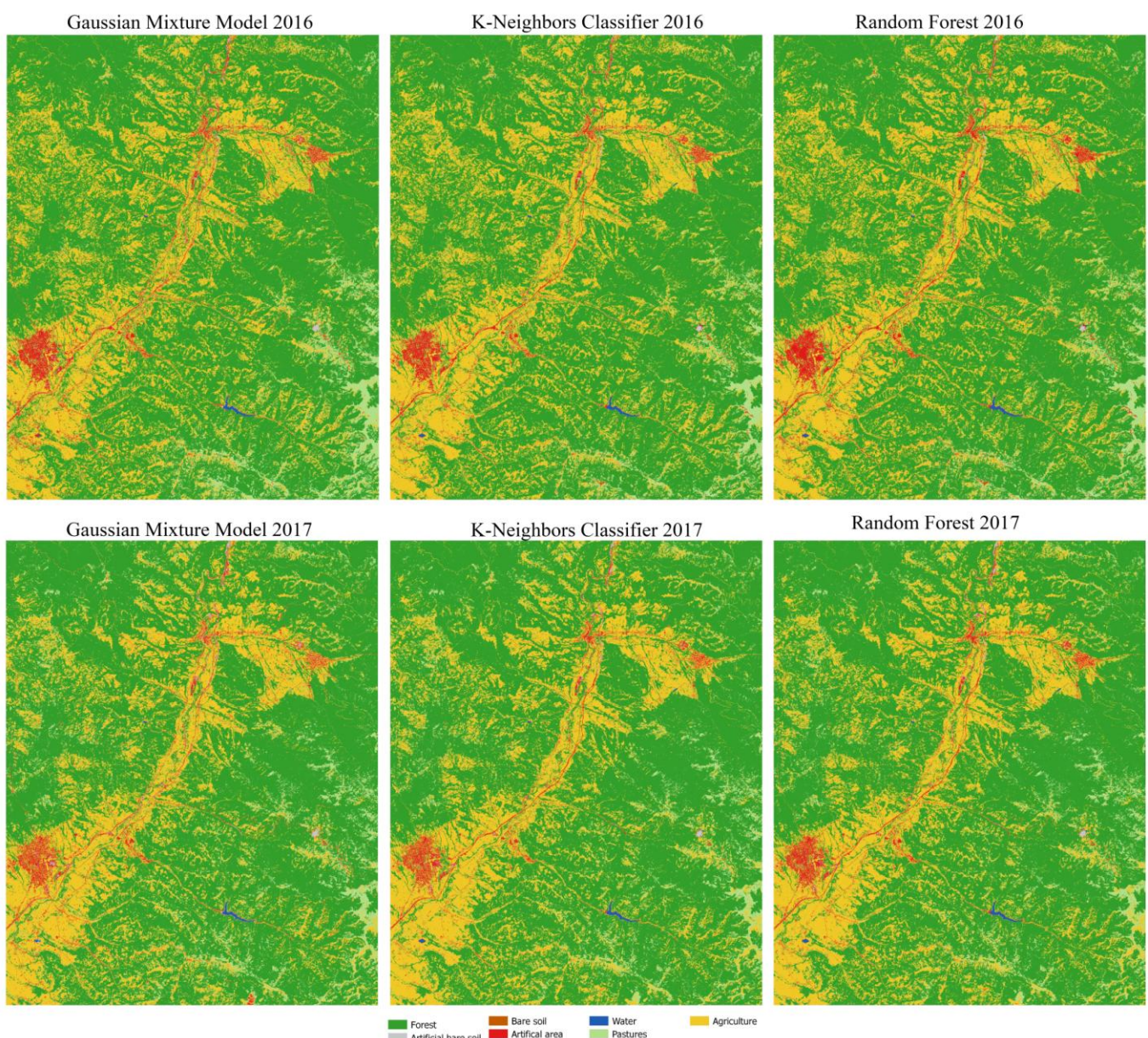
Confusion matrix was created and presents overall accuracy and kappa hat.

The land cover change was performed using SCP plugin in QGIS.

## NUMERICAL RESULTS

After applying the algorithms, three different land cover maps for each year were created (Figure 4).

Accuracy assessment for created land cover maps is presented in Tables 1-3. As it can be seen, ML algorithms gave very decent results where Random Forests goes up to 100% of accuracy.



**Figure 4.** Land cover maps for the area of interest created using different ML algorithms.

**Table 1.** Confusion matrix for K-Neighbors Classifier.

Class 2016	Forest	Artificial bare soil	Bare soil	Artificial area	Water	Pastures	Agriculture	Class 2017	Forest	Artificial bare soil	Bare soil	Artificial area	Water	Pastures	Agriculture
Forest	115307	0	5	39	0	263	1601	Forest	116280	0	3	11	0	153	515
Artificial bare soil	0	2059	0	286	0	0	1	Artificial bare soil	0	2548	0	296	0	0	0
Bare soil	0	0	239	39	0	35	16	Bare soil	0	0	308	56	0	0	92
Artificial area	7	499	20	11002	0	9	313	Artificial area	0	227	33	5135	0	0	162
Water	0	0	0	8	2256	0	0	Water	0	0	0	1	2259	0	0
Pastures	212	1	26	363	0	13726	621	Pastures	180	0	22	0	0	4922	515
Agriculture	1139	58	1147	2085	0	1902	56578	Agriculture	240	19	1078	2168	0	458	56406
<b>Kappa</b>	91.63%							<b>Kappa</b>	94.09%						
<b>Overall</b>	94.95%							<b>Overall</b>	96.79%						

**Table 2.** Confusion matrix for Gaussian Mixture Model Classifier.

Class 2016	Forest	Artificial bare soil	Bare soil	Artificial area	Water	Pastures	Agriculture	Class 2017	Forest	Artificial bare soil	Bare soil	Artificial area	Water	Pastures	Agriculture
Forest	114235	2	5	17	51	489	2879	Forest	115280	0	3	1	7	748	511
Artificial bare soil	0	2083	0	1059	57	0	0	Artificial bare soil	0	2401	0	727	108	0	0
Bare soil	0	3	0	102	0	23	99	Bare soil	78	0	55	119	0	0	185
Artificial area	42	498	24	10270	3	3	1533	Artificial area	116	383	174	4509	0	3	665
Water	0	0	0	0	2145	0	0	Water	0	0	0	0	2144	0	0
Pastures	478	0	232	368	0	13085	922	Pastures	227	0	24	2	0	11892	869
Agriculture	1910	31	1176	2006	0	2335	53697	Agriculture	999	10	1188	2309	0	2553	56905
<b>Kappa</b>				87.23%				<b>Kappa</b>				89.93%			
<b>Overall</b>				92.28%				<b>Overall</b>				94.15%			

**Table 3.** Confusion matrix for Random Forest Classifier.

Class 2016	Forest	Artificial bare soil	Bare soil	Artificial area	Water	Pastures	Agriculture	Class 2017	Forest	Artificial bare soil	Bare soil	Artificial area	Water	Pastures	Agriculture
Forest	116665	0	0	0	0	0	2	Forest	116453	0	2	8	0	485	298
Artificial bare soil	0	2617	0	0	0	0	0	Artificial bare soil	0	2647	0	150	1	0	2
Bare soil	0	0	1437	0	0	0	0	Bare soil	0	0	825	13	0	0	42
Artificial area	0	0	0	13821	0	0	0	Artificial area	0	138	59	6526	0	0	187
Water	0	0	0	0	2256	0	0	Water	0	0	0	0	2258	0	0
Pastures	0	0	0	0	0	15933	0	Pastures	93	0	10	0	0	10611	213
Agriculture	0	0	0	1	0	2	59128	Agriculture	154	9	548	970	0	4100	58393
<b>Kappa</b>				100%				<b>Kappa</b>				93.71%			
<b>Overall</b>				100%				<b>Overall</b>				96.35%			

Accuracy assessment results demonstrate how those ML algorithms execute the classification. The best result is given by the Random Forest algorithm with perfect accuracy of 100% for 2016 and 96.35% for 2017. In next part of this research, Random Forest land cover map will be used for the final analysis. Classification results are presented in Table 4:

**Table 4.** Classification results for RF land cover maps.

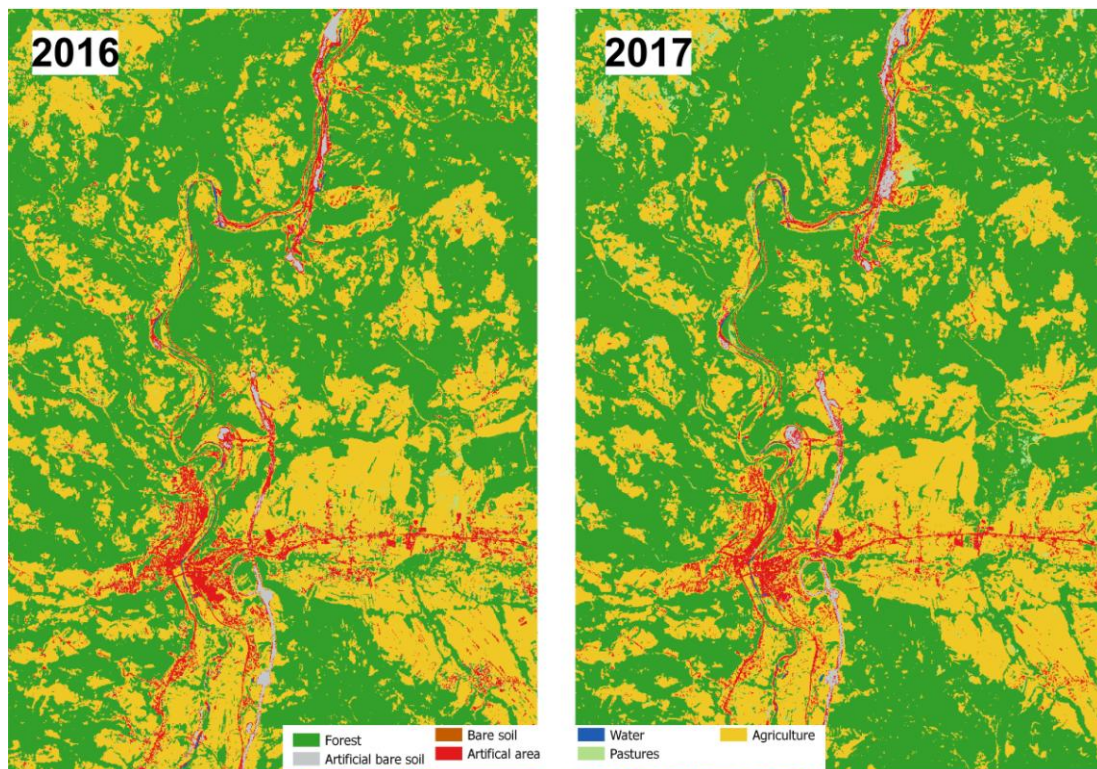
2016 Class	Pixel Sum	Percentage (%)	Area (km <sup>2</sup> )
Forest	7209286	65.89	720.93
Artificial bare soil	10320	0.09	1.03
Bare soil	7955	0.07	0.80
Artificial area	267578	2.45	26.76
Water	9883	0.09	0.99
Pastures	428576	3.92	42.86
Agriculture	3008299	27.49	300.83
2017 Class	Pixel Sum	Percentage (%)	Area (km <sup>2</sup> )
Forest	7419839	67.85	741.98
Artificial bare soil	17226	0.16	1.72
Bare soil	4049	0.04	0.40
Artificial area	192010	1.76	19.20
Water	9860	0.09	0.99
Pastures	543802	4.97	54.38
Agriculture	2748450	25.13	274.85

The results show that two classes are dominant with more than 90% of the research area: Forest with 65.9% in 2016 and 67.6% in 2017 and Agriculture with 27.9% and 25.1% respectively. Percentage of change is presented in Table 5.

**Table 5.** Change in classes. The positive values represent the increase of pixels in 2017 while negative values present decrease in 2017.

Class	Area (km <sup>2</sup> )	Percentage (%)
Forest	21.06	2.92
Artificial bare soil	0.69	66.92
Bare soil	-0.39	-49.10
Artificial area	-7.56	-28.24
Water	0.00	-0.23
Pastures	11.52	26.89
Agriculture	-25.98	-8.64

Change detection data in table 5 confirms the table 4 data and presents how much each class has changed. The highest increase has the Artificial bare soil (where our primary goal of this work belongs – Corridor 10 under construction), and Pasture classes versus the Bare Soil, Agriculture, and Artificial classes which decrease in area percentage cover. Figure 5 shows the difference in the northern part of the research area where the construction of Corridor 10 is in its full swing.



**Figure 5.** Northern part of the research area - Corridor 10 ongoing construction site.

## CONCLUSION

As table 5 is presenting, the class of interest in this research is within Artificial bare soil which presents the construction area of new Corridor 10. It can be seen that there is an increase of the area covered by this class which indicates that in one year there were changes in the environment. Since the land cover is still presented with same class and did not change into an Artificial area where constructed – paved highway belongs, we can conclude that the motorway is still under construction. This data acquired using remote sensing analysis of Sentinel 2 satellite imagery can be of great help in monitoring changes of the environment and big construction projects. Since the satellite data are widely accessible and have satisfying ground resolution with low, or no cost, we cannot exclude the remote sensing techniques from the environmental research, but we must expand the knowledge and capabilities provided. Random Forest machine learning algorithm used in this paper confirms that the classifying algorithms have advanced to the level when they can be of great help to the environment analysts. High accuracy of classified data obtained using Classification Tree algorithm gives new perspective to remote sensing. Furthermore, different machine learning algorithms (Random Forest, Gaussian Mixture Model, K-Neighbors Classifier, and other) along with the Artificial Neural Networks and Object Based Image Analysis (OBIA) classification are in the focus of remote sensing professionals and researchers, while rapid development and improvement of the algorithms is in progress.

With this methodology, it is possible to perform a broad spectrum of analysis, such as environmental stress detection (landslides, wildfires, flooding, etc.) or land cover map creation and other, with the very high percentage of accuracy while we save time and money in the process that used to last much longer.

## REFERENCES

- Belgrade Net. (2017).  
Retrieved from <http://www.belgradenet.com/business/>
- Canziani, G., Ferrati, R., Marinelli, C., & Dukatz, F. 2008. Artificial Neural Networks and Remote Sensing in the Analysis of the Highly Variable Pampean Shallow Lakes. *Mathematical Biosciences and Engineering*, 5(4). doi:10.3934/mbe.2008.5.691
- Copernicus Open Access Hub. (2017).  
Retrieved from <https://scihub.copernicus.eu/dhus/#/home>
- Duro, D.C., Franklin, S.E., & Dubé, M.G. 2012. A Comparison of Pixel-Based and Object-Based Image Analysis with Selected Machine Learning Algorithms for the Classification of Agricultural Landscapes Using SPOT-5 HRG Imagery. *Remote Sensing of Environment*, 118, pp. 259-272. doi:10.1016/j.rse.2011.11.020
- Elite Data Science. (2017). Retrieved from <https://elitedatascience.com/machine-learning-algorithms>
- ESA STEP. (2017).  
Retrieved from <http://step.esa.int/main/toolboxes/snap/>
- ESA. 2017. Sentinel 2 MSI.  
Retrieved from <https://earth.esa.int/web/sentinel/user-guides>
- Jensen, R.R., Hardin, P.J., & Yu, G. 2009. Artificial Neural Networks and Remote Sensing. *Geography Compass*, 3, pp. 630-646. doi:10.1111/j.1749-8198.2008.00215.x
- Koridori Srbije. (2017). Retrieved from <http://www.koridor10.rs>
- Lary, D.J., Alavi, A.H., Gandomi, A.H., & Walker, A.L. 2016. Machine Learning in Geosciences and Remote Sensing. *Geoscience Frontiers*, 7(1), pp. 3-10. doi:10.1016/j.gsf.2015.07.003
- Mas, J.F., & Flores, J.J. 2008. The Application of Artificial Neural Networks to the Analysis of Remotely Sensed Data. *International Journal of Remote Sensing*, Iss., 29(3). doi:10.1080/01431160701352154
- Scikit learn. (2014).  
Retrieved from <http://scikit-learn.org/0.15/modules/>

# SPATIAL FUNCTIONAL TRANSFORMATION AND TYPOLOGY OF THE SETTLEMENT SYSTEM OF TOPLICA DISTRICT

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## ABSTRACT

Contemporary processes of industrialization, urbanization, deagrarianization, the polarization and globalization contribute socio-economic transformation of the observed space as well as the creation of new carrier of functional relationships in space. Towns with its own influences enrich the network of surrounding settlements, strengthen their mutual relations and create a whole functional settlement system of one area, or the gravity of the urban core. By dividing the functions of the primary, secondary and tertiary, the basis and types of settlements are created by functional criteria according to the type of economic activity and the primary content in them. In this area in the second half of the twentieth and early twenty-first century witnessed substantial changes in almost all components of demographic structure, which resulted in the transformation of functional types of settlement, when the predominantly agrarian settlement characteristic of the area of Toplica road went up mixed and service settlement. The idea behind the study is for the geographically complex area to be displayed in the light of socio-economic development, and as a basis for further economic development of this part of the Republic of Serbia.

**Keyword:** urbanization, rural settlements, urban settlements, functions, typology, sustainable development, District of Toplica

## INTRODUCTION

Toplica district is located in the southern part of the Republic of Serbia and occupies a historical and geographic area, known as Toplica. Its northern border is Rasina district, Nišava district in the east, in southeast is Jablanica, and in the south Kosovo district. Toplica is a region which in the physical-geographical terms belongs to the depression-mountain-valley macro-region, or mezoregions of South Serbia.

The region is limited to the east by South Morava, a western border region bounded Kopaonik. The northern border region is represented by parts of the Great and Small Jastrebac, while the southern border connecting the corners of the highest parts of the Maidan, Djak, Radan, Petrovac, Bucumetski height, Greben and Dobre Glave.

If administrative regionalization was observed, Toplica region would be limited by the Toplica district, which consists of four municipalities: Prokuplje, Kuršumljaja, Blace and Žitotadja. The District has a total of 90,600 inhabitants according to the census of 2012, while 2002 had 111,813 inhabitants. Population density is 40.61 inhabitants / km<sup>2</sup>, which is below the national average. Its seat is a town Prokuplje. Borders of Toplica districts do not coincide with the natural, physical and geographical boundaries of the region and therefore county, as well as administrative spatial unit occupies only 73% of the region of Toplica, or an area of 2 231 km<sup>2</sup> (Rudić, 1978).

The largest territorial unit by area is Kuršumljaja municipality 952 km<sup>2</sup> (42.7% of Toplica region), then Prokuplje

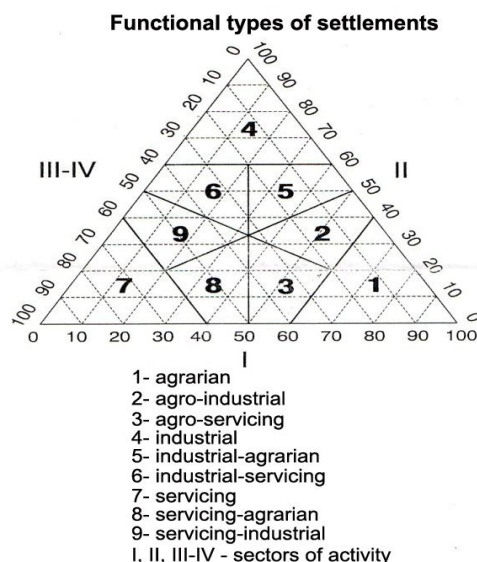
municipality with an area of 759 km<sup>2</sup> (34.0%), Blace municipality has an area of 306 km<sup>2</sup> (13.7%), and lowest per surface is the territory of the municipality of Žitotadja 214 km<sup>2</sup> (9.6%).

## MATERIAL AND METHODS

Significant contribution to the functional systematization of the settlements was given by (Grčić, 1999), who elaborated the methodology of the typological classification of the settlements. The concept of the economic structure of the population was applied in this work to determine the functional type of settlement of Toplica district, which is determined based on the structure of the active population according to activities, in other words, it is based on principle of dominant activity (functions). Apart from the structure of the active population the functional determination of the settlements regulates the overall activity with regards to the geographic position and socio-ecological conditions (Christaller, 1933). The method of tenar diagram is applied in this work, as one of grafical method of allocation of the functional type of settlements. It was presented by equilateral triangle whose sides are divided into sections that indicate the percentages of primary, secondary and tertiary activity. In order to fulfill the condition of participation of the active population by sectors up to 100%, in tertiary activities, a group of quaternary is included (health, education, information, local government). The interior of the diagram is divided into types, indicated by a combination of numbers and letters. The functional type will be defined in the intersection of three lines each of which has the value of a particular sector of activity (Matijević, 2009). Based on the economic composition of the population according to the

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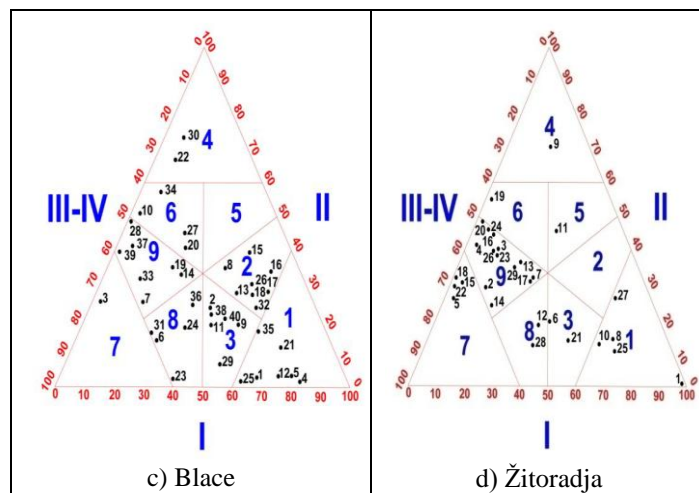
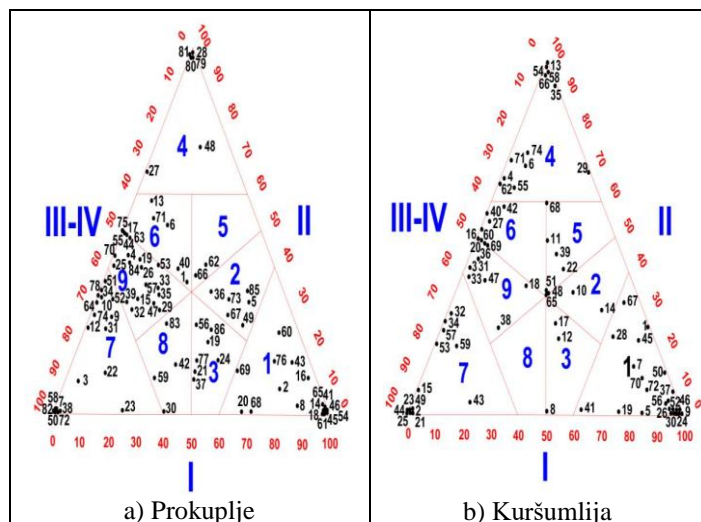
sectors of activity, functional types of settlements and their subtypes are distinguished (Myrdal, 1957).



**Figure 1.** Application of Horst-Fere triangle to determine the functional type of settlement. *Source:* handout "Types of population movements"

Geographical, traffic and administrative situation of the settlements affect the functional typology of the settlement, while the basic features are largely determined by population movements (Boudeville, 1966). The structure of population activities is an indicator that is used to determine the spatial and functional relationships that affect the transformation of the settlements (Vresk, 1990).

Method of Horst-Fere triangle (Fig. 1) involves separation of the nine types of settlements based on the participation of individual sectors of activity in the contingent of the population performing occupation. The method is adapted to our circumstances, and the available data. Figure 2. shows the tenar diagram for all four municipalities of Toplica district.



**Figure 2.** Horst-Fere's settlements triangle (a,b,c,d) municipalities of Toplica District.

The functional type of settlement is determined on the basis of the share of primary, secondary and tertiary sector activity in the total population performing occupation (Davies, 1967). The threshold for determining dominant belonging to a certain sector of activity is 60% (Table 1).

In Table 1, marked with Roman numerals, are sectors of activity: I - primary (agricultural) sector II - secondary (industrial) sector and III - tertiary-quaternary (service) sector.

**Table 1.** Model for extracting functional types of settlements.

Functional type of settlements	Criteria
Agrarian	$I \geq 60\%$
Agro-industrial	$I > II > III$
Agro-service	$I > III > II$
Industrial	$II \geq 60\%$
Industrial-agrarian	$II > I > III$
Industrial-service	$II > III > I$
Service	$III \geq 60\%$
Service-agrarian	$III > I > II$
Service-industrial	$III > II > I$

*Source:* Tošić, D. (2012): Principles of regionalization. Belgrade. University of Belgrade – Faculty of Geography.

## RESEARCH RESULTS AND DISCUSSION

The process of industrialization in Serbia started in the period between the two world wars, to its full momentum experienced in the years after World War II, mostly during the 60s and 70s of the twentieth century. (Stamenković, Bačević, 1992). Industrialization was the trigger for intensive deagrarianization and urbanization of our premises or relocation of the working age population from rural to urban areas, which have become industrial centers (Green, 2004). Although the process of industrialization greatly slowed to such an extent that

one can speak of deindustrialisation, agrarian reform and urbanization processes are still ongoing. The population continues to leave the rural areas and moved to urban and suburban areas. The same situation exists in the administrative

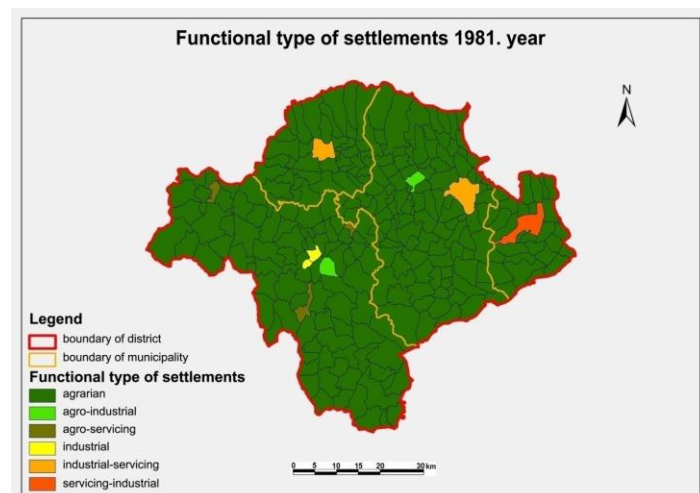
district of Toplica, except that the demographic base is pretty empty, which means that there are fewer people in rural areas, which could be moved to urban areas.

**Table 2.** The number of settlements by functional types of settlements.

Type	Municipality 1981, 2002. and 2011. Year												Total		
	Žitorada			Prokuplje			Blace			Kuršumljia					
	81	02	11	81	02	11	81	02	11	81	02	11	81	02	11
A	29	7	5	105	59	17	39	15	6	84	48	22	257	129	50
AI	0	3	0	1	10	5	0	13	8	1	8	6	2	34	19
AS	0	6	2	0	8	9	0	4	8	3	6	4	3	24	23
I	0	1	1	0	4	6	0	0	2	2	1	12	2	5	21
IA	0	3	1	0	5	1	0	2	0	0	4	3	0	14	5
IS	0	4	1	1	10	5	1	4	3	0	6	4	2	24	13
S	0	0	4	0	2	19	0	0	1	0	7	13	0	9	37
SA	0	2	2	0	3	4	0	0	4	0	3	1	0	8	11
SI	1	4	13	0	5	20	0	2	8	0	3	9	1	14	50
W	0	0	1	0	1	21	0	0	0	0	4	17	0	6	38

Source: National Bureau of Statistics

Functional types of settlements Toplica were analyzed during 1981, 2002. and 2011. (Table 2). Settlements of Toplica region in 1981 were predominantly agrarian. Even 257 from 267 settlements (96%) were distinctly agrarian functional type, which means that over 60% of the population engaged in its activities in the primary sector. Out of ten remaining settlements, five were the most common agrarian sector: two agrarian-industrial - Gelding (Kuršumljia municipality) and Mala Plana (municipality of Prokuplje) and three agro-service - Kuršumlijska Banja, Barlovo and Merčez (all in the municipality of Kuršumljia). (Spatial Plan of the Municipality of Kuršumljia).



**Figure 3.** Functional type of settlement in 1981. (Federal Statistical Yugoslavia, 1984).

Two settlements were an industrial functional type: Kuršumljia municipal center and its suburb Markoviće. Two settlements of industrial-service-municipal are centers of Blace and Prokuplje, while the only village with a dominant service function, and service-industrial type was the easternest and lowest municipal center – Žitoradja (Spatial Plan of the Municipality of Prokuplje).

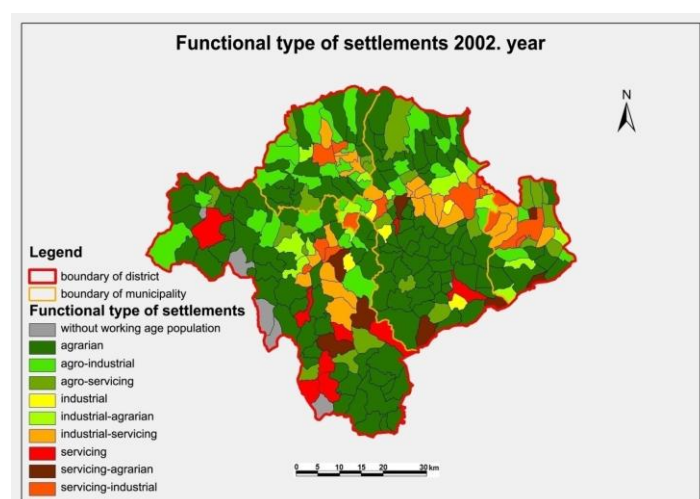
On the chart of the functional type of settlements in 1981 (Fig. 3) is notable that the scarce settlements that do not belong to the agrarian type distributed around the municipal centers, or at major traffic routes (Spatial plan of the Municipality of Blace).

In the 21-year period from 1981. to 2002. there have been significant changes in the functions of the Toplica settlement. The process of diversification of the economy is set and non-agricultural activities began to spread outside the municipal centers. Originally, it was industry, which was responsible for the development of this region, but it also increased the share of service sector activity (Dragojlović, 2017).

The largest number of settlements in 2002. and still belonged to the agrarian type, 129 or 48%, but this is a significant decline compared to 96% in 1981. In addition, 58 more settlements (22%) had a mixed function with predominantly agrarian function: 34 agro-industrial and agrarian-24 service. Significantly increased the number of settlements industrial-agrarian and industrial-type service, but also the settlement with primary service functions.

Fig. 4. shows that the generators changes in municipal centers and main roads in the district. Industrial and service activities are expanding from municipal centers in the suburbs,

where people travel to municipal centers to carry out activities. In addition, some manufacturing and service facilities are moved to the hinterland cities to reduce costs or due to changes in the needs of the population now lives practically urban way of life and in the settlements. The main road route Niš-Priština is also a major generator of change, because it is noticeable that a large number of settlements along this road developed service functions, but now belong to the service or the principal or mixed types with predominantly service functions. In addition to these settlements, services are dominant and in spas (Prolom, Kuršumlijska and Lukovska). The most intensive changes have gone in the municipality Žitoradja, which largely gravitates to Niš, but the impact of this great town felt and settlement functions of the municipality (Spatial Plan of the Municipality of Žitoradja).



**Figure 4.** Functional type of settlement, 2002. (Republican Statistical Office of Serbia, 2002).

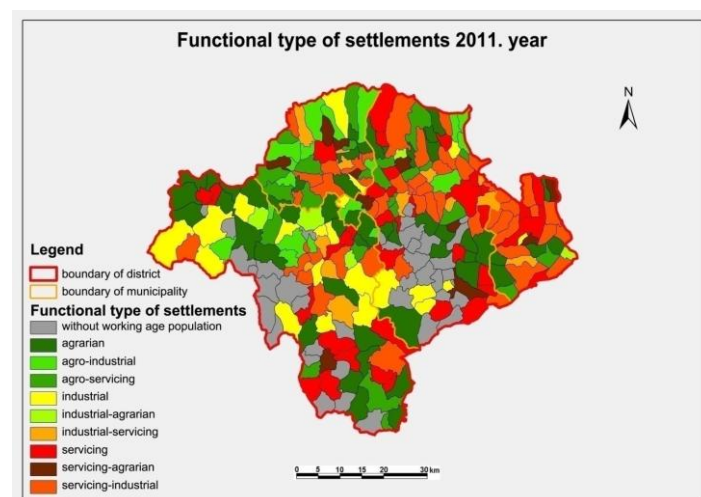
Demographic trends in Toplica are indicated by the fact that in 2002. there were 6 settlements without population performing occupation. Such settlements are mainly distributed along the administrative border with Kosovo and Metohija. Deagrarianization process has continued during the period 2002-2011. to such an extent that more settlements with predominantly service function (98 or 36.7% - services 37, 11 and 50 clerical agrarian-industrial service), but the village with predominantly agrarian function (92 or 34.5% - 50 agricultural, 19 agro-industrial and agrarian-23 service).

There is an evident increase in the number of industrial settlements, but the decline of the settlements of mixed-function type with periodical participation of the industrial sector. However, what is most disturbing is the increase in the number of settlements without population performing occupation. Such settlements in 2011. as many as 38 or 14% and are located in the higher parts of the municipality of Kuršumlija and Prokuplje.

On the Fig. 5. is noticeable that the municipal centers and the main road continues generators changes and land

reclamation, as well as to form a kind of agglomeration Prokuplje-Žitoradja whose main characteristic domination of tertiary and quaternary activities.

If we look at the demographic and economic trends, it is expected that in the future period should be increasing number of settlements without population performing occupation and to reduce the number of settlements of agrarian function type. Secondary sector activities, primarily wood-processing industry, in the case of expansion could not keep a determined amount of population and the challenges of further diversification of the economy, because the strong industrial sector has caused the development of roadside service activities.



**Figure 5.** Functional type of settlement, 2011. (Republican Statistical Office of Serbia, 2012).

## CONCLUSION

This paper presents a functional typology of Toplica District respectively Prokuplje, Kuršumlija, Blace and Žitoradja. In this area in the second half of the twentieth and early twenty-first century witnessed substantial changes in almost all components of demographic structure, which resulted in the transformation of functional types of settlement, when the predominantly agrarian settlement characteristic of the area of Toplica road went up and mixed noagrarian settlements.

In the district of Toplica, 215 agricultural settlements had agrarian as a dominant or complementary function, of which more than half had a pure agrarian function. Settlements are located on the southern slopes of Jastrebac, in the north of the county, then the village located on the slopes of Kopaonik and Radan mountain and village on the slopes Pasjače and Vidojevica are not affected by functional transformation. Character of these settlements is still agrarian, with agriculture only source of income. Agrarian functions are dominant in the mountainous areas of the municipality. In fact, this mountain village are not holders of agricultural production, already the

declared population as a farm used as a parameter in determining the functional typology of the village, according to the above-mentioned methodology. The analysis of the network of settlements and the functional characteristics of the village, there is a phenomenon of rural settlements, the so-called secondary centers where they developed the tool much more than in their environment.

The development of this area should be carried out exactly according to the model microdeveloping nucleus, and these secondary centers. To achieve functional transformation of rural settlements, it is necessary to develop some functions in some of them which it will exert influence on their environment and to encourage its development. New industries should be located according to space and the population potentials, and not displace existing drives industry. It is very important to connect them with high-quality network of roads, among themselves as well with the municipal center. These secondary centers need to be evenly chosen and transferred to some of the functions and activities (administrative, educational, health), which would allow a public affirmation of the population and kept him in the country. Thus, to achieve the qualitative transformation space.

## REFERENCES

- Boudeville, J. 1966. Problems of Regional Planning. Edinburgh: Edinburgh University Press.
- Christaller, W. 1933. Die Zentraleen Orte in Südeuschland. Eine ökonomischegeographische Untersuchung über die Gestmässigkeit der Verbreitung and Entwicklung der Siedlungen mit städtischen Functionen. Darmstadt. Doktor-Disertation (Gustav Fischer Jena). Reprinted 1980.
- Davies, E.K.D. 1967. Centrality and the Central Place Hierarchy. Urban Studies, 4.
- Dragojlović, J. 2017. The nodal centres of Toplica district. Belgrade: University of Belgrade - Faculty of Geography. Doctoral dissertation.
- Federal Statistical Yugoslavia (SZS). 1984. Census, households and apartments in 1981. Belgrade. Table 194 and 195.
- Grčić, M. 1999. Functional classification of the settlements Mačve, Šabačke Posavine i Pocerine. The messenger of the Serbian geographical society, notebook 1.
- Green, N. 2004. General Functional Polycentricity: A definition. Polinet-Working Paper.
- Matijević, D. 2009. Spatial functional connection of the settlements Municipalities Stara Pazova with the urban sistem of Belgrade. Belgrade: Geographical institute Jovan Cvijic.
- Myrdal, G. 1957. Economic Theory and Underdeveloped Regions. London: University Paperbacks, Methuen., pp. 12-13.
- Rudić, V. 1978. Population of Toplica. Belgrade: Belgrade Ethnographic Institute of SANU. Special edition, book 17.
- Republican Statistical Office of Serbia (RZS). 2002. Census, households and dwellings in the Republic of Serbia in 2002. Belgrade. 3.
- Republican Statistical Office of Serbia (RZS). 2012. Census of Population, Households and Dwellings in the Republic of Serbia, 2011. Belgrade. 11.
- Spatial plan of the Municipality of Blace.
- Spatial Plan of the Municipality of Kuršumlija.
- Spatial Plan of the Municipality of Prokuplje.
- Spatial Plan of the Municipality of Žitordja.
- Stamenković, S., & Bačević, M. 1992. Geography of the settlements, University textbook. Belgrade: University of Belgrade - Faculty of Geography - University of Belgrade - Faculty of Science.
- The regional spatial plan for the territory of Nišava, Toplica and Pirot District.
- Tošić, D. 2012. Principles of regionalization. Belgrade: University of Belgrade - Faculty of Geography.
- Vresk, M. 1990. A town in regional and urban planning. Zagreb: School books.

# ON THE AVAILABILITY VERSUS TRANSMITTED POWER OF WIRELESS SENSOR NETWORKS IN RAYLEIGH MULTIPATH FADING ENVIRONMENTS

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## ABSTRACT

**This paper presents simulation results for full availability of a wireless sensor network depending on the RF output power of the network nodes in Rayleigh multipath fading environment. Outage probability, relating to the situation in which any one of the sensors is unavailable, is computed by means of numerical simulation, and discussed. The results indicate that the trade-off between the outage probability and the energy efficiency of the network nodes should be sought, resulting in lowest required output power for the highest tolerable outage threshold.**

**Keywords:** Wireless sensor networks, Connectivity, Multipath fading, Covering range.

## INTRODUCTION

Wireless sensor networks (WSN), based on collaborative large number of sensor nodes, primarily handle with the information sensing, information processing and then forwarding to the final observer or a base station, (Li et al., 2009). Because of the possibilities that WSNs provide in wireless monitoring and controlling, their use in broadband communications has increased in recent years, (Li et al., 2009; Akyildiz et al., 2002; Patnaik et al., 2015). Today they are widely accepted as vital technology that enhance electrical power systems i.e. the smart grid, etc. (Gungor et al., 2010). Changes in the topology of the sensors' environment lead to changes in sensors' power allocation. In addition, the performance of WSNs is subjected to wireless channel characteristics, namely - multipath fading and shadowing phenomena.

According to gained experimental data, fading environment of WSN can be described as Rayleigh or close to Rayleigh fading. Relying on this, numerous published works have dealt with Rayleigh fading as a main disturbance in WSN leading to signal strength attenuation, (Bergamo & Mazzini, 2002; Puccinelli & Haenggi, 2006; Haenggi, 2003; Ren et al., 2011; Kumar & Lobiyal, 2013; Olofsson et al., 2016). The authors in Bergamo & Mazzini, (2002) have focused on optimal sensor localization in WSN, taking into account Rayleigh fading influence and possible sensor mobility. They proposed a simple schemes with low energy and computation complexity cost. In Puccinelli & Haenggi, (2006), the unreliability of sensor network caused by fading phenomena is discussed and adequate measurements and result interpretations are given.

The lifetime of sensors is one of primary concerns in WSN since the sensed data should be delivered to a single sink or specified destination in a certain time limit. According to this, five strategies that balance energy consumption in reaching required

sensors' lifetime in fading environment were proposed in Haenggi, (2003).

In addition to fading presence as a dominant nuisance in wireless networks, the shadowing phenomena influence on system performance degradation was considered in Ren et al., (2011); Kumar & Lobiyal, (2013). As a matter of fact, the impact of Rayleigh fading, path loss and shadowing on error performance was analyzed in Ren et al., (2011). Sensing coverage problem was discussed in Kumar & Lobiyal, (2013) relating to solutions with Elfes or shadowing sensing model. Furthermore, to define bit error rate as well as packet error rate of specific system, novel adaptive fading statistics in industrial sensor networks were proposed in Olofsson et al., (2016). Simulation model that characterize the wireless channel in industrial environments under Rician fading, was proposed in Gomes et al., (2017).

In this paper, we analyze full availability of a WSNs versus output power of sensors over Rayleigh fading channel. We assume log-distance path loss model in describing signal strength attenuation from node to node communication. Under given circumstances, the outage probability is defined as probability that any one of sensor nodes is unavailable in communication, and appropriate numerical results are given. In order to confirm presented analysis, simulation results are also presented.

The paper is structured as follows. In section 2, specific system model is presented and graphical illustration of coverage area with randomly placed sensors is given. This section also contains description of fading and path-loss model. Simulation setup is explained in section 3, and furthermore numerical and simulation results with appropriate discussion are shown in section 4. The paper is closed by main concluding remarks given in section 5.

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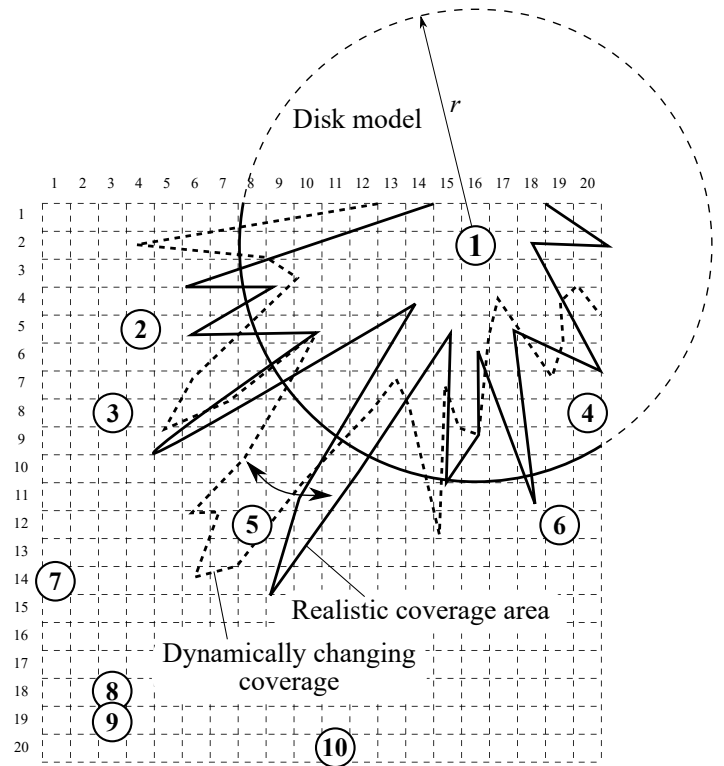
## SYSTEM MODEL

The system model assumes that a number of sensors are distributed randomly over a square closed space. Square geometry represents the simplest of cases, but it does not make the generality of the model decrease. In fact, it is relatively simple to modify the simulation model in order to include other geometries. Each sensor node represents a transceiver that incorporates transmitter and receiver blocks. It is assumed that the channels in nodes communication are orthogonal, i.e. that other nodes do not interfere with node-to-node communication. This can be achieved in a number of ways, for example - by using precise time synchronization and scheduling such that each sensor is assigned a time slot in which it transmits its data. Outside of the assigned time slot, the sensor receives the data from other sensors that are within its range and tries to decode and store their messages. If needed, it can resend, or relay, other sensors data so that in perspective, each sensor can communicate with every other sensor, whether directly, or through a number of relays. The sensor nodes acting as relays in this scenario have the role of so-called decode-and-forward (DF) relays, because they fully decode and then repeat the message to other nodes.

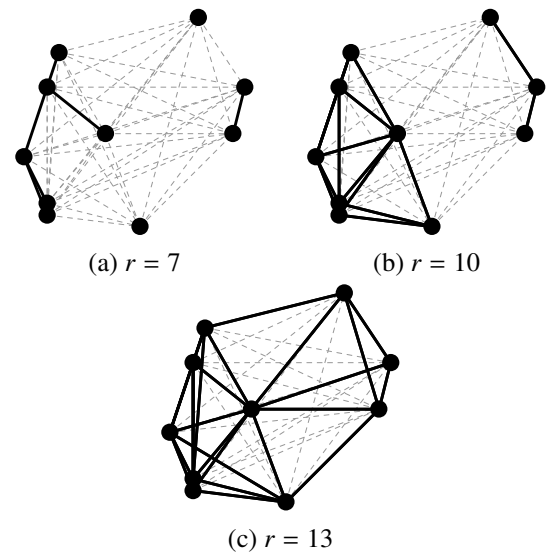
In any case, the network can certainly function as completely connected network, or the full availability network. This term denotes a network in which all the nodes can be reached through any one of the nodes, and the data from each sensor can be collected from a single point. In contrast, there might be networks that can not interconnect in such a complete way. For example, one can have two separate distant groups of sensors that are not able to interconnect with one another. In a sense, this can be viewed as a single sensor network with two fully connected clusters. Each cluster exhibits full availability on its own, but is unaware of the existence of the other cluster. Of course, this concept can be extended to multiple clusters in general.

One of the possible realizations of random sensor placement is shown in Fig. 1. This example shows ten sensors placed randomly in the matrix of dimensions  $20 \times 20$ , representing the square closed-space. The sensor nodes are numbered, so each sensors has its own unique identification number, or address, through which its messages can be identified. It is obvious that the distribution of sensors is not uniform in the space, and there are clusters in which the sensors are closer to each other. Each sensor coverage can be represented by a number of ways, for example - disk model assumes that each sensor can communicate with other sensors that are inside the disk centered at the sensor node, and with fixed radius proportional to sensor output power. However, this model is in many cases too simplistic to accurately describe the performance of sensor networks.

More realistic model takes into account complex propagation effects of the radio waves. Usually, the signal rarely propagates directly towards the destination node. In practice, emitted radio waves spread into all directions and reflect of objects in the closed space. Therefore, at the destination, we have a number



**Figure 1.** A realization of 10 random sensors distribution in  $20 \times 20$  matrix, and illustration of coverage models.



**Figure 2.** Illustration of sensors connectivity using simple disk coverage model with indicated diameter  $r$

of radio waves coming from different directions, reflected from a number of objects, and converging onto the receiver antenna. Each of the waves have different phase, according to its path, and all the radio waves with different phases and amplitudes interfere with one another, producing the signal level at the receiver antenna. Although it is possible to accurately simulate the realistic effects of such propagation, it is a very complex task. Instead, the propagation effects are modeled on the basis of statistical description, which is in turn based on the series of precise signal level mea-

measurements. Realistic node coverage area has more or less irregular shape, as illustrated in Fig. 1. Also, as conditions change over time, due to differences in temperatures, presence or absence of people, or slight changes in geometry and positions of surrounding objects, coverage area changes its shape dynamically. In such conditions, stochastic fading models are well-suited for performance analysis of wireless networks.

One of the most commonly used multipath fading models is the Rayleigh model. It assumes that the received signal envelope  $R$  over the wireless channel has Rayleigh probability density function (PDF), (Panić et al., 2013):

$$p_R(r) = \frac{2r}{\Omega} e^{-r^2/\Omega}, \quad (1)$$

where  $\Omega$  is the mean-square value of the envelope. The Rayleigh model has been used extensively in research literature to model the fading channels in wireless communications, including the sensor networks. It is considered the simplest of fading channel models, yet its potentials are not yet exhausted as the sensor networks represent complex systems, and the required computational complexity for the performance analysis is not neglectable.

Each sensor tries to demodulate the received signal according to the modulation format used. To facilitate demodulation, each transmitted packet of data has a pre-determined structure. Usually, it starts with a pre-amble, which is a fixed length sequence of alternating bits. Receiver uses the pre-amble to set up the time synchronization circuits and estimate the signal level, and therefore estimate the signal-to-noise ratio (SNR). Most of the noise in the channel originates from the thermal noise in the pre-amplifier circuitry of the receiver. Therefore, at the constant temperature, the noise power spectrum density is also constant. Accordingly, a lot of wireless integrated circuits include a digital thermometer, and the receiver is able to estimate the noise levels based on the temperature and receiver bandwidth. During the pre-amble period, receiver estimates the Received Signal Strength Indicator (RSSI) and SNR values. If the SNR is above the pre-determined theoretical threshold value, the receiver concludes that it is able to successfully decode the message, otherwise the rest of the message is ignored.

Following the pre-amble, there is a synchro-sequence, used to delimit the pre-amble from the useful data in a synchronous way. Address sequence identifying the transmitting sensor node follows next, accompanied by the data message. Finally, the message ends with cyclic-redundancy check (CRC) byte enabling transfer error detection. The message can be forward-error correction (FEC) coded, allowing error correction in addition to detection of errors, but this is usually optional.

After this review of the packet format, we continue with the appropriate simple model of receiver: if the SNR is above the threshold, the receiver is able to decode the message with very high probability of success. Otherwise, successful decoding can not be expected and the receiving node does not attempt to do so.

Packer-error rate is then:

$$\text{PER} = \begin{cases} 0, & \text{SNR} > t_h \\ 1, & \text{SNR} \leq t_h \end{cases} \quad (2)$$

Therefore, the threshold is very significant parameter for the sensors, and it defines the sensitivity of the nodes. If we assume that all the nodes transmit at the same pre-determined signal level, the node coverage is determined by the sensitivity threshold. If the SNR at the receiver is lower than the threshold, we say that the receiver experiences signal outage.

It is of vital interest to maintain the signal levels above the outage threshold. However, this can always be guaranteed if the nodes transmit at excessively high power. Of course this is not practical, and the nodes usually transmit at levels just above the required level, including a safety margin. Subsequently, outage-free operation can not be guaranteed, because the signals exhibit dynamic behavior, as discussed earlier. Instead of strict guarantee, probabilistic measures are used and the probability of outage is kept under the pre-defined low percentage value.

Another important parameter in considering the node coverage is the path loss. Obviously, the signal strength decreases with increasing distance from the transmitter, and this decrease - or loss, should also be included into the model, regardless that the distances are relatively small. There are multiple propagation models that take path loss into account differently, but ultimately the models are based on experience and analysis of large measurement datasets. For the sake of simplicity of formulation, we use the log-distance path loss model for indoor environments. When considering relevant parameters for the specific system model, we get the following form the path loss  $P_L$  at distance  $d$  from the transmitter (Ren et al., 2011):

$$P_L[\text{dB}] = P_{L_0} + 10\gamma \log_{10} \frac{d}{d_0}, \quad (3)$$

where the distance  $d_0$  is the reference distance (usually 1 m),  $P_{L_0}$  is the loss over reference distance, and  $\gamma$  represents the path loss exponent. The exponent  $\gamma$  can have different values, depending on the characteristics of the propagation environment, but is usually in the range of 2.0 – 3.0 for the closed-space office environments. We adopt the value of 2.5 for further simulations. Therefore, signal power  $P_{rx}$  at the receiver is:

$$P_{rx} = P_0 \left( \frac{d_0}{d} \right)^\gamma, \quad (4)$$

where the power  $P_0$  at close distance  $d_0$  is a constant fraction of the total output power  $P_0 = \alpha P_{tx}$ .

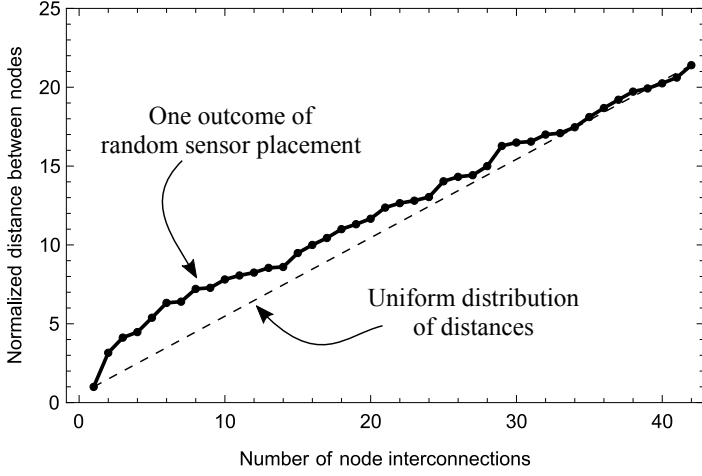
## SIMULATION SETUP

Based on the model from the previous section, we set up a simulation to numerically evaluate the availability of the network. In first step of the simulation, it is necessary to calculate mutual distances between the sensors. As the number of sensors

if  $N = 10$ , it is obvious that there are 45 interconnections in total, or  $N(N - 1)/2$ . For the specific case we use the node distribution shown in Fig. 1. Euclidean distances between nodes:

$$d_{i,j} = \left( (x_i - x_j)^2 + (y_i - y_j)^2 \right)^{1/2}, \quad (5)$$

are calculated, arranged in increasing order, and shown in Fig. 3.

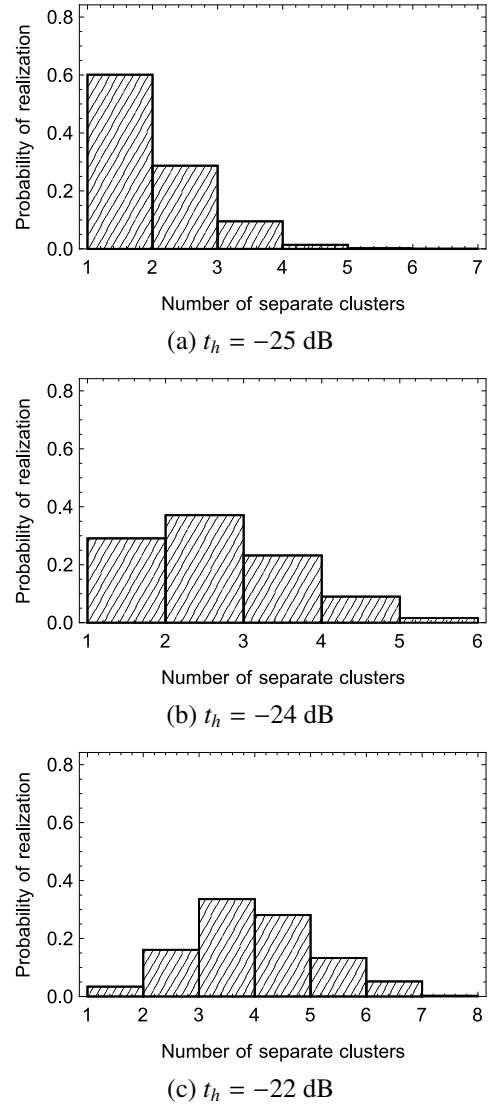


**Figure 3.** Distribution of internode distances for the sensor distribution in Fig. 1.

We further assume that all nodes transmit at the same output power. According to path-loss model, mean signal power that from one node reaches the other node is given by (4). Due to multipath fading, signal amplitude can vary according to (1). Since signal power relates to amplitude  $R$  as:  $P_{rx} = R^2/2$ , we need to use  $\Omega = P_{rx}/2$ . Single outcome of multipath fading affects specific link, or interconnection between two nodes. According to (2), if the resulting signal power at the receiver is above the threshold, we consider the link to be functional, otherwise the particular link is down.

When the procedure of simulating the individual links is repeated for each of the 45 interconnections, the links are divided into groups of passed and failed, and the clusters of interconnected nodes are identified. If the total number of clusters in the network is exactly one, then the network is fully available. In other cases, we have more than one separate clusters that can not communicate with each other. Therefore, the number of clusters is also a random variable and can have different values for the specific fading realizations. With sensor placement as in Fig. 1, and for a number of 1000 simulation runs, we have computed the number of clusters for different outage threshold values, and the results are shown in Fig. 4. Results indicate that the low threshold values would result mostly in a single cluster, while higher thresholds favor multiple clusters. Therefore, it is obvious that the receiver sensitivity should be as high as possible, allowing lower outage thresholds and thus higher availability of nodes. In this point of view, threshold corresponds reciprocally to receiver sensitivity. The higher the sensitivity of the receiver is, the lower is the outage threshold value.

The results can also be interpreted based on the output power of nodes. If we assume that the receiver sensitivity is fixed, and is

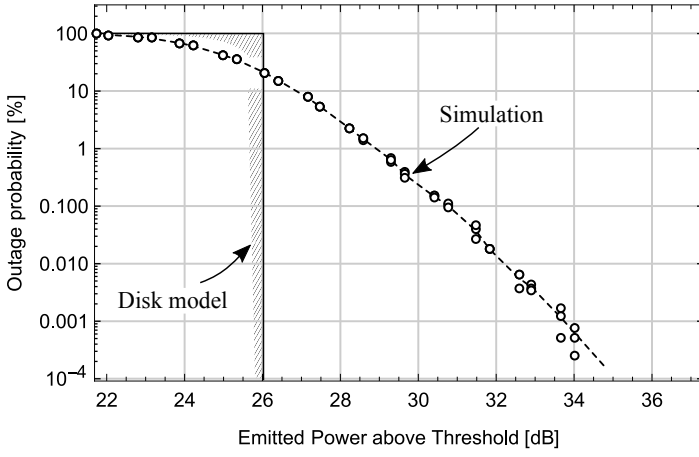


**Figure 4.** Example of cluster distribution for different outage thresholds

not subject to changes, than the output power should be increased to allow higher signal levels at reception. From this point of view, the outage threshold is fixed, and the values of  $t_h$  indicate the relative position of threshold in comparison to node output power. In other words, absolute values of  $t_h$  indicate the output power level over the outage threshold value.

## NUMERICAL RESULTS AND DISCUSSION

Simulations run until the total number of iterations is reached, or the number of positive outcomes is higher than 20. We take the fully available network as a positive outcome. In this way, the unnecessary usage of computer resources is avoided for the cases where percentage of positive outcomes is relatively high, i.e. there is no need to continue simulations when the result already reaches certain degree of precision. On the other hand, for the rare occurrences of positive outcomes, the maximum number of simulations will be run without reaching the requested precision.



**Figure 5.** Outage probability percentage versus node output power

We denote the ratio of emitted power to the outage threshold as:

$$\xi = \frac{P_0}{t_h} \quad (6)$$

Power threshold at the receiver is taken to be the limit of the node sensitivity  $t_h = P_x$ , but can also be set above that value as a safety margin. Therefore,  $\xi$  represents the portion of emitted power level that is above the receiver sensitivity limit. According to (4) and the disk model, we get:

$$\xi[\text{dB}] = 10\gamma \log_{10} \frac{d_{(2 \rightarrow 1)}}{d_0}, \quad (7)$$

From the Figs (1) and (2), we conclude that the crossing point between the two clusters and fully available network occurs when the disk radius reaches  $d_{(2 \rightarrow 1)} = 11$ , connecting the node 6 to nodes 5 and 10. Therefore, (7) predicts that the emitted power should be at least 26.03 dB above the node sensitivity in order to form the network with full availability. Furthermore, according to the model there is a sharp transition step between the two clusters and fully available network, and the network is perfectly functional after crossing this transition.

However, propagation model that takes into account the Rayleigh fading predicts more detailed results than the deterministic disk model. Main difference in predictions is that there is not an abrupt transition between the multiple clusters and full availability. Instead, the stochastic model exhibits smooth transition. The model uses continuous outage probability as a measure of network availability, which is the probability that the network will not be fully available. Thus, the network is better connected when the outage probability is lower.

When the transmitted power is 26 dB over the node sensitivity, the outage probability reads at about 20%, meaning that the network may not be available for 20% of the time. If we need this outage probability to be much lower, for example: 1%, we need higher output power of 29 dB, which is exactly double the required value we got for the disk model. From the data we estimate that after the break point, increase in power by factor 1.65, i.e. power increase by 65% leads to ten-fold reduction in outage probability.

For the break point we take the output power for which the outage probability is 50%, and for the specific case this value is 24.6 dB.

For other particular distributions of sensor nodes, one should expect slightly different break points, and also somewhat different slopes of the outage probability curves, although the general shape of the curves will be similar. This is due to the fact that the maximal cluster distance is limited by the closed-space geometry and the density of sensor nodes. Disk model can serve as a rough estimate of the break point, providing that a critical link connecting two clusters into a single cluster network can be identified.

## CONCLUSION

We have set up a simulation model that is capable of describing the connectivity of wireless sensor networks in indoor spaces with multipath Rayleigh fading. Simulation uses log-distance model to account for average signal decay with distance from network node. Although simple in its assumptions, simulation model is capable of giving insight into complex interplay between the network nodes in a dynamical environment. The numerical results obtained are focused on the full availability of the network, and the transmitter power that is required to attain it. Results of simulation are encouraging, and more efforts should be invested into deriving the analytical model to match the results. Outage probability versus the transmitter power curve can be obtained by means of simulation for any particular distribution of sensors. The results indicate that the trade-of between the outage probability and the energy efficiency of the network nodes should be sought, resulting in lowest required output power for the highest tolerable outage threshold.

## ACKNOWLEDGEMENT

D. Milić and J. Anastasov were supported in part by the Ministry of Science of Republic of Serbia under grant III44006.

## REFERENCES

- Akyildiz, I. F., Su, W., Sankarasubramaniam, Y., & Cayirci, E. 2002. Wireless sensor networks: A survey. *Computer networks*, 38(4), pp. 393–422.
- Bergamo, P. & Mazzini, G. 2002. Localization in sensor networks with fading and mobility. In: *The 13th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, IEEE, pp. 750–754 vol. 2.
- Gomes, R. D., Queiroz, D. V., Fonseca, I. E., & Alencar, M. S. 2017. A simulation model for industrial multi-channel wireless sensor networks. *Journal of Communication and Information Systems*, 32(1).
- Gungor, V. C., Lu, B., & Hancke, G. P. 2010. Opportunities and challenges of wireless sensor networks in smart grid. *IEEE transactions on industrial electronics*, 57(10), pp. 3557–3564.
- Haenggi, M. 2003. Energy-balancing strategies for wireless sensor networks. In: *Proceedings of the International Symposium Circuits and Systems ISCAS'03*, IEEE, pp. 4–4 vol. 4.

- Kumar, S. & Lobiyal, D. 2013. Sensing coverage prediction for wireless sensor networks in shadowed and multipath environment. *The Scientific World Journal*, 2013.
- Li, J., Andrew, L. L., Foh, C. H., Zukerman, M., & Chen, H.-H. 2009. Connectivity, coverage and placement in wireless sensor networks. *Sensors*, 9(10), pp. 7664–7693. Pmid:22408474
- Olofsson, T., Ahlén, A., & Gidlund, M. 2016. Modeling of the fading statistics of wireless sensor network channels in industrial environments. *IEEE Transactions on Signal Processing*, 64(12), pp. 3021–3034.
- Panić, S., Stefanović, M., Anastasov, J., & Spalević, P. 2013. Fading and interference mitigation in wireless communications. CRC Press.
- Patnaik, S., Li, X., & Yang, Y.-M. 2015. Recent Development in Wireless Sensor and Ad-hoc Networks. Springer.
- Puccinelli, D. & Haenggi, M. 2006. Multipath fading in wireless sensor networks: Measurements and interpretation. In: *Proceedings of the 2006 international conference on Wireless communications and mobile computing*, ACM, pp. 1039–1044 .
- Ren, Z., Wang, G., Chen, Q., & Li, H. 2011. Modelling and simulation of Rayleigh fading, path loss, and shadowing fading for wireless mobile networks. *Simulation Modelling Practice and Theory*, 19(2), pp. 626–637.

# INFORMATION AND COMMUNICATIONS TECHNOLOGIES IN FUNCTION OF TEACHING PROCESS

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## ABSTRACT

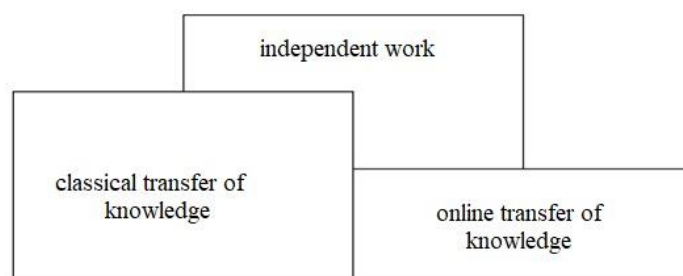
This paper, based on the extensive research of relevant professional literature, will present and elaborate possible aspects of influence which information-communications technologies (ICT) have on educational process. Contemporary information-communications technologies and their use in teaching process resulted in numerous enhancements of teaching process altering traditional methods of learning and teaching. With the aim of confirming theoretical postulates researches were conducted on candidates with specific educational degrees in various forms of traditional teaching and teaching with educational software. The survey was conducted on a control and experimental group, on a sample of 1542 candidates. The candidates were observed in a period of approximately five months. The results of this research indicate that the candidates taught by use of educational software have shown far greater level of achieved knowledge and significantly higher motivation for learning. The participants of the experimental group actively participated in the acquisition of their competences. The contribution of this paper is in the new methodological approach to the organization of education, which significantly improves the teaching process, its goals and outcomes.

**Keywords:** Education, teaching, educational software.

## INTRODUCTION

For many years the process of teaching was considered as the knowledge transfer process and the learning process as the process of receiving the aforementioned. The development of information and communications technologies and their use in teaching process resulted in fundamental role change in both teachers and students. This is exactly why the method of transfer and adopting knowledge changes continually. Rapid development of internet and contemporary sophisticated technologies is the main driving force and initiator of these changes. In relevant professional literature eminent authors use different determinants for concepts of use of educational software in education such as: distance learning, e-education, virtual education, e-learning, online learning (Moore et al., 2011). Based on their researches, the aforementioned authors concluded that almost all definitions of distance learning are similar in being a model of learning which includes two or more people, students and teachers, in different time intervals, locations and forms. Some authors, e.g. Nichols, (2003), define e-learning as a strictly technological tool based on web technologies, online transfer or online options. However, authors such as Benson, (2004), provided a clearer definition stating that online learning is a new and improved form of distance learning. Therefore, according to particular authors, the combination of traditional learning and distance learning deems most appropriate for majority of target groups (Bose, 2003). With regard the

utilization, authors (Georgouli et al., 2008) pointed out that the combined learning should be used to explain learning on several different ways and methods of learning as presented in figure 1.



**Figure 1.** Combine learning.

Source: Georgouli et al., A Framework for Adopting LMS methods - to introduce e-Learning in traditional, 2008, str. 227.

The object and purpose of this research is to examine the applicability of educational software and how their application affects the motivation and active role of students in acquiring knowledge, as well as the quality of the acquired knowledge. The hypotheses of this research are:

- the application of educational software in the learning process motivates students to participate actively in acquiring their own competencies.
- knowledge gained through learning through educational software is more applicable and more extensive.

The thorough analysis of the application of educational software in the immediate teaching process, conducted during this research, is a guideline for the development of new

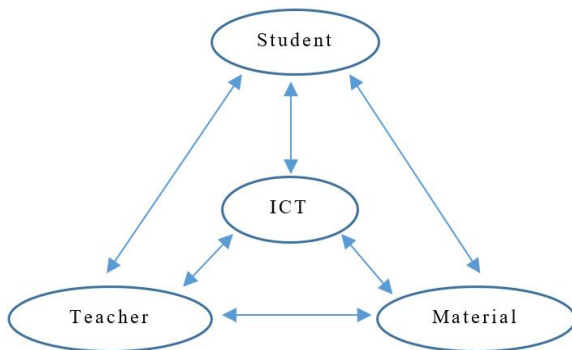
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methodological approaches and the improvement of teaching methodology.

## CONCEPT OF TEACHING WITH USE OF ICT

In contemporary use of teaching process information and communications technologies represent, strictly speaking, unavoidable teaching asset as a support to teachers in traditional ways of learning and, broadly speaking, they represent a new methodological approach through various ways of realization of learning and teaching. "Multimedia teaching and learning according to eminent authors provide possibility for successful gaining of knowledge to all students in the class" (Chiou, 2008).

ICT implemented in teaching process as support to traditional ways of learning does not alter educational paradigm founded on transfer of knowledge from teacher to student, where teacher possesses knowledge and student is a passive receiver of knowledge. Such gained knowledge is, mostly, reduced to the model of reproduction of knowledge. The relationship between student, teacher and curriculum can be represented with the didactic triangle in figure 2. Element knowledge was omitted from this triangle because it is very difficult to determine quantity and quality of gained knowledge during the teaching process itself, when this type of teaching is conducted. Teaching is not adjusted to an individual but follows previously determined set of actions.

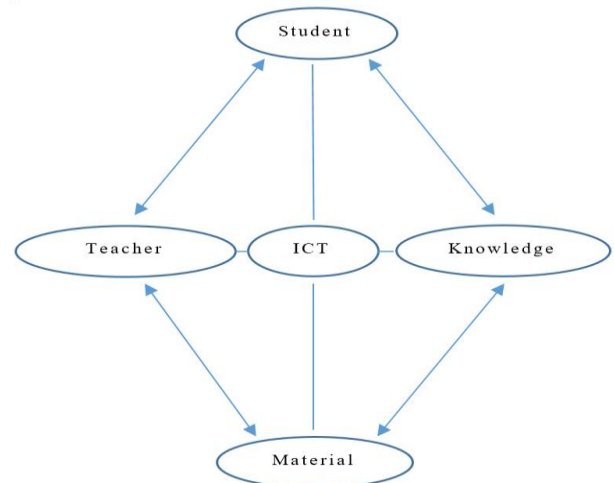


**Figure 2.** Didactic triangle.

New teaching methods of education with implementation of ICT imply an active student's role in his/her own education, and put the teacher in a position of professional adviser and coordinator of learning process. The model of reproduction of knowledge transforms into the model of understanding, upgrading and exploring new knowledge. The learning process is implemented with the utilization of various educational software and online platforms which require individual involvement of each student at his/her own pace. This type of gaining knowledge is possible with immediate mastering of teaching curriculum required for class continuation and adopting of new knowledge. The relationship between the student, teacher and curriculum is

enhanced with the knowledge component and can be represented with didactic square in figure 3.

This paper includes a comparison of these two methods of teaching with regard the aspects of motivation and adopted knowledge.



**Figure 3.** Didactic square.

## RESEARCH DESCRIPTION

ECDL training for less employable groups was conducted within IPA 2020 project for Serbia. Candidates of younger age with III and IV degree of professional qualifications were selected within this program and, among other things, trained for work in Word and Excel. Training was conducted with the implementation of ECDL online educational software, and lecturer's presence. Trainees learned individually on ECDL online platform, with the use of educational software, while lecturer doubled as both mentor and coordinator of this process. This research included several municipalities of Raska district and an experimental group was founded in this way.

The second (control) group was consisted of the first year Higher Education Institutions' students from the same area, who learned to use Word and Excel during their first semester of Computer Science and Informatics class. The teaching process was traditional with the use of MS Office program packages (MS Word and MS Excel).

A computer lab was required for both groups in order for teaching to be adequate. The experimental group demanded free access to internet network, while the control group needed the required MS Office program package. Both groups had approximately the same number of classes during the implementation of these two modules.

The conducted research included method of analyses and synthesis and comparative method aiming at theoretical conclusions, also a poll and testing were used as a method for gathering data. Accumulated data were statistically processed in MS Excel program.

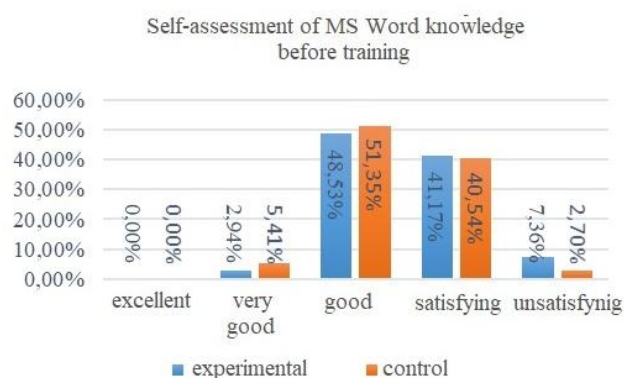
## RESULTS AND DISCUSSION

The implementation of ITC in teaching process has, undeniably, brought much news. The results of this research, among many others, pointed out several characteristics that make learning with the use of educational software different than traditional learning with ITC support.

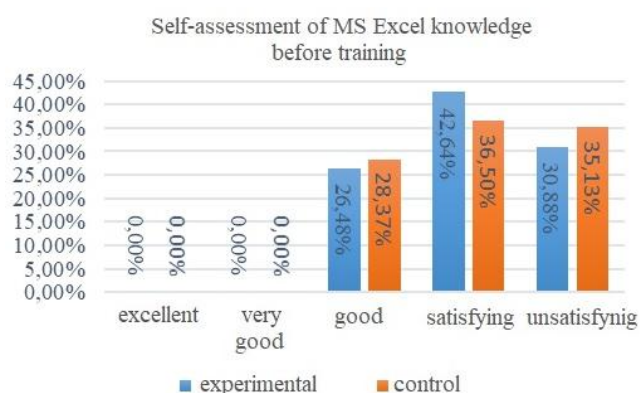
Two groups were formed for the needs of this research:

- control group – consisted of 74 first year Higher Education Institutions' students
- experimental group – consisted of 68 ECDL trainees

Members of both groups have, prior to training, assessed their knowledge of MS Word text processing program and MS Excel table calculation program. According to figures 4 and 5, the level of previous knowledge, prior to training, was similar in both groups. Also, it was apparent that the group members assessed their knowledge of MS Word significantly better than MS Excel.

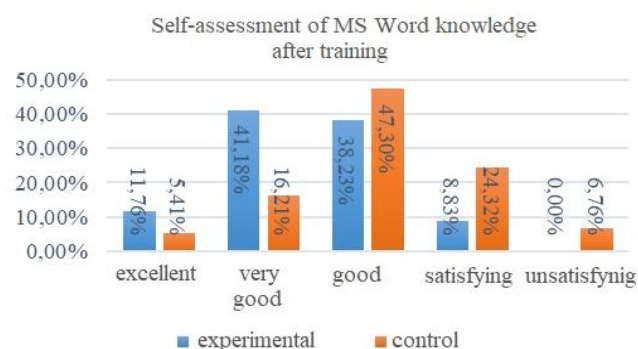


**Figure 4.** Self-assessment of MS Word knowledge before training.

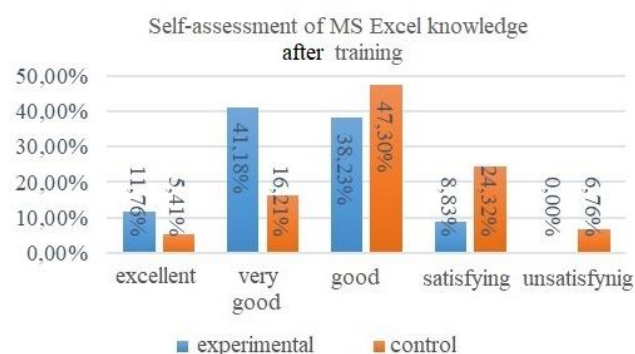


**Figure 5.** Self-assessment of MS Excel knowledge before training.

After training, the candidates re-assessed their knowledge. The results greatly differ, as shown on figures 6 and 7.



**Figure 6.** Self-assessment of MS Word knowledge after training



**Figure 7.** Self-assessment of MS Excel knowledge after training.

Based on the self-assessment of attendees from both groups, it is evident that members of the experimental group assessed their knowledge, upon completion of training, as significantly better than members of the control group.

In addition to having the group members self-assess their knowledge, the level of their knowledge was checked by expert teachers and lecturers. The results of this control are presented in table format, for each particular module – Table 1 and Table 2, and summed up for entire training program – Table 3. Members of the control group were assessed on 1-100 scale; with basic level is from 51-65, medium level from 66-85 and higher level from 86-100. Assessment scale for members of the experimental group was from 1-36 points. Basic level was attained with 27-29 points, medium from 30-36 and higher from 34-36 points.

**Table 1.** Expert's assessment for results in Word.

MS Word	Did not pass	Basic level	Medium level	Higher level
Control group	8	38	21	7
Experimental group	0	8	41	19

Results concerning answers to several important questions are shown in figures 8, 9 and 10. Those results stress a major difference between teaching in control and experimental group, i.e. key differences between traditional teaching and teaching with the use of educational software can be seen.

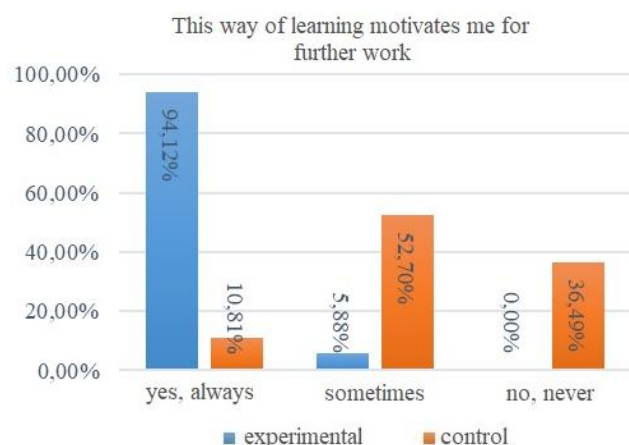
**Table 2.** Expert's assessment for results in Excel.

MS Excel	Did not pass	Basic level	Medium level	Higher level
Control group	11	34	24	5
Experimental group	2	13	37	16

**Table 3.** Expert's assessment for results – in total.

Total	Did not pass	Basic level	Medium level	Higher level
Control group	12	33	24	5
Experimental group	2	11	39	16

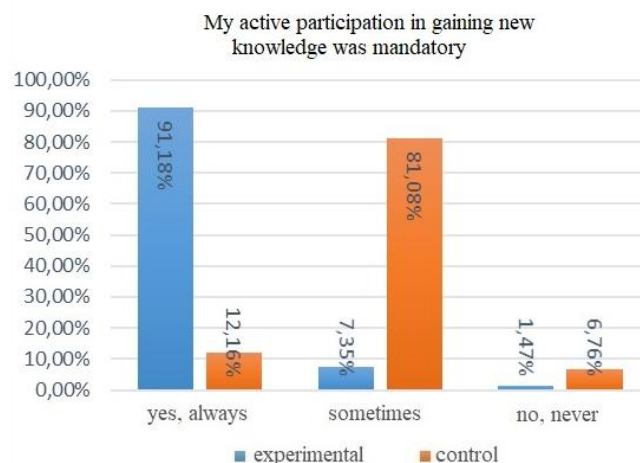
It can also be seen that members of the experimental group, learning with the use of educational software, were significantly more motivated for work than members of the control group learning traditionally with the use of ICT.



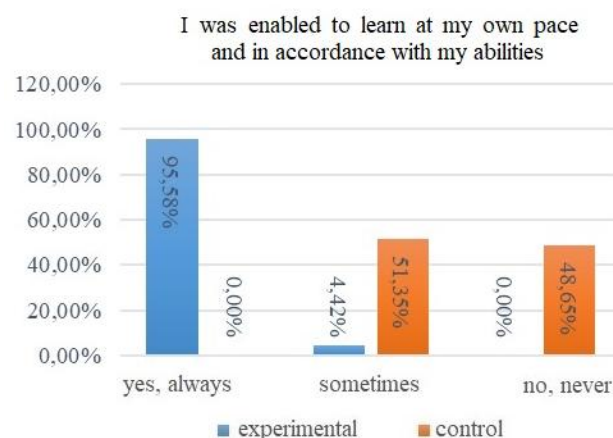
**Figure 8.** Motivation.

It becomes apparent that the teaching organized with the use of educational software demands major active participation of each individual and enables attendees to learn at their own pace and in accord with their own abilities. Muller et al., (2006) also confirmed these postulates pointing out that implementation

of ITC, i.e. educational software, in teaching, provides opportunity for each pupil-student to process content and make progress in accordance with his/her own individual affinities and abilities (Muller et al., 2006). The fact, confirmed with this research, that the quality of knowledge adopted with the help of educational software is much better could be added to the existing eminent literature. Namely, the educational software arouses the research spirit and requires of students to use previously acquired knowledge while acquiring new one at all times.



**Figure 9.** Active teaching.



**Figure 10.** Individual teaching.

Members of both groups stated that they were able to gain new knowledge regardless of their foreknowledge, and that interlinking of obtained knowledge was mandatory in order to receive additional knowledge. Members of control group stated that teacher's assistance was mandatory for gaining new knowledge, while members of experimental group stressed they were able to obtain new knowledge without teacher's key assistance, with the use of educational software.

Concerning the quality of gained knowledge, the difference between groups is evident. With regard MS Word, the experimental group achieved better results, with 60 members

reaching medium and higher level; whilst 59 attendees of control group reached basic and medium level. With regard MS Excel, the results are almost the same: 58 attendees in control group reached basic and medium level and 63 attendees of experimental group reached medium and higher level. Finally, summarizing both modules results in the following: the level of gained knowledge in control group is focused on basic level with 44,59%, while medium level was reached by 32,43%. The highest level was reached by 6,76% only. Medium and higher level was reached by 80,88% in experimental group, wherein 57,35% reached medium and 23,53% reached higher level.

The aforementioned results were confirmed with theoretical researches conducted by numerous reputable international authors (Chaney-Cullen & Duffy, 1999; Muller et al., 2006; Forgasz, 2006; Hwanga et al, 2007), pointing out immense, indisputable advantages of utilization of ICT and educational software in teaching and also regarding quality of gained and adopted knowledge, motivation of students for work and improvement of overall teaching process in general. As reasons for insufficient and inadequate implementation of ICT i.e. computers and educational software in teaching; particularly in chemistry, biology and physics classes; teachers stressed inadequate and insufficient providing of related IT equipment to educational institutions – computers, software, access to internet and stable internet connection and the lack of suitable multimedia materials etc.

## CONCLUSION

Results of research presented in this paper point out what should be the goal for contemporary teaching – enabling pupils and students to obtain new information which they will be able to understand and critically analyze by themselves, from different sources of knowledge. Implementing new technologies, ICT and educational software in particular, is highly important for contemporary teaching. With the utilization of ICT everyone can share their data and information with other users that use Information and communications technologies (ICT), i.e. as much as one is IT literate or enabled. In general, it can be said that information literacy represents a set of knowledge and skills that users need to obtain and which can be improved and enhanced from available sources, creating new knowledge, skills and abilities. The results of this research point out, beyond doubt, that implementation of ICT in learning process and teaching requires well and clearly defined goals, and that learning should be less about memorizing facts and data and more about acquiring skills and practically applicable scientific knowledge. Based on theoretical studies and conducted researches in educational institutions in the Republic of Serbia it can be concluded that in order to have a successful implementation of ICT in teaching process, the following activities are mandatory: improving of conditions for work and learning in educational institutions of the Republic of Serbia, which significantly

increases the quality of teaching and learning process; acquiring those skills and knowledge related to contemporary conceptual technological processes. In addition to the aforementioned, professional and methodical education of teaching personnel involved in education of students in the areas of basic ICT knowledge, effects and consequences its implementation has on an individual and society in general, is also required. In accordance with the IT Development Strategy in the Republic of Serbia, it is required to have extensive organization of training which would provide educational workers with appropriate instructions, guidelines and knowledge on fulfillment of conditions and ways to implement ICT integration and educational software in teaching process; furthermore, to stimulate lecturers and educational workers to improve their knowledge and skills, favoring team work and cooperation both within teachers and educators as well as in pupils, i.e. students. In order to overcome difficulties and deficiencies in educational work, transforming educational institutions into research facilities is necessary, wherein pupils/students will not only copy data and information but become active researchers and well trained and sufficiently educated for resolving specific situations and problems. That being said, it is certainly vital to enable exchange of knowledge between educational workers and educators on related use of ICT and educational software in teaching as well as coordinating the needs of national institutions and facilities for teaching personnel which will become fully literate in this manner. Results of researches conducted in neighboring countries undoubtedly indicate that labor market has utmost need for experts armed with knowledge and skills, guaranteeing success in society and that education is a continual process and a category that should last throughout the life cycle through the life-long process of learning implemented more and more in the world.

## REFERENCES

- Benson, P.E. 2004. Online learning: A means to enhance professional development. *Crit Care Nurse*, 24(1), pp. 60-3. PMID:15007894
- Bose, K. 2003. An e-learning experience-a written analysis based on my experience in an e-learning pilot project. *Campus-Wide Information Systems*, 20(5), pp. 193-199.
- Chaney-Cullen, T., & Duffy, M.T. 1999. Strategic Teaching Framework: Multimedia to Support Teacher Change. *The Journal of the Learning Sciences*, 1, pp. 1-40. Taylor & Francis. doi:<http://www.jstor.org/stable/1466703>
- Chiou, C. 2008. The effect of concept mapping on students' learning achievements and interests. *Innovations in Education & Teaching International*, 45(4), pp. 375-387.
- Forgasz, H. 2006. Factors That Encourage Or Inhibit Computer Use For Secondary Mathematics Teaching. *Journal Of Computer Use For Secondary Mathematics Teaching*, 25(1).
- Georgouli, K., Skalkidis, I., & Guerreiro, P. 2008. A Framework for Adopting LMS to Introduce Learning in a

- Traditional. *Educational Technology & Society*, 11(2), pp. 227-240.
- Hwanga, W.Y., Wang, C.Y., & Sharples, M. 2007. A study of multimedia annotation of Webbased materials. *Computers & Education*, 48, pp. 680-699. Retrieved from [www.sciencedirect.com](http://www.sciencedirect.com)
- Moore, J., Dickson-Deane, C., & Galyen, K. 2011. E-Learning, online learning, and distance learning environments: Are they the same? *Internet and Higher Education*, 14, pp. 129-135.
- Muller, D., Eklund, J., & Shanna, M. 2006. The future of multimedia learning. In *Essential Issues for Research*. Australia: University of Sydney NSW.
- Nichols, M. 2003. A theory for eLearning. *Journal of Educational Technology & Society*, 6(2), p. 76.

# RESOLVING SYSTEMS OF NONLINEAR INTEGRAL EQUATIONS VIA C-CLASS FUNCTIONS

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## ABSTRACT

Fixed point theorems for monotone operators in ordered metric spaces are widely investigated and have found various applications in differential and integral equations, see Nieto & Rodríguez-López (2005); Nieto & Rodríguez-López (2007); Wu (2008). Motivated by the work in Agarwal et al. (2012); Luong & Thuan (2011) we study the existence of solutions for a system of nonlinear integral equations in ordered b-metric spaces using the results given in this paper.

**Key words:** b-metric spaces, compatible mappings, P-increasing, integral equation, C-class functions.

## INTRODUCTION

In Guo & Lakshmikantham (1987) studied the concept of coupled fixed points. Later, in Bhaskar & Lakshmikantham (2006) studied monotone property and supported this by providing an application to the existence of periodic boundary value problems.

In Bakhtin (1989) introduced the concept of a b-metric space as a generalization of metric spaces. In Czerwik (1993) and Czerwik (1998) extended many results related to the b-metric spaces. Since then, several papers have been published on the fixed point theory of various classes of single-valued and multi-valued operators in b-metric spaces (Akkouchi, 2011; Aydi et al., 2012; Boriceanu, 2009; Boriceanu et al., 2010; Bota et al., 2011; Hussain & Shah, 2011; Olatinwo, 2008; Huang et al., 2015; Mustafa et al., 2013; Mustafa et al., 2014; Ansari et al., 2014; Aghajani et al., 2014).

In this paper, we use the notion of C-class of function which is generalization of altering distance function and by using this definition we have improved the results of Hussain, Abbas, Azam and Ahmad (Hussain et al., 2014) in the setting of b-metric space. Also we provide an application to an integral equation to support our results presented here.

Consistent with Bakhtin (1989); Czerwik (1993); Czerwik (1998); Huang et al. (2015) the following definitions and results will be needed in the sequel.

**Definition 1.1.** (Bakhtin, 1989; Czerwik, 1993) Let  $M$  be a nonempty set and  $B \geq 1$  be a given real number. A function  $d_b : M \times M \rightarrow R^+$  is a b-metric if the following conditions are satisfied:

- (b1)  $d_b(\bar{x}, \bar{y}) = 0$  if and only if  $\bar{x} = \bar{y}$ ;
- (b2)  $d_b(\bar{x}, \bar{y}) = d_b(\bar{y}, \bar{x})$ ;
- (b3)  $d_b(\bar{x}, \bar{z}) \leq B[d_b(\bar{x}, \bar{y}) + d_b(\bar{y}, \bar{z})]$

for all  $\bar{x}, \bar{y}, \bar{z} \in M$ .

The pair  $(M, d_b)$  is called a b-metric spaces. It is well known that each metric space is also b-metric (with  $B = 1$ ) while the converse is not true.

**Definition 1.2.** Let  $M$  be a nonempty set. Then  $(M, \leq, d_b)$  is called a partially ordered b-metric space if and only if  $d_b$  is a b-metric on a partially ordered set  $(M, \leq)$ .

**Definition 1.3.** (Boriceanu et al., 2010) Let  $(M, d_b)$  be a b-metric space. Then a sequence  $\{x_p\}$  in  $M$  is called b-convergent if and only if there exists  $\bar{x} \in M$  such that  $d_b(x_p, \bar{x}) \rightarrow 0$  as  $p \rightarrow +\infty$ . In this case, we write  $\lim_{p \rightarrow \infty} x_p = \bar{x}$ .

**Definition 1.4.** (Boriceanu et al., 2010) Let  $(M, d_b)$  be a b-metric space. Then a sequence  $\{x_p\}$  in  $M$  is called b-Cauchy if and only if  $d_b(x_p, x_q) \rightarrow 0$ , as  $p, q \rightarrow +\infty$ .

**Definition 1.5.** (Boriceanu et al., 2010) The b-metric space  $(M, d_b)$  is b-complete if every b-Cauchy sequence in  $M$  b-converges.

In Ansari et al. (2014) introduced the concept of C-class functions which cover a large class of contractive conditions.

**Definition 1.6.** (Ansari et al., 2014) A mapping  $W : [0, \infty)^2 \rightarrow R$  is called C-class function if it is continuous and satisfies following axioms:

- (1)  $W(\bar{s}, \bar{t}) \leq \bar{s}$ ;
- (2)  $W(\bar{s}, \bar{t}) = \bar{s}$  implies that either  $\bar{s} = 0$  or  $\bar{t} = 0$  for all  $\bar{s}, \bar{t} \in [0, \infty)$ .

We denote C-class functions as  $C$ .

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**Definition 1.7.** ((Hussain et al., 2014), Definition 7) Suppose that  $H, P: M \times M \rightarrow M$  are two mappings. Then  $H$  is said to be  $P$ -increasing with respect to  $\leq$  if for all  $\bar{x}, \bar{y}, \xi, g \in M$  with  $P(\bar{x}, \bar{y}) \leq P(\xi, g)$  we have  $H(\bar{x}, \bar{y}) \leq H(\xi, g)$ .

**Definition 1.8.** ((Hussain et al., 2014), Definition 10) An element  $(\bar{x}, \bar{y}) \in M \times M$  is called a coupled coincidence point of mappings  $H, P: M \times M \rightarrow M$  if  $H(\bar{x}, \bar{y}) = P(\bar{x}, \bar{y})$  and  $H(\bar{y}, \bar{x}) = P(\bar{y}, \bar{x})$ .

**Definition 1.9.** ((Hussain et al., 2014), Definition 12) Let  $H, P: M \times M \rightarrow M$ . We say that the pair  $\{H, P\}$  is generalized compatible if

$$\begin{cases} d_b(H(P(x_p, y_p), P(y_p, x_p)), P(H(x_p, y_p), H(y_p, x_p))) \\ \rightarrow 0 \text{ as } p \rightarrow \infty; \\ d_b(H(P(y_p, x_p), P(x_p, y_p)), P(H(y_p, x_p), H(x_p, y_p))) \\ \rightarrow 0 \text{ as } p \rightarrow \infty; \end{cases}$$

whenever  $(x_p)$  and  $(y_p)$  are sequences in  $M$  such that

$$\begin{cases} \lim_{p \rightarrow \infty} H(x_p, y_p) = \lim_{p \rightarrow \infty} P(x_p, y_p) = t_1 \\ \lim_{p \rightarrow \infty} H(y_p, x_p) = \lim_{p \rightarrow \infty} P(y_p, x_p) = t_2. \end{cases}$$

**Definition 1.10.** (Bhaskar & Lakshmikantham, 2006) Let  $(M, \leq)$  be a partially ordered set. Then the mapping  $H: M \times M \rightarrow M$  is said to have the mixed monotone property if  $H$  is monotone non-decreasing in its first argument and is monotone non-increasing in its second argument, that is, for all  $x_1, x_2 \in M$ ,  $x_1 \leq x_2$  implies  $H(x_1, \bar{y}) \leq H(x_2, \bar{y})$ , for any  $\bar{y} \in M$  and for all  $y_1, y_2 \in M$ ,  $y_1 \leq y_2$  implies  $H(\bar{x}, y_1) \geq H(\bar{x}, y_2)$ , for any  $\bar{x} \in M$ .

In the paper Hussain et al. (2014), in Theorem 15, it is shown the example when  $(M, \leq)$  is a partially ordered set such that there exists a complete metric  $d$  on  $M$ .

Therefore, in this paper we give Theorem 1.11 which represents the continuation of Theorem 15 from Hussain et al. (2014) on b-metric spaces with the implementation of the functions of the C-class. However, in our case, in the following Theorem 1.11 the condition (1) highly differs from the condition in Theorem 15 (Hussain et al., 2014).

**Theorem 1.11.** Let  $(M, \leq, d_b)$  be an ordered complete b-metric space (with parameter  $B > 1$ ). Assume that  $H, P: M \times M \rightarrow M$  are two generalized compatible mappings such that  $H$  is  $P$ -increasing with respect to  $\leq$ ,  $P$  is continuous and has the mixed

monotone property, and there exist two elements  $x_0, y_0 \in M$  with

$$P(x_0, y_0) \leq H(x_0, y_0) \text{ and } P(y_0, x_0) \geq H(y_0, x_0).$$

Suppose that there exist  $\varphi \in \Phi$  and  $\psi \in \Psi$ ,  $W$  is C-class such that

$$\begin{aligned} & \varphi(\bar{s}^a d_b(H(\bar{x}, \bar{y}), H(\xi, g))) \\ & \leq W(\varphi(\frac{d_b(P(\bar{x}, \bar{y}), P(\xi, g)) + d_b(P(\bar{y}, \bar{x}), P(g, \xi))}{2}, \\ & \psi(\frac{d_b(P(\bar{x}, \bar{y}), P(\xi, g)) + d_b(P(\bar{y}, \bar{x}), P(g, \xi))}{2})). \end{aligned} \quad (1)$$

for all  $\bar{x}, \bar{y}, \xi, g \in M$ ,  $a > 1$  with  $P(\bar{x}, \bar{y}) \leq P(\xi, g)$  and  $P(\bar{y}, \bar{x}) \geq P(g, \xi)$ . Suppose that for any  $\bar{x}, \bar{y} \in M$ , there exist  $\xi, g \in M$ , such that

$$\begin{cases} H(\bar{x}, \bar{y}) = P(\xi, g) \\ H(\bar{y}, \bar{x}) = P(g, \xi). \end{cases}$$

Also suppose that either

- (a)  $H$  is continuous or
- (b)  $M$  has the following property
  - (i) If a non-decreasing sequence  $\{x_p\} \rightarrow \bar{x}$ , then  $x_p \leq \bar{x}$  for all  $p$
  - (ii) If a non-increasing sequence  $\{y_p\} \rightarrow \bar{y}$ , then  $\bar{y} \leq y_p$  for all  $p$ .

Then,  $H$  and  $P$  have a coupled coincidence point in  $M$ .

The proof of this theorem follows directly.

Hussain, Abbas, Azam and Ahmad in Hussain et al. (2014) in Corollary 22 gives the situation when  $(M, \leq)$  is a partially ordered set and suppose there is a metric  $d$  on  $M$  such that  $(M, d)$  is a complete metric space.

In this paper we give Corollary 1.12 that shows the broadening of Corollary 22 in Hussain et al. (2014) on b-metric spaces by using the C-class functions. It should be noted that in our example, in Corollary 1.12 which follows the condition (2) there is a great difference from the condition in Corollary 22 in Hussain et al. (2014).

**Corollary 1.12.** Let  $(M, \leq, d_b)$  be an ordered complete b-metric space (with parameter  $B > 1$ ) and suppose there is a metric  $d_b$  on  $M$  such that  $(M, d_b)$  is a complete metric space. Assume that  $H: M \times M \rightarrow M$  be an increasing map with respect to  $\leq$  and there exist two elements  $x_0, y_0 \in M$  with  $x_0 \leq H(x_0, y_0)$  and  $y_0 \geq H(y_0, x_0)$ .

Suppose there exists  $\psi \in \Psi$ ,  $W$  is C-class such that

$$\begin{aligned} & \bar{s}^a(d_b(H(\bar{x}, \bar{y}), H(\xi, g))) \\ & \leq W\left(\frac{d_b(\bar{x}, \xi) + d_b(\bar{y}, g)}{2}, \right. \\ & \left. \psi\left(\frac{d_b(\bar{x}, \xi) + d_b(\bar{y}, g)}{2}\right)\right) \end{aligned} \quad (2)$$

for all  $\bar{x}, \bar{y}, \xi, g \in M$ ,  $a > 1$  with  $\bar{x} \leq \xi$  and  $\bar{y} \geq g$ . Also suppose that either

- (a)  $H$  is continuous or
  - (b)  $M$  has the following property
    - (i) If a non-decreasing sequence  $\{x_p\} \rightarrow \bar{x}$ , then  $x_p \leq \bar{x}$  for all  $p$
    - (ii) If a non-increasing sequence  $\{y_p\} \rightarrow \bar{y}$ , then  $\bar{y} \leq y_p$  for all  $p$ .
- Then  $H$  has a coupled fixed point.

## RESULTS

Fixed point theorems for monotone operators in ordered metric spaces are widely investigated and have found various applications in differential and integral equations see Nieto & Rodríguez-López (2005); Nieto & Rodríguez-López (2007); Wu (2008)). Motivated by the work in Agarwal et al. (2012); Luong & Thuan (2011) we study the existence of solutions for a system of nonlinear integral equations using the results given in the previous section.

Let  $E$  denote the class of those functions  $\varepsilon: [0, \infty) \rightarrow [0, \infty)$  which satisfies the following conditions:

- (a<sub>1</sub>)  $\varepsilon$  is increasing
- (b<sub>1</sub>) there exists  $\psi \in \Psi, W \in C$  such that

$$\varepsilon(\tau) = \frac{1}{2m} W\left(\frac{\tau}{2}, \psi\left(\frac{\tau}{2}\right)\right) \text{ for all } \tau \in [0, \infty)$$

Consider the integral equation

$$\begin{aligned} \bar{x}(\tau) = & \int_a^b (K_1(\tau, \phi) + K_2(\tau, \phi)) (h(\phi, \bar{x}(\phi)) + l(\phi, \bar{x}(\phi))) d\phi \\ & + k(\tau) \end{aligned} \quad (3)$$

for all  $\tau \in [a, b]$ . We suppose that  $K_1, K_2, h$  and  $l$  satisfy the following conditions:

- (i)  $0 \leq K_1(\tau, \phi); 0 \leq K_2(\tau, \phi)$  for all  $\tau, \phi \in [a, b]$ ;
- (ii) There exist  $\lambda, \mu > 0$  and  $\varepsilon \in E$  such that for all  $\bar{x}, \bar{y} \in R$ ,  $\bar{x} \geq \bar{y}$ ,
 
$$0 \leq h(\tau, \bar{x}) - h(\tau, \bar{y}) \leq \lambda \varepsilon(\bar{x} - \bar{y})$$
 and
 
$$0 \leq l(\tau, \bar{x}) - l(\tau, \bar{y}) \leq \mu \varepsilon(\bar{x} - \bar{y}).$$
- (iii) There is

$$\max\{\lambda^2, \mu^2\} \left\{ \sup_{\tau \in [a, b]} \int_a^b (K_1(\tau, \phi) + K_2(\tau, \phi)) d\phi \right\}^2 \leq m^2,$$

for  $m > 0$ .

- (iv) There exist continuous functions  $\bar{z}, \varpi: [a, b] \rightarrow R$

such as

$$\begin{aligned} \bar{z}(\tau) \leq & \int_a^b K_1(\tau, \phi) (h(\phi, \bar{z}(\phi)) + l(\phi, \varpi(\phi))) d\phi \\ & + \int_a^b K_2(\tau, \phi) (h(\phi, \varpi(\phi)) + l(\phi, \bar{z}(\phi))) d\phi + k(\tau) \end{aligned}$$

$$\begin{aligned} \text{and } \varpi(\tau) \geq & \int_a^b K_1(\tau, \phi) (h(\phi, \varpi(\phi)) + l(\phi, \bar{z}(\phi))) d\phi \\ & + \int_a^b K_2(\tau, \phi) (h(\phi, \bar{z}(\phi)) + l(\phi, \varpi(\phi))) d\phi + k(\tau) \end{aligned}$$

for all  $\tau \in [a, b]$ .

In Hussain et al. (2014), Theorem 26) we studied the integral equation (3) and proved that the mapping  $H$  has a coupled fixed point that is a solution in  $M = C([a, b], R)$ .

Also, we overviewed the same integral equation (3) and proved that the mapping  $H$  has a coupled fixed point that is a solution in  $M = C([a, b], R)$ , too, but in the range of b-metric spaces by the implementation of C-class functions. Our case gives different conditions (b<sub>1</sub>) and (iii) when compared to the conditions given in Theorem 26 in Hussain et al. (2014).

**Theorem 2.1.** Consider the integral equation (3) with  $K_1, K_2 \in C([a, b] \times [a, b], R)$ ,  $h, l \in C([a, b] \times R, R)$  and  $k \in C([a, b], R)$  and suppose that the conditions (i)-(iv) are satisfied. Then the integral equation (3) has a solution in  $C([a, b], R)$ .

**Proof:** The first part of proof of this theorem is very similar to the proof of Theorem 26 in Hussain et al. (2014). For the proof we use the following approach. Let  $M = C([a, b], R)$  denote the space of all continuous functions defined on the interval  $[a, b]$  to  $R$ . We endowed  $M$  with the metric  $d_b: M \times M \rightarrow R$  defined by

$$d_b(\bar{x}, \bar{y}) = \sup_{\tau \in [a, b]} |\bar{x}(\tau) - \bar{y}(\tau)|^2, \text{ for all } \bar{x}, \bar{y} \in M.$$

Compared to Theorem 26 in Hussain et al. (2014) where  $(M, d)$  is a complete metric space and  $(M, d, \leq)$  is a complete ordered metric space, in our case  $(M, d_b)$  is a complete b-metric space and  $(M, d_b, \leq)$  is a complete ordered b-metric space if  $\bar{x} \leq \bar{y}$  whenever  $\bar{x}(\tau) \leq \bar{y}(\tau)$  for all  $\tau \in [a, b]$ . Suppose  $\{\xi_p\}$

is a monotone non-decreasing in  $M$  that converges to  $\xi \in M$ .

Then for every  $\tau \in [a, b]$  the sequence of real numbers

$$\xi_1(\tau) \leq \xi_2(\tau) \leq \dots \leq \xi_n(\tau) \leq \dots$$

converges to  $\xi(\tau)$ . Therefore for all  $\tau \in [a, b]$ ,  $p \in N$ ,  $\xi_p(\tau) \leq \xi(\tau)$ . Hence  $\xi_p \leq \xi$  for all  $p$ . Similarly, we can verify that  $\lim_p g_p(\tau)$  of a monotone non-increasing sequence  $g_p(\tau)$  in  $M$  is a lower bound for all the elements in the sequence. That is,  $g \leq g_p$  for all  $p$ . Therefore, condition (b) of Corollary (1.12) holds.

Also,  $M \times M = C([a, b], R) \times C([a, b], R)$  is a partially ordered set if we define the following order relation on  $M \times M$ , for all  $\bar{x}, \bar{y}, \xi, g \in M$ , with  $\bar{x} \leq \xi$  and  $\bar{y} \geq g$ .

A mapping  $H : M \times M \rightarrow M$  define by

$$H(\bar{x}, \bar{y})(\tau) = \int_a^b K_1(\tau, \phi) (h(\phi, \bar{x}(\phi)) + l(\phi, \bar{y}(\phi))) d\phi \\ + \int_a^b K_2(\tau, \phi) (h(\phi, \bar{y}(\phi)) + l(\phi, \bar{x}(\phi))) d\phi + kh(\tau) \text{ for all } \tau \in [a, b].$$

Now we will prove that  $H$  is increasing. For  $x_1 \leq x_2$ , that is  $x_1(\tau) \leq x_2(\tau)$ , for all  $\tau \in [a, b]$ , we have

$$H(x_1, \bar{y})(\tau) - H(x_2, \bar{y})(\tau) \\ = \int_a^b K_1(\tau, \phi) (h(\phi, x_1(\phi)) + l(\phi, \bar{y}(\phi))) d\phi \\ + \int_a^b K_2(\tau, \phi) (h(\phi, \bar{y}(\phi)) + l(\phi, x_1(\phi))) d\phi + k(\tau) \\ - \int_a^b K_1(\tau, \phi) (h(\phi, x_2(\phi)) + l(\phi, \bar{y}(\phi))) d\phi \\ - \int_a^b K_2(\tau, \phi) (h(\phi, \bar{y}(\phi)) + l(\phi, x_2(\phi))) d\phi - k(\tau) \\ = \int_a^b K_1(\tau, \phi) (h(\phi, x_1(\phi)) - h(\phi, x_2(\phi))) d\phi \\ + \int_a^b K_2(\tau, \phi) (l(\phi, x_1(\phi)) - l(\phi, x_2(\phi))) d\phi \leq 0.$$

Hence  $H(x_1, \bar{y})(\tau) \leq H(x_2, \bar{y})(\tau)$  for all  $\tau \in [a, b]$ , that is  $H(x_1, \bar{y}) \leq H(x_2, \bar{y})$ . Similarly, if  $y_1 \leq y_2$ , that is  $y_1(\tau) \leq y_2(\tau)$  for all  $\tau \in [a, b]$ , we have

$$H(\bar{x}, y_1)(\tau) - H(\bar{x}, y_2)(\tau) \\ = \int_a^b K_1(\tau, \phi) (h(\phi, \bar{x}(\phi)) + l(\phi, y_1(\phi))) d\phi \\ + \int_a^b K_2(\tau, \phi) (h(\phi, y_1(\phi)) + l(\phi, \bar{x}(\phi))) d\phi + k(\tau) \\ - \int_a^b K_1(\tau, \phi) (h(\phi, \bar{x}(\phi)) + l(\phi, y_2(\phi))) d\phi \\ - \int_a^b K_2(\tau, \phi) (h(\phi, y_2(\phi)) + l(\phi, \bar{x}(\phi))) d\phi + k(\tau) \\ = \int_a^b K_2(\tau, \phi) (h(\phi, y_1(\phi)) - h(\phi, y_2(\phi))) d\phi \leq 0.$$

$$- \int_a^b K_1(\tau, \phi) (h(\phi, \bar{x}(\phi)) + l(\phi, y_2(\phi))) d\phi \\ - \int_a^b K_2(\tau, \phi) (h(\phi, y_2(\phi)) + l(\phi, \bar{x}(\phi))) d\phi - k(\tau) \\ = \int_a^b K_1(\tau, \phi) (l(\phi, y_1(\phi)) - l(\phi, y_2(\phi))) d\phi \\ + \int_a^b K_2(\tau, \phi) (h(\phi, y_1(\phi)) - h(\phi, y_2(\phi))) d\phi \leq 0.$$

Hence,  $H(\bar{x}, y_1)(\tau) \leq H(\bar{x}, y_2)(\tau)$  for all  $\tau \in [a, b]$ , that is  $H(\bar{x}, y_1) \leq H(\bar{x}, y_2)$ . Thus  $H(\bar{x}, \bar{y})$  is increasing. Now, for  $\bar{x}, \bar{y}, \xi, g \in M$  such that  $\bar{x} \leq \xi$  and  $g \leq \bar{y}$ , we conclude that

$$d_b(H(\bar{x}, \bar{y})(\tau) - H(\xi, g)(\tau)) \\ = \sup_{\tau \in [a, b]} |H(\bar{x}, \bar{y})(\tau) - H(\xi, g)(\tau)|^2 \\ = \left\{ \sup_{\tau \in [a, b]} \left| \int_a^b K_1(\tau, \phi) (h(\phi, \bar{x}(\phi)) + l(\phi, \bar{y}(\phi))) d\phi \right. \right. \\ \left. \left. + \int_a^b K_2(\tau, \phi) (h(\phi, \bar{y}(\phi)) + l(\phi, \bar{x}(\phi))) d\phi + k(\tau) \right. \right. \\ \left. \left. - \left( \int_a^b K_1(\tau, \phi) (h(\phi, \xi(\phi)) + l(\phi, g(\phi))) d\phi \right. \right. \right. \\ \left. \left. \left. + \int_a^b K_2(\tau, \phi) (h(\phi, g(\phi)) + l(\phi, \xi(\phi))) d\phi + k(\tau) \right) \right| \right\}^2 \\ = \left\{ \sup_{\tau \in [a, b]} \left| \int_a^b K_1(\tau, \phi) [h(\phi, \bar{x}(\phi)) - h(\phi, \xi(\phi))] \right. \right. \\ \left. \left. + [l(\phi, \bar{y}(\phi)) - l(\phi, g(\phi))] d\phi \right. \right. \\ \left. \left. + \int_a^b K_2(\tau, \phi) [h(\phi, \bar{y}(\phi)) - h(\phi, g(\phi))] \right. \right. \\ \left. \left. + [l(\phi, \bar{x}(\phi)) - l(\phi, \xi(\phi))] d\phi \right| \right\}^2 \\ \leq \left\{ \sup_{\tau \in [a, b]} \left| \int_a^b K_1(\tau, \phi) \right. \right. \\ \left. \left. \times [\lambda \varepsilon(\bar{x}(\phi) - \xi(\phi)) + \mu \varepsilon(\bar{y}(\phi) - g(\phi))] d\phi + \right. \right. \\ \left. \left. \int_a^b K_2(\tau, \phi) [\lambda \varepsilon(\bar{y}(\phi) - g(\phi)) + \mu \varepsilon(\bar{x}(\phi) - \xi(\phi))] d\phi \right| \right\}^2 \\ \leq \max \{ \lambda^2, \mu^2 \} \left\{ \sup_{\tau \in [a, b]} \left| \int_a^b [K_1(\tau, \phi) + K_2(\tau, \phi)] \right. \right. \\ \left. \left. [\varepsilon |(\bar{x}(\phi) - \xi(\phi))| + \varepsilon |(\bar{y}(\phi) - g(\phi))|] d\phi \right| \right\}^2$$

As the function  $\varepsilon$  is increasing and  $\bar{y}(\tau) \geq g(\tau)$  for all  $\tau \in [a, b]$ , then

$$\varepsilon(\bar{x}(\phi) - \xi(\phi)) \leq \varepsilon(d_b(\bar{x}, \xi)), \quad \varepsilon(\bar{y}(\phi) - g(\phi)) \leq \varepsilon(d_b(\bar{y}, g)),$$

for all  $\phi \in [a, b]$ , we obtain

$$\begin{aligned}
& d_b(H(\bar{x}, \bar{y}), H(\xi, g)) \\
& \leq \max\{\lambda^2, \mu^2\} \left[ \varepsilon^2(d_b(\bar{x}, \xi)) + \varepsilon^2(d_b(\bar{y}, g)) \right] \\
& \times \left\{ \sup_{\tau \in [a, b]} \int_a^b (K_1(\tau, \phi) + K_2(\tau, \phi)) d\phi \right\}^2 \\
& \leq m^2 \left[ \varepsilon^2(d_b(\bar{x}, \xi)) + \varepsilon^2(d_b(\bar{y}, g)) \right] \\
& \leq m^2 \varepsilon^2(d_b(\bar{x}, \xi) + d_b(\bar{y}, g)) \\
& \leq \frac{m^2}{4m^2} W \left( \frac{d_b(\bar{x}, \xi) + d_b(\bar{y}, g)}{2}, \psi \left( \frac{d_b(\bar{x}, \xi) + d_b(\bar{y}, g)}{2} \right) \right).
\end{aligned}$$

So, for  $\bar{x} \leq \xi$  and  $g \leq \bar{y}$ , are following

$$\begin{aligned}
& 2^2(d_b(H(\bar{x}, \bar{y}), H(\xi, g))) \\
& \leq W \left( \frac{d_b(\bar{x}, \xi) + d_b(\bar{y}, g)}{2}, \psi \left( \frac{d_b(\bar{x}, \xi) + d_b(\bar{y}, g)}{2} \right) \right).
\end{aligned}$$

Now by (iv) it follows that  $\bar{z}(\tau) \leq H(\bar{z}, \bar{\omega})(\tau)$  and  $H(\bar{\omega}, \bar{z})(\tau) \leq \bar{\omega}(\tau)$  for all  $\tau \in [a, b]$ , that is  $\bar{z} \leq H(\bar{z}, \bar{\omega})$  and  $H(\bar{\omega}, \bar{z}) \leq \bar{\omega}$ . Thus all of the hypotheses of Corollary (1.12) are satisfied and the mapping  $H$  has a coupled fixed point that is a solution in  $M = C([a, b], R)$  of the integral equation (3).

## REFERENCES

- Agarwal, R.P., Hussain, N., & Taoudi, M.A. 2012. Fixed point theorems in ordered Banach spaces and applications to nonlinear integral equations. *Abstr. Appl. Anal.*, . Article ID 245872, 15 pages.
- Akkouchi, M. 2011. Common fixed point theorems for two selfmappings of a b-metric space under an implicit relation. *Hacet. J. Math. Statist.*, 40(6), pp. 805-810.
- Aydi, H., Bota, M., Karapinar, E., & Mitrović, S. 2012. A fixed point theorem for set-valued quasi-contractions in b-metric spaces. *Fixed Point Theory Appl.*, 88.
- Ansari, A.H., Chandok, S., & Ionescu, C. 2014. Fixed point theorems on b-metric spaces for weak contractions with auxiliary functions. *J. Inequal. Appl.*, 429.
- Aghajani, A., Abbas, M., & Roshan, J.R. 2014. Common fixed point of generalized weak contractive mappings in partially ordered b-metric spaces. *Math. Slov.*, 4, pp. 941-960.
- Bhaskar, T.G., & Lakshmikantham, V. 2006. Fixed point theorems in partially ordered metric spaces and applications. *Nonlinear Anal.*, 65, pp. 1379-1393.
- Boriceanu, M. 2009. Strict fixed point theorems for multivalued operators in b-metric spaces. *Int. J. Mod. Math.*, 4(3), pp. 285-301.
- Boriceanu, M., Bota, M., & Petrusel, A. 2010. Multivalued fractals in b-metric spaces. *Cent. Eur. J. Math.*, 8(2), pp. 367-377.
- Bota, M., Molnar, A., & Varga, C. 2011. On Ekelandsvariational principle in b-metric spaces. *Fixed Point Theory*, 12(2), pp. 21-28.
- Bakhtin, I.A. 1989. The contraction mapping principle in quasimetric spaces. *Funct. Anal., Unianowsk Gos. Ped. Inst.*, 30, pp. 26-37.
- Czerwik, S. 1993. Contraction mappings in b-metric spaces. *Acta. Math. Inform. Univ. Ostraviensis*, 1, pp. 5-11.
- Czerwik, S. 1998. Nonlinear set-valued contraction mappings in b-metric spaces. *Atti. Sem. Mat. Fis. Univ. Modena*, 46, pp. 263-276.
- Guo, D., & Lakshmikantham, V. 1987. Coupled fixed points of nonlinear operators with applications. *Nonlinear Anal.*, 11, pp. 623-632.
- Hussain, N., Abbas, M., Azam, A., & Ahmad, J. 2014. Coupled coincidence point results for a generalized compatible pair with applications. *Fixed Point Theory and Applications*, 62.
- Hussain, N., & Shah, M.H. 2011. KKM mapping in cone b-metric spaces. *Comput. Math. Appl.*, 62, pp. 1677-1684.
- Huang, H., Paunović, Lj., & Radenović, S. 2015. On some new fixed point results for rational Geraghty contractive mappings in ordered b-metric spaces. *J. Nonlinear Sci. Appl.*, 8, pp. 800-807.
- Luong, N.V., & Thuan, N.X. 2011. Coupled fixed point in partially ordered metric spaces and applications. *Nonlinear Anal.*, 74, pp. 983-992.
- Mustafa, Z., Roshan, J.R., Parvaneh, V., & Kadelburg, Z. 2013. Some common fixed point results in ordered partial b-metric spaces. *J. Inequal. Appl.*, 562.
- Mustafa, Z., Roshan, J.R., Parvaneh, V., & Kadelburg, Z. 2014. Fixed point theorems for Weakly T-Chatterjea and weakly T-Kannan contractions in b-metric spaces. *J. Inequal. Appl.*, 46.
- Mustafa, Z., Jaradat, M.M.M., Ansari, A.H., Popović, B.Z., & Jaradat, H.M. 2016. C-class functions with new approach on coincidence point results for generalized [Formula: see text]-weakly contractions in ordered b-metric spaces. *Springerplus*, 5(1), p. 802. PMID:27390643
- Nieto, J.J., & Rodríguez-López, R. 2005. Contractive Mapping Theorems in Partially Ordered Sets and Applications to Ordinary Differential Equations. *Order*, 22(3), pp. 223-239. doi:10.1007/s11083-005-9018-5
- Nieto, J.J., & Rodríguez-López, R. 2007. Existence and Uniqueness of Fixed Point in Partially Ordered Sets and Applications to Ordinary Differential Equations. *Acta Mathematica Sinica, English Series*, 23(12), pp. 2205-2212. doi:10.1007/s10114-005-0769-0
- Olatinwo, M.O. 2008. Some results on multi-valued weakly jungck mappings in b-metric space. *Central European Journal of Mathematics*, 6(4), pp. 610-621. doi:10.2478/s11533-008-0047-3
- Wu, Y. 2008. New fixed point theorems and applications of mixed monotone operator. *J. Math. Anal. Appl.*, 341, pp. 883-893.

# NOTES ON THE MOORE-PENROSE INVERSE OF A LINEAR COMBINATION OF COMMUTING GENERALIZED AND HYPERGENERALIZED PROJECTORS

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## ABSTRACT

The aim of this paper is to give alternate representations of the Moore-Penrose inverse of a linear combination of generalized and hypergeneralized projectors and to provide alternate proofs of the invertibility of some linear combination of commuting generalized and hypergeneralized projectors.

**Keywords:** Idempotent, Projector, Generalized projector, Hypergeneralized projector, Moore-Penrose inverse.

**2000 Mathematics Subject Classification.** 15A09

## INTRODUCTION

Let  $C^{n \times m}$  denote the set of all  $n \times m$  complex matrices. The symbols  $A^*$ ,  $R(A)$ ,  $N(A)$  and  $r(A)$  will denote the conjugate transpose, the range (column space), the null space and the rank of a matrix  $A \in C^{n \times m}$ , respectively. The Moore-Penrose inverse of  $A$ , is the unique matrix satisfying the equations.

$$AA^\dagger A = A, A^\dagger AA^\dagger = A^\dagger, AA^\dagger = (AA^\dagger)^*, A^\dagger A = (A^\dagger A)^*.$$

More about the Moore-Penrose inverse can be found in (Ben-Israel & Greville, 1974).

$I_n$  will denote the identity matrix of order  $n$ .  $P_S$  denotes the orthogonal projector onto subspace  $S$ . We use the notations  $C_n^P, C_n^{OP}, C_n^{EP}, C_n^{GP}$  and  $C_n^{HGP}$  for the subsets of  $C^{n \times n}$  consisting of projectors (idempotent matrices), orthogonal projectors (Hermitian idempotent matrices), EP (range-Hermitian) matrices, generalized and hypergeneralized projectors, respectively, i.e.

$$C_n^P = \{A \in C^{n \times n} : A^2 = A\},$$

$$C_n^{OP} = \{A \in C^{n \times n} : A^2 = A = A^*\},$$

$$C_n^{EP} = \{A \in C^{n \times n} : R(A) = R(A^*)\} = \{A \in C^{n \times n} : AA^\dagger = A^\dagger A\},$$

$$C_n^{GP} = \{A \in C^{n \times n} : A^2 = A^*\},$$

$$C_n^{HGP} = \{A \in C^{n \times n} : A^2 = A^\dagger\}.$$

The concepts of generalized and hypergeneralized projectors were introduced by Groß & Trenkler (1997), who presented very interesting properties of the classes of generalized and hypergeneralized projectors. Interesting results concerning generalized and hypergeneralized projectors can be found in the papers (Baksalary, 2009; Baksalary et al., 2008; Radosavljević & Djordjević, 2013; Stewart, 2006).

In this paper we give alternate representations of the Moore-Penrose inverse of a linear combination of generalized and hypergeneralized projectors and to provide alternate proofs of the invertibility of some linear combination of commuting generalized and hypergeneralized projectors of the paper (Tošić et al., 2011). We provide alternate proofs of the nonsingularity of  $\alpha_1 I_n + \alpha_2 A^s + \alpha_3 B^k$  and  $\alpha_1 A^s + \alpha_2 B^k + \alpha_3 C^l$  where  $s, k, l \in \mathbb{N}$ ,  $\alpha_1, \alpha_2, \alpha_3 \in C$  and  $A, B$  and  $C$  are commuting generalized or hypergeneralized projectors under various conditions. Also, we give the alternate form of Moore-Penrose inverse of a linear combination  $\alpha_1 A^s + \alpha_2 B^k$  of two generalized or hypergeneralized matrices.

## THE INVERSES OF GENERALIZED PROJECTORS OR HYPERGENERALIZED PROJECTIONS

In (Baksalary et al., 2008) authors proved that the generalized projector  $A \in C_r^{n \times n}$  can be represented by

$$A = U \begin{bmatrix} K & 0 \\ 0 & 0 \end{bmatrix} U^*, \quad (1)$$

where,  $U \in C^{n \times n}$  is unitary,  $\Sigma = \text{diag}(\sigma_1 I_{r_1}, \dots, \sigma_t I_{r_t})$  is a diagonal matrix of singular values of  $A$ ,

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$\sigma_1 > \sigma_2 > \dots > \sigma_t > 0$ ,  $r_1 + r_2 + \dots + r_t = r$  and  $K \in C^{r \times r}$  satisfies  $(\Sigma K)^3 = I_r$  and  $KK^* = I_r$ .

In the following theorem, we use the star-orthogonality. The notion of star-orthogonality introduced by Hestenes (1961). Let us recall that matrices  $A, B \in C^{n \times m}$  are star-orthogonal, denoted by  $A \perp B$ , if  $AB^* = 0$  and  $A^*B = 0$ . It is well-known that for  $A, B \in C_n^{EP}$ ,

$$A \perp B \Leftrightarrow AB = 0 \Leftrightarrow BA = 0.$$

If  $A, B$  are hypergeneralized projectors, then  $A \perp B$  or  $AB = BA = 0$  is sufficient for  $A + B$  to be a hypergeneralized projector (see (Groß & Trenkler, 1997)).

If we consider a finite commuting family where all of the members are generalized or hypergeneralized projectors, then  $\prod_{i=1}^m A_i^{k_i}$ , where  $m, k_1, \dots, k_m \in \mathbb{N}$  is also a generalized or hypergeneralized projector. Hence, the following theorem is equivalent to Theorem 2.11 (Tošić et al., 2011) and shows that  $\alpha_1 I_n + \alpha_2 A^s + \alpha_3 B^k$  is nonsingular, in the case when  $A, B$  are generalized projectors such that  $A + B \in C_n^{GP}$  or when  $A, B$  are hypergeneralized projectors such that  $A \perp B$ .

First, we will state an auxiliary result.

**Lemma 2.1** (Tošić & Cvetković-Ilić, 2013). Let  $K \in C^{r \times r}$  be such that  $K^3 = I_r$  and let  $\alpha_1, \alpha_2 \in \mathbb{C}$ . If  $\alpha_1^3 + \alpha_2^3 \neq 0$ , then  $\alpha_1 K + \alpha_2 I_r$  and  $\alpha_1 K^2 + \alpha_2 I_r$  are nonsingular and

$$(\alpha_1 K + \alpha_2 I_r)^{-1} = \frac{1}{\alpha_1^3 + \alpha_2^3} (\alpha_1^2 K^2 - \alpha_1 \alpha_2 K + \alpha_2^2 I_r)$$

and

$$(\alpha_1 K^2 + \alpha_2 I_r)^{-1} = \frac{1}{\alpha_1^3 + \alpha_2^3} (\alpha_1^2 K - \alpha_1 \alpha_2 K^2 + \alpha_2^2 I_r).$$

**Theorem 2.1** Let  $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{C}$ ,  $\alpha_1 \neq 0$ ,  $\alpha_1^3 + \alpha_2^3 \neq 0$ ,  $\alpha_1^3 + \alpha_3^3 \neq 0$  and  $s, k \in \mathbb{N}$ . If  $A, B \in C^{n \times n}$  be commuting generalized projectors such that  $A + B \in C_n^{GP}$  or  $A, B \in C^{n \times n}$  be commuting hypergeneralized projectors such that  $A \perp B$ ,  $A \perp B$  then  $\alpha_1 I_n + \alpha_2 A^s + \alpha_3 B^k$  is nonsingular and

$$(\alpha_1 I_n + \alpha_2 A^s + \alpha_3 B^k)^{-1} = \frac{1}{\alpha_1^3 + \alpha_2^3} [\alpha_1^2 (A^s)^3 + \alpha_2^2 (A^s)^2 - \alpha_1 \alpha_2 A^s] + (I - AA^\dagger) [\alpha_1 I_n + \alpha_3 B^k]^{-1}. \quad (3)$$

**Proof.** (1) Let  $A, B \in C^{n \times n}$  be commuting generalized projectors such that  $A + B \in C_n^{GP}$ . By Theorem 5 in (Groß & Trenkler, 1997), we have that for generalized projectors  $A, B$   $A + B \in C_n^{GP} \Leftrightarrow AB = 0 = BA$ .

If  $A$  is given by (1) and  $r(A) = r$ , then  $B$  has the form

$$B = U \begin{bmatrix} 0 & 0 \\ 0 & G \end{bmatrix} U^*, \quad (4)$$

where  $G \in C^{(n-r) \times (n-r)}$  is a generalized projector. Then

$$\alpha_1 I_n + \alpha_2 A^s + \alpha_3 B^k = U \begin{bmatrix} \alpha_1 I_r + \alpha_2 K^s & 0 \\ 0 & \alpha_1 I_{n-r} + \alpha_3 G^k \end{bmatrix} U^*,$$

where

$$K^s = \begin{cases} I_r, & s \equiv_3 0 \\ K, & s \equiv_3 1 \\ K^2, & s \equiv_3 2. \end{cases} \quad (5)$$

and

$$G^k = \begin{cases} P_{R(G)}, & k \equiv_3 0 \\ G, & k \equiv_3 1 \\ G^2, & k \equiv_3 2. \end{cases} \quad (6)$$

Obviously,  $\alpha_1 I_n + \alpha_2 A^s + \alpha_3 B^k$  is nonsingular if and only if  $\alpha_1 I_r + \alpha_2 K^s$  and  $\alpha_1 I_{n-r} + \alpha_3 G^k$  are nonsingular. By Lemma 2.1 it follows that  $\alpha_1 I_r + \alpha_2 K^s$  is nonsingular for every  $s \in \mathbb{N}$  and

$$(\alpha_1 I_r + \alpha_2 K^s)^{-1} = \begin{cases} (\alpha_1 + \alpha_2)^{-1} I_r, & s \equiv_3 0 \\ \frac{1}{\alpha_1^3 + \alpha_2^3} (\alpha_2^2 K^* - \alpha_1 \alpha_2 K + \alpha_1^2 I_r), & s \equiv_3 1 \\ \frac{1}{\alpha_1^3 + \alpha_2^3} (\alpha_2^2 K - \alpha_1 \alpha_2 K^* + \alpha_1^2 I_r), & s \equiv_3 2 \end{cases}. \quad (7)$$

Since  $G^3$  is an orthogonal projector and  $(\alpha_1 I_{n-r})^3 + (\alpha_3 G^k)^3 = \alpha_1^3 I_{n-r} + \alpha_3^3 G^3$ , we get that  $(\alpha_1 I_{n-r})^3 + (\alpha_3 G^k)^3$  is nonsingular for all constants  $\alpha_1, \alpha_3 \in \mathbb{C} \setminus \{0\}$  such that  $\alpha_1^3 + \alpha_3^3 \neq 0$ .

From the invertibility of  $(\alpha_1 I_{n-r})^3 + (\alpha_3 G^k)^3$ , it follows that  $\alpha_1 I_{n-r} + \alpha_3 G^k$  is nonsingular. Now,

$$(\alpha_1 I_n + \alpha_2 A^s)^{-1} = U \begin{bmatrix} (\alpha_1 I_r + \alpha_2 K^s)^{-1} & 0 \\ 0 & (\alpha_1 I_{n-r} + \alpha_3 G^k)^{-1} \end{bmatrix} U^*, \quad (8)$$

where  $(\alpha_1 I_r + \alpha_2 K^s)^{-1}$  is given by (7). Obviously, the form (8) is equivalent to the form (3).

(2) Let  $A, B \in C^{n \times n}$  be commuting hypergeneralized projectors such that  $A \perp B$ . Since  $A, B \in C_n^{EP}$  it follows that  $A \perp B \Leftrightarrow AB = 0 \Leftrightarrow BA = 0$ .

The proof is similar to item (1).  $\square$

In subsequent consideration, the first part of Theorem 2.5 in (Tošić & Cvetković-Ilić, 2013) plays a crucial role.

**Theorem 2.2** (Tošić & Cvetković-Ilić, 2013). Let  $A \in C_r^{n \times n}$  and  $B \in C^{n \times n}$  be generalized projectors and let  $k, l \in N$ ,  $\alpha_1, \alpha_2 \in \mathbb{C} \setminus \{0\}$ . If  $A + B \in C_n^{GP}$ , then the following conditions are equivalent:

- (i)  $R(A) \oplus R(B) = \mathbb{C}^{n \times 1}$
- (ii)  $N(A) \oplus N(B) = \mathbb{C}^{n \times 1}$
- (iii)  $R(A) \cap R(B) = \{0\}$  and  $N(A) \cap N(B) = \{0\}$
- (iv)  $\alpha_1 A^k + \alpha_2 B^l$  is nonsingular.

As a corollary we get the following result:

**Corollary 2.4** Let  $A \in C_r^{n \times n}$  and  $B \in C^{n \times n}$  be generalized projectors and let  $k, l \in N$ ,  $\alpha_1, \alpha_2 \in \mathbb{C} \setminus \{0\}$ . If  $A + B \in C_n^{GP}$ , then the following conditions are equivalent:

- (i)  $\alpha_1 A^k + \alpha_2 B^l$  is nonsingular,
- (ii)  $A + B$  is nonsingular.

Also, we need the following lemma:

**Lemma 2.2** (Mišić et al., 2016). Let  $P_1 \in C_r^{n \times n}$  and  $P_2 \in C^{n \times n}$  be orthogonal projectors,  $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{C}$ ,  $\alpha_1 \neq 0$ ,  $\alpha_1 - \alpha_2 \neq 0$  and  $\alpha_1 - \alpha_3 \neq 0$ . If  $P_1 P_2 = 0 = P_2 P$ , then  $\alpha_1 I_n - \alpha_2 P_1 - \alpha_3 P_2$  is nonsingular.

Theorem 2.10 in (Tošić et al., 2011) presents necessary and sufficient conditions for the invertibility of  $\alpha_1 A + \alpha_2 B + \alpha_3 C$ . The following theorem presents also necessary and sufficient conditions for the invertibility of  $\alpha_1 A^s + \alpha_2 B^k + \alpha_3 C^l$ .

**Theorem 2.3** Let  $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{C} \setminus \{0\}$ ,  $\alpha_1^3 + \alpha_2^3 \neq 0$ ,  $\alpha_1^3 + \alpha_3^3 \neq 0$  and  $s, k, l \in N$ . If  $A, B, C$  are generalized

projectors such that  $B + C \in C_n^{GP}$  or  $A, B, C$  are hypergeneralized projectors such that  $B \perp C$ , then  $\alpha_1 A^s + \alpha_2 B^k + \alpha_3 C^l$  is nonsingular if and only if  $(I_n - AA^\dagger)(B + C) + AA^\dagger$  is nonsingular.

**Proof. (1)** Let  $A, B, C$  are generalized projectors. By Theorem 5 in (Groß & Trenkler, 1997), we have that for generalized projectors  $B, C$ ,  $B + C \in C_n^{GP} \Leftrightarrow BC = 0 = CB$ .

Suppose that  $A$  has the form (1) and  $r(A) = r$ . Then  $B$  has the form

$$B = U \begin{bmatrix} D & 0 \\ 0 & G \end{bmatrix} U^*, \quad (9)$$

where  $D \in C^{r \times r}$  and  $G \in C^{(n-r) \times (n-r)}$  are generalized projectors and  $KD = DK$ .

Also,  $C$  has the form

$$C = U \begin{bmatrix} M & 0 \\ 0 & N \end{bmatrix} U^*, \quad (10)$$

where  $M \in C^{r \times r}$  and  $N \in C^{(n-r) \times (n-r)}$  are generalized projectors and  $KM = MK$ . From  $BC = 0 = CB$  it follows that  $DM = 0 = MD$  and  $GN = 0 = NG$ , i.e.  $D + M \in C_r^{GP}$  and  $G + N \in C_{n-r}^{GP}$ , respectively.

Now,

$$\alpha_1 A^s + \alpha_2 B^k + \alpha_3 C^l = U \begin{bmatrix} \alpha_1 K^s + \alpha_2 D^k + \alpha_3 M^l & 0 \\ 0 & \alpha_2 G^k + \alpha_3 N^l \end{bmatrix} U^*,$$

where,  $K^s$  is given by (5),  $D^k$ ,  $M^l$ ,  $G^k$  and  $N^l$  are given by (6).

Remark that:

$$(\alpha_1 K^s)^3 + (\alpha_2 D^k + \alpha_3 M^l)^3 = \alpha_1^3 K^3 + \alpha_2^3 D^3 + \alpha_3^3 M^3.$$

Since  $D^3$  and  $M^l$  are orthogonal projectors, then  $\alpha_1^3 I_r + \alpha_2^3 D^3 + \alpha_3^3 M^3$ , i.e.  $(\alpha_1 K^m)^3 + (\alpha_2 D^k + \alpha_3 M^l)^3$  is nonsingular for every constants  $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{C}$  such that  $\alpha_1 \neq 0$ ,  $\alpha_1^3 + \alpha_2^3 \neq 0$  and  $\alpha_1^3 + \alpha_3^3 \neq 0$  (by Lemma 2.2).

From the invertibility of  $(\alpha_1 K^s)^3 + (\alpha_2 D^k + \alpha_3 M^l)^3$ , it follows that  $\alpha_1 K^s + \alpha_2 D^k + \alpha_3 M^l$  is nonsingular.

Also,

$$(I_n - AA^\dagger)(B + C) + AA^\dagger = U \begin{bmatrix} I_r & 0 \\ 0 & G + N \end{bmatrix} U^*,$$

Remark that the invertibility of  $\alpha_2 G^k + \alpha_3 N^l$ , is equivalent to the invertibility of  $G + N$  for every constants  $\alpha_2, \alpha_3 \in \mathbb{C} \setminus \{0\}$  (by Corollary 2.4). Hence,  $\alpha_1 A^s + \alpha_2 B^k + \alpha_3 C^l$  is nonsingular if and only if  $(I_n - AA^\dagger)(B + C) + AA^\dagger$  is nonsingular.

(2) Let  $A, B, C$  are hypergeneralized projectors. Since  $B, C \in C_n^{EP}$  it follows that:

$$B \perp C \Leftrightarrow BC = 0 \Leftrightarrow CB = 0,$$

so the proof is similar as the item (1).  $\square$

## THE MOORE-PENROSE INVERSES OF GENERALIZED PROJECTORS OR HYPERGENERALIZED PROJECTIONS

In this section, we first present the form of the Moore-Penrose inverse of  $\alpha_1 A^s + \alpha_2 B^k$ , where  $s, k \in \mathbb{N}$  and  $A, B$  are commuting generalized projectors or commuting hypergeneralized projectors. Remark that Theorem 2.1 in (Tošić et al., 2011) presents the form of the Moore-Penrose inverse of  $\alpha_1 A + \alpha_2 B$ , where  $A$  and  $B$  are commuting generalized projectors or commuting hypergeneralized projectors.

**Theorem 3.1** Let  $A \in C_r^{n \times n}$  and  $B \in C^{n \times n}$  be commuting generalized projectors or commuting hypergeneralized projectors, and let  $s, k \in \mathbb{N}$ ,  $\alpha_1, \alpha_2 \in \mathbb{C} \setminus \{0\}$  and  $\alpha_1^3 + \alpha_2^3 \neq 0$ . Then

$$(\alpha_1 A^s + \alpha_2 B^k)^\dagger = (\alpha_1 A^s + \alpha_2 AA^\dagger B^k)^\dagger + \alpha_2^{-1} (I_n - AA^\dagger) (B^k)^2. \quad (11)$$

**Proof.** We only prove that the result holds when  $A \in C_r^{n \times n}$  and  $B \in C^{n \times n}$  are commuting generalized projectors. As for the case that  $A \in C_r^{n \times n}$  and  $B \in C^{n \times n}$  are commuting hypergeneralized projectors, the proof is similar so we will omit them.

Let  $A \in C_n^{GP}$  be of the form (1) and  $r(A) = r$ . We get that the condition  $AB = BA$  is equivalent to the fact that  $B$  has the form (9). Now,

$$\alpha_1 A^s + \alpha_2 B^k = U \begin{bmatrix} \alpha_1 K^s + \alpha_2 D^k & 0 \\ 0 & \alpha_2 G^k \end{bmatrix} U^*,$$

where  $U \in C^{n \times n}$  is unitary,  $K, D \in C^{r \times r}$  are such that  $K^3 = I_r$ ,  $K^* = K^{-1}$ ,  $D^2 = D^*$ ,  $KD = DK$  and  $G \in C^{(n-r) \times (n-r)}$  is a generalized projector such that:

$$G^k = \begin{cases} P_{R(G)}, & k \equiv_3 0 \\ G, & k \equiv_3 1 \\ G^*, & k \equiv_3 2. \end{cases}$$

Similarly as in Theorem 2.1 we conclude that  $\alpha_1 K^s + \alpha_2 D^k$  is nonsingular.

Let

$$W = U \begin{bmatrix} (\alpha_1 K^s + \alpha_2 D^k)^{-1} & 0 \\ 0 & \alpha_2^{-1} (G^k)^\dagger \end{bmatrix} U^*,$$

where

$$(G^k)^\dagger = \begin{cases} P_{R(G)}, & k \equiv_3 0 \\ G^*, & k \equiv_3 1 \\ G, & k \equiv_3 2, \end{cases} \quad (12)$$

i.e. the right hand side of (11). Obviously,  $W$  is the Moore-Penrose inverse of  $\alpha_1 A^s + \alpha_2 B^k$ .  $\square$

With the additional requirements of Theorem 3.1 it is possible to give a more precise form of Moore-Penrose inverse. The following theorem is a generalization of Corollary 2.4 in (Tošić et al., 2011).

**Theorem 3.2** Let  $s, k \in \mathbb{N}$ ,  $\alpha_1, \alpha_2 \in \mathbb{C} \setminus \{0\}$ . If  $A, B \in C_n^{GP}$  such that  $A + B \in C_n^{GP}$  or  $A, B \in C_n^{HGP}$  such that  $A \perp B$ , then:

$$(\alpha_1 A^s + \alpha_2 B^k)^\dagger = \alpha_1^{-1} (A^s)^2 + \alpha_2^{-1} (B^k)^2. \quad (13)$$

**Proof.** (1)  $A, B \in C_n^{GP}$  be such that  $A + B \in C_n^{GP}$ . Similarly as in Theorem 2.1, we can suppose that  $A$  and  $B$  have the form given by (1) and (4), respectively.

Since  $(\alpha_1 A^s + \alpha_2 B^k) = U \begin{bmatrix} \alpha_1 K^s & 0 \\ 0 & \alpha_2 G^k \end{bmatrix} U^*$ , where  $G^k$  is defined by (6), we get that:

$$(\alpha_1 A^s + \alpha_2 B^k)^\dagger = U \begin{bmatrix} \alpha_1^{-1} K^{-s} & 0 \\ 0 & \alpha_2^{-1} (G^k)^\dagger \end{bmatrix} U^*,$$

where

$$K^{-s} = \begin{cases} I_r, & s \equiv_3 0 \\ K^*, & s \equiv_3 1 \\ K, & s \equiv_3 2 \end{cases} \quad (14)$$

and  $G^\dagger$  is defined by (12), i.e.  $(\alpha_1 A^s + \alpha_2 B^k)^\dagger$  is defined by (13).

(2) Let  $A, B \in C_n^{HGP}$  be such that  $A \perp^* B$ . Since  $A, B \in C_n^{EP}$ , it follows that  $A \perp B \Leftrightarrow AB = 0 \Leftrightarrow BA = 0$ , so the proof is similar to the item (1).  $\square$

In the next theorem, we present the form of Moore-Penrose inverse of  $\alpha_1 A^s + \alpha_2 A^k$ , where  $s, k \in \mathbb{N}$  and  $A$  is a generalized or hypergeneralized projector. Remark that it is a corollary of Theorem 3.1 and that it is also Corollary 2.5 in (Tošić et al., 2011).

**Theorem 3.3** Let  $A \in C_r^{n \times n}$  be a generalized or hypergeneralized projector and let  $\alpha_1, \alpha_2 \in \mathbb{C}$ ,  $\alpha_1^3 + \alpha_2^3 \neq 0$  and  $s, k \in \mathbb{N}$ . Then:

$$(\alpha_1 A^s + \alpha_2 A^k)^\dagger = \frac{1}{\alpha_1^3 + \alpha_2^3} [\alpha_1^2 (A^s)^2 + \alpha_2^2 (A^k)^2 - \alpha_1 \alpha_2 A^s A^k]. \quad (15)$$

Proof. Suppose that generalized projector  $A$  has the form (1). Then  $\alpha_1 A^s + \alpha_2 A^k$  has the form

$$\alpha_1 A^s + \alpha_2 A^k = U \begin{bmatrix} \alpha_1 K^s + \alpha_2 K^k & 0 \\ 0 & 0 \end{bmatrix} U^*,$$

where

$$\alpha_1 K^s + \alpha_2 K^k = \begin{cases} (\alpha_1 + \alpha_2)I_r, & s \equiv_3 0, k \equiv_3 0 \\ \alpha_1 I_r + \alpha_2 K, & s \equiv_3 0, k \equiv_3 1 \\ \alpha_1 I_r + \alpha_2 K^*, & s \equiv_3 0, k \equiv_3 2 \\ \alpha_1 K + \alpha_2 I_r, & s \equiv_3 1, k \equiv_3 0 \\ (\alpha_1 + \alpha_2)K, & s \equiv_3 1, k \equiv_3 1 \\ \alpha_1 K + \alpha_2 K^*, & s \equiv_3 1, k \equiv_3 2 \\ \alpha_1 K^* + \alpha_2 I_r, & s \equiv_3 2, k \equiv_3 0 \\ \alpha_1 K^* + \alpha_2 K, & s \equiv_3 2, k \equiv_3 1 \\ (\alpha_1 + \alpha_2)K^*, & s \equiv_3 2, k \equiv_3 2 \end{cases}$$

By Lemma 2.1 and Lemma 2 in (Baksalary et al., 2008) it follows that  $\alpha_1 K^s + \alpha_2 K^k$  is nonsingular for every  $s, k \in \mathbb{N}$  and

$$(\alpha_1 K^s + \alpha_2 K^k)^{-1} = \begin{cases} (\alpha_1 + \alpha_2)^{-1} I_r, & s \equiv_3 0, k \equiv_3 0 \\ \frac{1}{\alpha_1 + \alpha_2} (\alpha_2^2 K^* - \alpha_1 \alpha_2 K + \alpha_1^2 I_r), & s \equiv_3 0, k \equiv_3 1 \\ \frac{1}{\alpha_1 + \alpha_2} (\alpha_2^2 K - \alpha_1 \alpha_2 K^* + \alpha_1^2 I_r), & s \equiv_3 0, k \equiv_3 2 \\ \frac{1}{\alpha_1 + \alpha_2} (\alpha_1^2 K^* - \alpha_1 \alpha_2 K + \alpha_2^2 I_r), & s \equiv_3 1, k \equiv_3 0 \\ (\alpha_1 + \alpha_2)^{-1} K^*, & s \equiv_3 1, k \equiv_3 1 \\ \frac{1}{\alpha_1 + \alpha_2} (\alpha_2^2 K + \alpha_1^2 K^* - \alpha_1 \alpha_2 I_r), & s \equiv_3 1, k \equiv_3 2 \\ \frac{1}{\alpha_1 + \alpha_2} (\alpha_1^2 K - \alpha_1 \alpha_2 K^* + \alpha_2^2 I_r), & s \equiv_3 2, k \equiv_3 0 \\ \frac{1}{\alpha_1 + \alpha_2} (\alpha_1^2 K + \alpha_2^2 K^* - \alpha_1 \alpha_2 I_r), & s \equiv_3 2, k \equiv_3 1 \\ (\alpha_1 + \alpha_2)^{-1} K, & s \equiv_3 2, k \equiv_3 2 \end{cases} \quad (17)$$

Let

$$W = U \begin{bmatrix} (\alpha_1 K^s + \alpha_2 K^k)^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^*,$$

i.e. the right hand side of (15). Obviously,  $W$  is Moore-Penrose inverse of  $\alpha_1 A^s + \alpha_2 A^k$ . As for the case that  $A$  is a hypergeneralized projector, the proof is similar so we omit it.  $\square$

Also, we will consider the star partial ordering, introduced by Drazin (1978). Let us recall that for the matrices  $A, B \in C^{n \times n}$ , a matrix  $A$  is less or equal than  $B$  with respect to the star partial ordering, denoted by  $A \leq^* B$ , if  $A^* A = A^* B$  and  $AA^* = BA^*$ . If  $A \in C_n^{EP}$ , then for any  $B \in C^{n \times n}$ ,  $A \leq^* B \Leftrightarrow AB = A^2 = BA$ .

Theorem 2.7 in (Tošić et al., 2011) presents the form of Moore-Penrose inverse of  $\alpha_1 A^s + \alpha_2 B^k$ . In the next theorem, we give the alternate form of Moore-Penrose inverse of  $\alpha_1 A^s + \alpha_2 B^k$ .

**Theorem 3.4** Let  $\alpha_1, \alpha_2 \in \mathbb{C}$ ,  $\alpha_2 \neq 0$ ,  $\alpha_1^3 + \alpha_2^3 \neq 0$  and  $s, k \in \mathbb{N}$ . If  $A \in C_r^{n \times n}$  and  $B \in C^{n \times n}$  be generalized projectors such that  $B - A \in C_n^{GP}$  or  $A \in C_n^{EP}$ ,  $B \in C_n^{HGP}$  such that  $A \leq^* B$ , then:

$$(\alpha_1 A^s + \alpha_2 B^k)^\dagger = \frac{1}{\alpha_1^3 + \alpha_2^3} [\alpha_1^2 (A^s)^2 + \alpha_2^2 (A^k)^2 - \alpha_1 \alpha_2 A^s A^k] + \alpha_2^{-1} (I - AA^\dagger)(B^k)^2.$$

**Proof.** (1) By Theorem 6 [5], it follows that  $B - A \in C_n^{GP}$  if and only if  $AB = A^2 = BA$ . Suppose that  $A$  has the form (1) and  $B$  has the form given by (9). From  $AB = A^2 = BA$ , we get that

$$B = U \begin{bmatrix} K & 0 \\ 0 & G \end{bmatrix} U^*,$$

where  $G \in C^{(n-r) \times (n-r)}$  is a generalized projector. Now  $\alpha_1 A^s + \alpha_2 B^k$  has the form

$$\alpha_1 A^s + \alpha_2 B^k = U \begin{bmatrix} \alpha_1 K^s + \alpha_2 K^k & 0 \\ 0 & \alpha_2 G^k \end{bmatrix} U^*,$$

where  $\alpha_1 K^s + \alpha_2 K^k$  is given by (16) and  $G^k$  is given by (6). By Lemma 2.1 and Lemma 2 in (Baksalary et al., 2008) it follows that  $\alpha_1 K^s + \alpha_2 K^k$  is nonsingular for every  $s, k \in \mathbb{N}$  and  $(\alpha_1 K^s + \alpha_2 K^k)^{-1}$  is given by (17). Obviously

$$(\alpha_1 A^s + \alpha_2 B^k)^\dagger = U \begin{bmatrix} (\alpha_1 K^s + \alpha_2 K^k)^{-1} & 0 \\ 0 & \alpha_2^{-1} (G^k)^\dagger \end{bmatrix} U^*.$$

(2) Under the assumptions  $A \in C_n^{EP}$ ,  $B \in C_n^{HGP}$  and  $A \leq B$ , we get  $A \in C_n^{HGP}$  by Theorem 3 in (Groß & Trenkler, 1997). Then  $B$  has the form  $B = U \begin{bmatrix} \Sigma K & 0 \\ 0 & G \end{bmatrix} U^*$ . Since  $B \in C_n^{HGP}$ , then  $G \in C_{n-r}^{HGP}$ . Now the proof is similar to the item (1).  $\square$

## ACKNOWLEDGMENTS

The author would like to thank the anonymous reviewers for his/their very useful comments that helped to improve the presentation of this paper.

## REFERENCES

- Agarwal, R.P., Hussain, N., & Taoudi, M.A. 2012. Fixed point theorems in ordered Banach spaces and applications to nonlinear integral equations. *Abstr. Appl. Anal.*, Article ID 245872, 15 pages.
- Akkouchi, M. 2011. Common fixed point theorems for two selfmappings of a b-metric space under an implicit relation. *Hacet. J. Math. Statist.*, 40(6), pp. 805-810.
- Aydi, H., Bota, M., Karapinar, E., & Mitrović, S. 2012. A fixed point theorem for set-valued quasi-contractions in b-metric spaces. *Fixed Point Theory Appl.*, 88.
- Ansari, A.H., Chandok, S., & Ionescu, C. 2014. Fixed point theorems on b-metric spaces for weak contractions with auxiliary functions. *J. Inequal. Appl.*, 429.

- Aghajani, A., Abbas, M., & Roshan, J.R. 2014. Common fixed point of generalized weak contractive mappings in partially ordered b-metric spaces. *Math. Slov.*, 4, pp. 941-960.
- Bhaskar, T.G., & Lakshmikantham, V. 2006. Fixed point theorems in partially ordered metric spaces and applications. *Nonlinear Anal.*, 65, pp. 1379-1393.
- Boriceanu, M. 2009. Strict fixed point theorems for multivalued operators in b-metric spaces. *Int. J. Mod. Math.*, 4(3), pp. 285-301.
- Boriceanu, M., Bota, M., & Petrusel, A. 2010. Multivalued fractals in b-metric spaces. *Cent. Eur. J. Math.*, 8(2), pp. 367-377.
- Bota, M., Molnar, A., & Varga, C. 2011. On Ekelandsvariational principle in b-metric spaces. *Fixed Point Theory*, 12(2), pp. 21-28.
- Bakhtin, I.A. 1989. The contraction mapping principle in quasimetric spaces. *Funct. Anal., Unianowsk Gos. Ped. Inst.*, 30, pp. 26-37.
- Czerwik, S. 1993. Contraction mappings in b-metric spaces. *Acta. Math. Inform. Univ. Ostraviensis*, 1, pp. 5-11.
- Czerwik, S. 1998. Nonlinear set-valued contraction mappings in b-metric spaces. *Atti. Sem. Mat. Fis. Univ. Modena*, 46, pp. 263-276.
- Guo, D., & Lakshmikantham, V. 1987. Coupled fixed points of nonlinear operators with applications. *Nonlinear Anal.*, 11, pp. 623-632.
- Hussain, N., Abbas, M., Azam, A., & Ahmad, J. 2014. Coupled coincidence point results for a generalized compatible pair with applications. *Fixed Point Theory and Applications*, 62.
- Hussain, N., & Shah, M.H. 2011. KKM mapping in cone b-metric spaces. *Comput. Math. Appl.*, 62, pp. 1677-1684.
- Huang, H., Paunović, Lj., & Radenović, S. 2015. On some new fixed point results for rational Geraghty contractive mappings in ordered b-metric spaces. *J. Nonlinear Sci. Appl.*, 8, pp. 800-807.
- Luong, N.V., & Thuan, N.X. 2011. Coupled fixed point in partially ordered metric spaces and applications. *Nonlinear Anal.*, 74, pp. 983-992.
- Lakshmikantham, V., & Ćirić, L.B. 2009. Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces. *Nonlinear Anal.*, 70, pp. 4341-4349.
- Mustafa, Z., Roshan, J.R., Parvaneh, V., & Kadelburg, Z. 2013. Some common fixed point results in ordered partial b-metric spaces. *J. Inequal. Appl.*, 562.
- Mustafa, Z., Roshan, J.R., Parvaneh, V., & Kadelburg, Z. 2014. Fixed point theorems for Weakly T-Chatterjea and weakly T-Kannan contractions in b-metric spaces. *J. Inequal. Appl.*, 46.
- Mustafa, Z., Jaradat, M.M.M., Ansari, A.H., Popović, B.Z., & Jaradat, H.M. 2016. C-class functions with new approach on coincidence point results for generalized [Formula: see text]-weakly contractions in ordered b-metric spaces. *Springerplus*, 5(1), p. 802. pmid:27390643
- Nieto, J.J., & Rodríguez-López, R. 2005. Contractive Mapping Theorems in Partially Ordered Sets and Applications to Ordinary Differential Equations. *Order*, 22(3), pp. 223-239. doi:10.1007/s11083-005-9018-5
- Nieto, J.J., & Rodríguez-López, R. 2007. Existence and Uniqueness of Fixed Point in Partially Ordered Sets and Applications to Ordinary Differential Equations. *Acta*

- Mathematica Sinica, English Series, 23(12), pp. 2205-2212. doi:10.1007/s10114-005-0769-0
- Olatinwo, M.O. 2008. Some results on multi-valued weakly jungck mappings in b-metric space. Central European Journal of Mathematics, 6(4), pp. 610-621. doi:10.2478/s11533-008-0047-3
- Pacurar, M. 2010. Sequences of almost contractions and fixed points in b-metric spaces. Analele Universitatii de Vest. Timisoara Seria Matematica Informatica, 48(3), pp. 125-137.
- Roshan, J.R., Parvaneh, V., Shobkolaei, N., Sedghi, S., & Shatanawi, W. 2013. Common fixed points of almost generalized  $(\psi, \phi)$ -contractive mappings in ordered b-metric spaces. Fixed Point Theory Appl., 159, p. 23.
- Wu, Y. 2008. New fixed point theorems and applications of mixed monotone operator. J. Math. Anal. Appl., 341, pp. 883-893.

# STARK WIDTHS OF Sc IV SPECTRAL LINES WITHIN 4s-4p TRANSITION ARRAY

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## ABSTRACT

**Stark full widths at half maximum of six 4s - 4p Sc IV spectral lines, broadened by collisions with electrons, have been calculated for electron density of  $10^{17} \text{ cm}^{-3}$  by using the modified semiempirical method. The results are provided for temperatures from 10 000 K to 160 000 K. They are used also to discuss the similarities of Sc IV spectral lines within multiplet, supermultiplet and transition array. Obtained results will be included in the STARK-B database which is also a part of Virtual atomic and molecular data center(VAMDC).**

**Keywords:** Stark widths, spectral lines, line profiles, Sc IV.

## INTRODUCTION

Broadening of spectral lines by fluctuating electric microfield produced by charged particles moving near an emitter/absorber, known as Stark broadening, is important for many topics in physics and astronomy. Data on Stark broadening are needed for laboratory plasmas diagnostic and investigation (Konjević, 1999; Torres et al., 2006; Peláez et al., 2009), laser produced plasma analysis, modelling and diagnostics (Richou & Molitor, 1970; Gornushkin et al., 1999; Sorge et al., 2000; Seidel et al., 2001) and inertial fusion plasma (Keane et al., 1990; Griem, 1992). Stark broadening data are also useful in technology, as for example for laser welding and piercing of various metals (Dimitrijević & Sahal-Bréchet, 2014), design, investigation and improvement of effectivity of various light sources based on plasma (Wieser et al., 1997; Seidel et al., 2001), and the design and developing of lasers (Csillag & Dimitrijević, 2004; Dimitrijević & Sahal-Bréchet, 2014). In many astrophysical plasmas Stark broadening of spectral lines is very important or at least non negligible and should be taken into account (Beauchamp et al., 1997; Popović et al., 2001; Dimitrijević, 2003; Dimitrijević & Sahal-Bréchet, 2014).

Stark broadening data are particularly important for stellar plasma research, and for analysis and synthesis of stellar spectra (Beauchamp et al., 1997; Dimitrijević & Sahal-Bréchet, 2014). In atmospheres of white dwarfs, pre white dwarf stars, and post AGB(Asymptotic Giant Branch) stars this is usually the main broadening mechanism (Tankosić et al., 2003; Milovanović et al., 2004; Simić et al., 2006; Dufour et al., 2011). It can not be neglected and in some atmospheric layers of A and late B stars (Simić et al., 2005b,a, 2009). As an example, (Popović et al., 2001) have shown that, for A-type star atmospheres, the inclusion of Stark broadening can change the equivalent widths by 10-45%.

Consequently, for determination of abundances, the importance of Stark broadening of analysed spectral lines should be checked.

For some astrophysical problems, like modelling of stellar atmospheres, radiative transfer or derivation of accurate atmospheric parameters, we need a very large number of various atomic data, including Stark broadening (Dufour et al., 2011). We note as well that (Rauch et al., 2007) emphasized the crucial importance of an accurate and as much as possible complete Stark broadening data set for large number of atoms and ions, “for sophisticated analysis of stellar spectra by means of NLTE model atmospheres”.

The development of satellite-born astronomy, enabling to obtain stellar spectra with earlier not possible resolution, as well as development of computers enabling very sophisticated NLTE synthesis and modelling of stellar spectra, made that earlier astrophysically insignificant data on trace elements now are important. Scandium is present and observed in stellar spectra (Adelman & Pintado, 2000; Adelman et al., 2001; Kahraman Alicavus et al., 2017) so that its Stark broadening parameters are needed. However, there is no at all experimental data for Stark broadening of scandium lines. For Sc IV there is no neither theoretical data. Results of Stark broadening parameter calculations exist only for Sc II (Popović & Dimitrijević, 1996, 1997), Sc III (Dimitrijević & Sahal-Bréchet, 1992), Sc X (Dimitrijević & Sahal-Bréchet, 1998a,b; Elabidi & Sahal-Bréchet, 2011) and Sc XI (Dimitrijević & Sahal-Bréchet, 1998a,b).

In order to provide the Stark broadening data, needed first of all for stellar astrophysics, for Sc IV spectral lines, which are absent completely in the literature, Stark Full Widths at Half intensity Maximum(FWHM)  $W$  for six transitions have been calculated here, by using the modified semiempirical method (MSE, Dimitrijević & Konjević (1980); Dimitrijević & Kršljanin (1986); Dimitrijević & Popović (2001)) for collisions of Sc IV ions with electrons. The obtained results will be used for the discussion of simi-

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larities of Stark widths within a supermultiplet. Also, the obtained results will be implemented in the STARK-B database (Sahal-Br  chot et al., 2015, 2017), a repository for spectral line broadening and shifts due to collisions with charged particles, and also a node of Virtual Atomic and Molecular Data Center - VAMDC (?Rixon et al., 2011; Dubernet et al., 2016).

## THE MODIFIED SEMIEMPIRICAL METHOD

Within the frame of the modified semiempirical(MSE) approach (Dimitrijevi   & Konjevi  , 1980; Dimitrijevi   & Kr  ljanin, 1986; Popovi   et al., 2001) the electron impact full width at half intensity maximum(FHWM) of an isolated non hydrogenic ion line is given as

I

$$W_{MSE} = N \frac{8\pi}{3} \hbar^{22} \left(\frac{2m}{\hbar}\right)^{1/2} \frac{\pi}{\sqrt{3}} \frac{\lambda^2}{2\pi c} \times$$

$$\times \left\{ \sum_{\ell_i \pm 1} \sum_{L_i' J_i'} \mathfrak{R}^2[n_i \ell_i L_i J_i, n_i(\ell_i \pm 1) L_i' J_i'] \bar{g}(x_{\ell_i, \ell_i \pm 1}) + \right.$$

$$+ \sum_{\ell_f \pm 1} \sum_{L_f' J_f'} \mathfrak{R}^2[n_f \ell_f L_f J_f, n_f(\ell_f \pm 1) L_f' J_f'] \bar{g}(x_{\ell_f, \ell_f \pm 1}) +$$

$$\left. + \left( \sum_{i'} \mathfrak{R}_{ii'}^2 \right)_{\Delta n \neq 0} g(x_{n_i, n_i+1}) + \left( \sum_{f'} \mathfrak{R}_{ff'}^2 \right)_{\Delta n \neq 0} g(x_{n_f, n_f+1}) \right\}, \quad (1)$$

where the initial level is denoted as  $i$  and the final one as  $f$ . The square of the matrix element  $\{\mathfrak{R}^2[n_k \ell_k L_k J_k, (\ell_k \pm 1) L_{k'} J_{k'}], \quad k = i, f\}$  is given by the expression

$$\mathfrak{R}^2[n_k \ell_k L_k J_k, n_k(\ell_k \pm 1) L_{k'} J_{k'}] =$$

$$= \frac{\ell_{>}}{2J_k + 1} Q[\ell_k L_k, (\ell_k \pm 1) L_{k'}] Q(J_k, J_{k'}) [R_{n_k \ell_k}^{n_k^* (\ell_k \pm 1)}]^2. \quad (2).$$

Here,  $\ell_{>} = \max(\ell_k, \ell_k \pm 1)$  and

$$\left( \sum_{k'} \mathfrak{R}_{kk'}^2 \right)_{\Delta n \neq 0} = \left( \frac{3n_k^*}{2Z} \right)^2 \frac{1}{9} (n_k^{*2} + 3\ell_k^2 + 3\ell_k + 11) \quad (3)$$

In Eq.(1)

$$x_{\ell_k, \ell_{k'}} = \frac{E}{\Delta E_{\ell_k, \ell_{k'}}}, \quad k = i, f$$

$E = \frac{3}{2} kT$  is the electron kinetic energy and  $\Delta E_{\ell_k, \ell_{k'}} = |E_{\ell_k} - E_{\ell_{k'}}|$  is the energy difference between levels  $\ell_k$  and  $\ell_{k \pm 1} (k = i, f)$ ,

$$x_{n_k, n_{k+1}} \approx \frac{E}{\Delta E_{n_k, n_{k+1}}},$$

where for  $\Delta n \neq 0$ , the energy difference between energy levels with  $n_k$  and  $n_{k+1}$ ,  $\Delta E_{n_k, n_{k+1}}$  is approximated as

$$\Delta E_{n_k, n_{k+1}} = 2Z^2 E_H / n_k^{*3}, \quad (4)$$

$n_k^* = [E_H Z^2 / (E_{ion} - E_k)]^{1/2}$  is the effective principal quantum number,  $Z$  is the residual ionic charge(charge “seen” by optical electron; for example  $Z=1$  for neutrals) and  $E_{ion}$  is the appropriate spectral series limit.  $N$  and  $T$  are electron density and temperature, respectively and  $Q(\ell L, \ell' L')$ ,  $Q(J, J')$  multiplet and line factors. With  $g(x)$  (Griem, 1968, 1974) and  $\bar{g}(x)$  (Dimitrijevi   & Konjevi  , 1980) are denoted the corresponding Gaunt factors. The needed radial integrals  $[R_{n_k \ell_k}^{n_k^* \ell_k \pm 1}]$  have been calculated here within the Coulomb approximation by using the method of (Bates & Damgaard, 1949) and the tables of (Oertel & Shomo, 1968). We note that if for the higher principal quantum numbers there is no the corresponding data in (Oertel & Shomo, 1968), the radial integrals may be calculated with the help of the article of (Van Regemorter et al., 1979).

## RESULTS AND DISCUSSION

Stark widths(FHWM) of six Sc IV spectral lines, broadened by collisions with electrons, have been calculated using Eqs.(1-4) within the frame of MSE method (Dimitrijevi   & Konjevi  , 1980; Popovi   et al., 2001). Energy levels and ionization energy needed for their calculations have been taken from (Sugar & Corliss, 2004).

In Table 1, the results of our MSE calculations of Stark widths(FHWM) for six spectral lines broadened by electron-impacts, for a perturber density of  $10^{17} \text{ cm}^{-3}$  and for a set of temperatures from 10 000 K to 160 000 K, are shown. The chosen temperature range is of interest in astrophysics, laboratory plasma, fusion research, technology and for lasers and laser produced plasma. If one needs perturber densities lower than  $10^{17} \text{ cm}^{-3}$  the extrapolation is linear. For higher perturber densities extrapolation is linear if the influence of Debye screening is negligible or reasonably small. In Table 1 are also given the observed wavelength and the multiplet number from the NIST database (Kramida et al., 2017). It is shown as well the quantity  $3kT/2\Delta E$ , representing the ratio of the average energy of free electrons,  $E = 3kT/2$ , and the energy difference of initial or final and the closest perturbing level,  $\Delta E$ . It is calculated for  $T=20\,000 \text{ K}$  in the following way:

$$\Delta E = \text{Max}[E/\Delta E_{i,i'}, E/\Delta E_{f,f'}, \Delta E_{n_i, n_i+1}, \Delta E_{n_f, n_f+1}] \quad (5)$$

This ratio shows which collisions are dominant.  $3kT/2\Delta E = 1$  is the threshold for the corresponding inelastic transition. If it is lower than one, elastic collisions are dominant and it is so called low temperature limit. If it is large, say larger than 50, high temperature limit approximation can be applied.

The calculated triplets belong to the same  $4s^3 P^o - 4p^3 S$  multiplet, singlets to the same  $4s^1 P^o - 4p^1 L (L=S, P, D)$  supermultiplet and all calculated transitions to the same,  $4s - 4p$  transition array. (Wiese & Konjevi  , 1982) concluded after examination of regularities and similarities in plasma broadened spectral line widths that line widths in angular frequency units in multiplets usually agree within a few per cent, in supermultiple within about 30 per

**Table 1.** This table gives electron-impact broadening(Stark broadening) Full Widths at Half Intensity Maximum(W) for Sc IV spectral lines, for a perturber density of  $10^{17} \text{ cm}^{-3}$  and temperatures from 10 000 to 160 000 K. Also are given multiplet numbers and wavelengths from NIST database (Kramida et al., 2017) and  $3kT/2\Delta E$  for  $T = 20\,000 \text{ K}$ , where  $\Delta E$  is the energy difference between closest perturbing level and the closer of initial and final levels.

Transition	T(K)	W[Å]	W[ $10^{12} \text{ s}^{-1}$ ]
Sc IV $3s^23p^54s^1P^o-3s^23p^54p^1P$	10000.	0.0735	0.294
(Mult. 100)	20000.	0.0519	0.208
2171.2 Å	40000.	0.0367	0.147
$3kT/2\Delta E = 0.541$	80000.	0.0260	0.104
	160000.	0.0194	0.0776
Sc IV $3s^23p^54s^1P^o-3s^23p^54p^1D$	10000.	0.0802	0.337
(Mult. 101)	20000.	0.0621	0.260
2119.0 Å	40000.	0.0479	0.201
$3kT/2\Delta E = 0.526$	80000.	0.0399	0.167
	160000.	0.0344	0.144
Sc IV $3s^23p^54s^1P^o-3s^23p^54p^1S$	10000.	0.0472	0.321
(Mult. 102)	20000.	0.0334	0.227
1665.9 Å	40000.	0.0240	0.163
$3kT/2\Delta E = 0.453$	80000.	0.0180	0.122
	160000.	0.0146	0.0989
Sc IV $3s^23p^54s^3P^o_2-3s^23p^54p^3S_1$	10000.	0.0964	0.271
(Mult. 85)	20000.	0.0682	0.192
2586.9 Å	40000.	0.0482	0.136
$3kT/2\Delta E = 0.606$	80000.	0.0344	0.0969
	160000.	0.0272	0.0765
Sc IV $3s^23p^54s^3P^o_1-3s^23p^54p^3S_1$	10000.	0.104	0.274
(Mult. 85)	20000.	0.0737	0.194
2678.0 Å	40000.	0.0521	0.137
$3kT/2\Delta E = 0.606$	80000.	0.0373	0.0980
	160000.	0.0295	0.0774
Sc IV $3s^23p^54s^3P^o_0-3s^23p^54p^3S_1$	10000.	0.125	0.279
(Mult. 85)	20000.	0.0886	0.197
2906.5 Å	40000.	0.0626	0.140
$3kT/2\Delta E = 0.606$	80000.	0.0451	0.101
	160000.	0.0357	0.0797

cent and within a transition array within about 40 per cent. The obtained here results give us an opportunity to check the similarities of Stark line widths within multiplet, supermultiplet and transition array for Sc IV in order to see if they could be used for derivation of missing values. The transformation of the Stark widths expressed in Å-units to the widths in angular frequency units may be performed by the following formula:

$$W(\text{Å}) = \frac{\lambda^2}{2\pi c} W(s^{-1}), \quad (6)$$

where  $c$  is the speed of light.

From Table 1 we obtained that within the Sc IV  $4s^3P^o-4p^3S$  multiplet the smallest Stark width value is 2.87% smaller from largest at  $T=10\,000 \text{ K}$  and 4.02% at  $T=160\,000 \text{ K}$ . Within Sc IV  $4s^1P^o-4p^1L$  ( $L=S, P, D$ ) supermultiplet, the smallest  $W$  is 12.76% smaller from the largest one at  $T=10\,000 \text{ K}$  and 46.11% at

$T=160\,000 \text{ K}$ . For  $4s-4p$  transition array these values are 19.58% and 46.88%. We can conclude that in average, these values are in agreement with (Wiese & Konjević, 1982) conclusions and that disagreement increases with the increase of temperature.

If we have Stark width value for one member of multiplet, supermultiplet or transition array, the needed width for another member, can be obtained with the help of the expression:

$$W_1 = \left(\frac{\lambda_1}{\lambda}\right)^2 W. \quad (7)$$

Here, with  $W_1$  is denoted the corrected width, for the spectral line with the wavelength  $\lambda_1$ , and  $\lambda$  is the wavelength of the line for which we have the Stark width value  $W$ .

The Stark FWHM for Sc IV spectral lines obtained within the modified semiempirical method and shown in Table 1, will be also implemented in the STARK-B database (Sahal-Bréchet et al.,

2015, 2017), created first of all for the investigations, modelling and diagnostics of the plasma of stellar atmospheres, but useful also for the diagnostics of laboratory plasmas, investigation of laser produced and inertial fusion plasma as well as for plasma technologies.

We want to draw attention that STARK-B database is one of the databases which enter also in the Virtual Atomic and Molecular Data Center - VAMDC (Rixon et al., 2011; Dubernet et al., 2016), created in order to enable an efficacious search and mining of atomic and molecular data. scattered in different databases and to make more convenient their adequate use. VAMDC portal with 30 databases with atomic and molecular data, including STARK-B, is on the web site: <http://portal.vamdc.org/>.

## CONCLUSION

Within the frame of MSE method we have calculated Stark widths for six Sc IV spectral lines broadened by collisions with electrons. The obtained data are used to check similarities of Stark widths within a multiplet, supermultiplet and transition array. Also the Stark widths for six Sc IV spectral lines will be implemented in STARK-B database. Other theoretical as well as experimental data for Stark broadening of Sc IV spectral lines do not exist, so that we hope that the obtained results will be of interest, for various problems, especially in stellar physics and laboratory plasma diagnostics.

## ACKNOWLEDGEMENT

This work is a part of the project 176002 "Influence of collisional processes on astrophysical plasma line shapes" supported by the Ministry of Education, Science and Technological Development of Serbia.

## REFERENCES

Adelman, S.J., & Pintaldo, O. 2000. Elemental abundance analyses with Complejo Astronomico El Leoncito REOSC echelle spectrograms. IV. Extensions of nine previous analyses. *Astronomy and Astrophysics*, 354, pp. 899-903.

Adelman, S.J., Snow, T.P., Wood, E.L., Ivans, I.I., Sneden, C., Ehrenfreund, P., & Foing, B.H. 2001. An elemental abundance analysis of the mercury manganese star HD 29647. *Monthly Notices of the Royal Astronomical Society*, 328(4), pp. 1144-1150.

Bates, D.R., & Damgaard, A. 1949. The Calculation of the Absolute Strengths of Spectral Lines. *Philosophical Transactions of the Royal Society of London. Series A. Mathematical and Physical Sciences*, 242(842), pp. 101-122.

Beauchamp, A., Wesemael, F., & Bergeron, P. 1997. Spectroscopic Studies of DB White Dwarfs: Improved Stark Profiles for Optical Transitions of Neutral Helium. *Astrophysical Journal Supplement*, 108, pp. 559-573.

Csillag, L., & Dimitrijević, M.S. 2004. On the Stark broadening of the 537.8 nm and 441.6 nm Cd<sup>+</sup> lines excited in a hollow cathode laser discharge. *Applied Physics B: Lasers and Optics*, 78(2), pp.221-223. doi:10.1007/s00340-003-1368-3

Dimitrijević, M.S. 2003. Stark broadening in Astrophysics: Applications of Belgrade School Results and collaboration of former Soviet republics. *Astronomical and Astrophysical Transactions*, 22, pp. 389-412.

Dimitrijević, M.S., & Konjević, N. 1980. Stark widths of doubly- and triply-ionized atom lines. *Journal of Quantitative Spectroscopy & Radiative Transfer*, 24, pp. 451-459.

Dimitrijević, M.S., & Kršljanin, V. 1986. Electron-impact shifts of ion lines: Modified semiempirical approach. *Astronomy and Astrophysics*, 165, pp. 269-274.

Dimitrijević, M.S., & Popović, L. Č. 2001. Modified Semiempirical Method. *Journal of Applied Spectroscopy*, 68(6), pp. 893-901. doi:10.1023/A:1014396826047

Dimitrijević, M.S., & Sahal-Bréchet, S. 1992. Stark broadening of spectral lines of multicharged ions of astrophysical interest. V. Sc III and Ti IV lines. *Astronomy and Astrophysics Supplement Series*, 95, pp. 121-128.

Dimitrijević, M.S., & Sahal-Bréchet, S. 1998a. Stark broadening of spectral lines of multicharged ions of astrophysical interest. XXI. Sc X, Sc XI, Ti XI and Ti XII spectral lines. *Astronomy and Astrophysics Supplement Series*, 131, pp. 143-144.

Dimitrijević, M.S., & Sahal-Bréchet, S. 1998b. Stark broadening parameter tables for Sc X, Sc XI, Ti XI and Ti XII. *Serbian Astronomical Journal*, 157, pp. 39-64.

Dimitrijević, M.S., & Sahal-Bréchet, S. 2014. On the Application of Stark Broadening Data Determined with a Semiclassical Perturbation Approach, *Atoms*, 2, pp. 357-377.

Dubernet, M.L., Antony, B.K., Ba, Y.A., & et al., 2016. The virtual atomic and molecular data centre(VAMDC) consortium. *Journal of Physics B: Atomic, Molecular and Optical Physics*, 49(7), p.074003.

Dufour, P., Ben Nessib, N., Sahal-Bréchet, S., Dimitrijević, M.S. 2011. Stark Broadening of Carbon and Oxygen Lines in Hot DQ White Dwarf Stars: Recent Results and Applications. *Baltic Astronomy*, 20, pp. 511-515.

Elabidi, H., & Sahal-Bréchet, S. 2011. Checking the dependence on the upper level ionization potential of electron impact widths using quantum calculations. *European Physical Journal D*, 61, pp. 285-290.

Gornushkin, I.B., King, L.A., Smith, B.W., Omenetto, N., & Winefordner, J.D. 1999. Line broadening mechanisms in the low pressure laser-induced plasma. *Spectrochimica Acta, Part B: Atomic Spectroscopy*, 54(8), pp. 1207-1217.

Griem, H.R. 1968. Semiempirical Formulas for the Electron-Impact Widths and Shifts of Isolated Ion Lines in Plasmas. *Physical Review*, 165(1), pp. 258-266.

Griem, H.R. 1974. Spectral line broadening by plasmas. New York: Academic Press, Inc.

- Griem, H.R. 1992. Plasma spectroscopy in inertial confinement fusion and soft x-ray laser research. *Physics of Fluids*, 4(7), pp. 2346-2361.
- Kahraman Alicavus, F., Niemczura, E., Polińska, M., Helminiak, K.G., Lampens, P., Molenda-Żakowicz, J.,... Kambe, E. 2017. High-resolution spectroscopy and abundance analysis of  $\delta$  Scuti stars near the  $\gamma$  Doradus instability strip. *Monthly Notices of the Royal Astronomical Society*, 470(4), pp. 4408-4420.
- Keane, C.J., Lee, R.W., Hammel, B.A., Osterheld, A.L., Suter, L.J., Calisti, A.,... Talin, B. 1990. Line broadening of Ne-like xenon as a diagnostic for high-density implosion experiments. *Review of Scientific Instruments*, 61(10), pp. 2780-2782.
- Konjević, N. 1999. Plasma broadening and shifting of non-hydrogenic spectral lines: present status and applications. *Physics Reports*, 316(6), pp. 339-401.
- Kramida, A., Ralchenko, Yu., Reader, J., & -NIST ASD Team. 2017. NIST Atomic Spectra Database. Gaithersburg, MD: National Institute of Standards and Technology.(ver. 5.5.1), Retrieved from <https://physics.nist.gov/asd> 2017, Nov 18.
- Milovanović, N., Dimitrijević, M.S., Popović, L.Č., & Simić, Z. 2004. Importance of collisions with charged particles for stellar UV line shapes: Cd III. *Astronomy and Astrophysics*, 417, pp. 375-380.
- Oertel, G.K., & Shomo, L.P. 1968. Tables for the Calculation of Radial Multipole Matrix Elements by the Coulomb Approximation. *Astrophysical Journal Supplement*, 16, pp. 175-218.
- Peláez, R.J., Ćirišan, M., Djurović, S., Aparicio, J.A., Mar, S. 2009. Stark broadening measurements of low-intensity singly and doubly ionized xenon spectral lines. *Astronomy and Astrophysics*, 507(3), pp. 1697-1705.
- Popović, L.Č., & Dimitrijević, M.S. 1996. Stark widths for astrophysically important ns-np transitions in Sc II, Y II and Zr II spectra. *Astronomy and Astrophysics*, 120, pp. 373-374. Supplement Series
- Popović, L.Č., & Dimitrijević, M.S. 1997. Stark broadening parameter tables for Xe II, Sc II, Y II and Zr II. *Bulletin Astronomique de Belgrade*, 155, pp. 159-163.
- Popović, L.Č., Simić, S., Milovanović, N., Dimitrijević, M.S. 2001. Stark Broadening Effect in Stellar Atmospheres: Nd II Lines. *Astrophysical Journal*, 135(1), pp. 109-114. Supplement Series. doi:10.1086/321778
- Rauch, T., Ziegler, M., Werner, K., & et al., 2007. High-resolution FUSE and HST ultraviolet spectroscopy of the white dwarf central star of Sh 2-216. *Astronomy and Astrophysics*, 470(1), pp. 317-329.
- Richou J., & Molitor A. 1970. Determination of the electron density of a xenon plasma, created in a shock tube, by laser interferometry and spectroscopy. *Comptes Rendus de L'Académie de Science, Serie B*, 271, pp. 753-756.
- Rixon, G., Dubernet, M.L., Piskunov, N., & et al., 2011. VAMDC-The Virtual Atomic and Molecular Data Centre: A New Way to Disseminate Atomic and Molecular Data-VAMDC Level 1 Release. . In: AIP Conference Proceedings., pp. 107-115 1344.
- Sahal-Bréchet, S., Dimitrijević, M.S., & Moreau, N. 2017. STARK-B database. Observatory of Paris / LERMA and Astronomical Observatory of Belgrade. Retrieved from <http://stark-b.obspm.fr> 2017 November 12.
- Sahal-Bréchet, S., Dimitrijević, M.S., Moreau, N., & Ben Nessib, N. 2015. The STARK-B database VAMDC node: a repository for spectral line broadening and shifts due to collisions with charged particles. *Physica Scripta*, 50, 054008.
- Seidel, S., Wrubel, Th., Roston, G., & Kunze, H.-J. 2001. Line profile measurements of  $(^4\text{S})6s^5\text{S}-(^4\text{S})6p^5\text{P}$  transitions of Xe III. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 71(2-6), pp. 703-709.
- Simić, Z., Dimitrijević, M.S., & Kovačević, A. 2009. Stark broadening of spectral lines in chemically peculiar stars: Te I lines and recent calculations for trace elements. *New Astronomy Review*, 53(7-10), pp. 246-251.
- Simić, Z., Dimitrijević, M.S., Milovanović, N., & Sahal-Bréchet, S. 2005a. Stark broadening of Cd I spectral lines. *Astronomy and Astrophysics*, 441(1), pp. 391-393.
- Simić, Z., Dimitrijević, M.S., Popović, L.Č., & Dačić, M. 2005b. Stark Broadening of F III Lines in Laboratory and Stellar Plasma. *Journal of Applied Spectroscopy*, 72(3), pp. 443-446. doi:10.1007/s10812-005-0095-4
- Simić, Z., Dimitrijević, M.S., Popović, L.Č., & Dačić, M. 2006. Stark broadening parameters for Cu III, Zn III and Se III lines in laboratory and stellar plasma. *New Astronomy*, 12(3), pp. 187-191.
- Sorge, S., Wierling, A., Röpke, G., Theobald, W., Suerbrey, R., & Wilhelm, T. 2000. Diagnostics of a laser-induced dense plasma by hydrogen-like carbon spectra. *Journal of Physics B: Atomic, Molecular and Optical Physics*, 33(16), pp. 2983-3000.
- Sugar, J., & Corliss, C. 2004. Atomic Energy Levels of the Iron-Period Elements: Potassium through Nickel. *Journal of Physical and Chemical Reference Data*, 14, pp. 1-664, Supplement 2.
- Tankosić, D., Popović, L.Č., & Dimitrijević, M.S. 2003. The electron-impact broadening parameters for Co III spectral lines. *Astronomy and Astrophysics*, 399, pp. 795-797.
- Torres, J., van de Sande, M.J., van der Mullen, J.J.A.M., Gamero, A., & Sola, A. 2006. Stark broadening for simultaneous diagnostics of the electron density and temperature in atmospheric microwave discharges. *Spectrochimica Acta, Part B: Atomic Spectroscopy*, 61(1), pp. 58-68.
- van Regemorter, H., Hoang Binh, D., & Prud'homme, M. 1979. Radial transition integrals involving low or high effective quantum numbers in the Coulomb approximation. *Journal of Physics B: Atomic, Molecular and Optical Physics*, 12, pp. 1053-1061.
- Wiese, W.L., & Konjević, N. 1982. Regularities and similarities in plasma broadened spectral line widths (Stark widths). *Journal of Quantitative Spectroscopy and Radiative Transfer*, 28, pp. 185-198.
- Wieser, J., Murnick, D.E., Ulrich, A., Huggins, H.A., Liddle, A., Brown, W.L. 1997. Vacuum ultraviolet rare gas excimer light source. *Review of Scientific Instruments*, 68(3), pp. 1360-1364.

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**The UNIVERSITY thought.** Publication in natural sciences / editor in chief Nebojša Živić. - Vol. 3, no. 1 (1996)- . - Kosovska Mitrovica : University of Priština, 1996- (Kosovska Mitrovica : Art studio KM). - 29 cm

Polugodišnje. - Prekid u izlaženju od 1999-2015. god. - Je наставак: Универзитетска мисао. Природне науке = ISSN 0354-3951  
ISSN 1450-7226 = The University thought. Publication in natural sciences  
COBISS.SR-ID 138095623

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